

*Dedicated to Professor Dr. Sorin Dan Anghel on His 65<sup>th</sup> Anniversary*

## SIMPLE METHOD FOR INVESTIGATION OF LOW FREQUENCY DAMPED OSCILLATIONS OF ELASTOMERS

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**ABSTRACT.** A simple method for digital data recording of mechanical low frequency oscillations has been proposed. The methods was applied to measurements the elastic constant, attenuation and viscosity coefficients of elastomers. Such parameters can be difficult measure by other methods for polymers with low flow, like the elastomers.

**Keywords:** *digital data recording, low frequency oscillations, elastomers*

### INTRODUCTION

Studding the mechanical oscillation in not a novelty, being performed with simplest methods since the eve of modern physics, but the methods of performing such study are subject of continuously change, in function of the development of new equipments for observation and data acquisition. The classic methods suppose the use of very simplest instruments, watch, for time measuring, ruler, for distance measurement, and scale, for mass measurements. Such method allows better observation and understanding of physical phenomena and measuring principles, but the precision of the measurement is low. Other disadvantage is the fact that the data must be collected and introduced manually in the computer for digital analyze. The modern methods are based on the use of complex equipments, containing a lot of different kind of sensors, which can be regarded as “black box”, with one entry and one output. The user starts the experiment, the equipment collects and processes the data and provides directly on its output the parameters of the

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physical phenomena. The user cannot see how the data were collected and processed. It is a great disadvantage. On the other hand such equipments are expensive and not accessible for all the scholar communities, especially for poor regions.

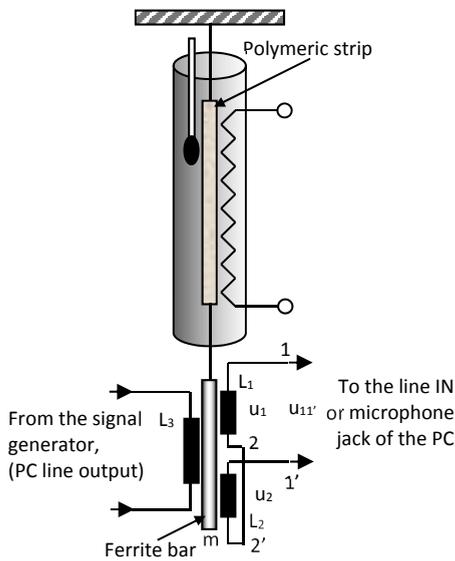
The aims of this work in to present a method of investigation of low frequency oscillations which combines the advantages of classic observation of the phenomena with the new method of digital data acquisition. The great advantage is that the equipment can be "home made", the most expensive component being an ordinary computer. In addition the user can themselves set the parameters of the experiment and data acquisition, fact that allows better understanding of the process. Finally the method can be used for effective investigation of very low frequency oscillation of polymeric systems, allowing the measurement of elastic modulus and viscosity coefficient.

The elastomers, below the glass transition temperature, have predominant elastic behavior and look like solid elastics, without flow, [1-4]. However they have also viscoelastic properties, but the viscosity parameters cannot be measured in solid state with standard viscosimetric methods. For viscosity measurements the material must be melts or dissolved in good solvent [5]. The parameters obtained by this way are not consistent for the characterization of the material in the solid state phase. A method for measuring these parameters in solid state is necessarily. That is a new feature of our experiment. The viscoelastic parameters can be obtained from the parameters of damped oscillations.

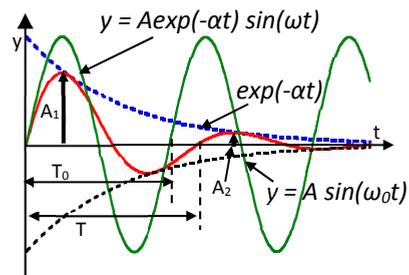
## EXPERIMENTAL

The main piece of the experimental set-up is the unbalanced transformers, home made. It has symmetrically structure consisting on three coils, wired linearly along a cylindrical support, 8 cm length and 1.1 cm inner diameter (Fig. 1). In the middle of the structure is the inductor coil  $L$ , 1 cm length, which contains 200 turns of cooper wire with  $\Phi = 0.35$  mm, (CuEm 0.35), wired on multi layers. Symmetrically, on the right and the left sides, there are the two coils,  $L_1$  and  $L_2$  of the induced part of the transformer. Each coil is 2.5 cm long and contains 200 turns, CuEm 0.35 mm, wired on multilayer. These coils are wired in the same direction, but theirs ends are connected in opposite position, so that the induced tensions will be in opposite phase. The core of the transformer consists on the ferrite bar with 8 mm diameter and 11 cm length. This bar represents the mass of the oscillating system. It is attached to the rubber strip of 30 cm length. The strip and the ferrite bar together form the

oscillating system. The polymeric strip is introduced inside a glass tube, with inner diameter 3 cm, containing a thermometer and a heating resistor. This represents a simple heating system, allowing the study of oscillations at many temperatures. The ferrite bar oscillates freely inside the transformer. All the components are placed in vertical position, avoiding the apparition of the friction between the ferrite bar and the inner part of the transformer. At equilibrium the ferrite bar is situated in the middle of the transformer. The inductor coil of the transformer is connected to the headphone jack or line output of sound card of the PC. It will be supplied by the audio frequency signal provided by the generator signal of the Scope software. The induced coils are connected to the microphone entry of the PC and will provide the signal which will be visualized and recorded by the Scope software.



**Fig. 1.** Experimental set-up of the oscillating system



**Fig. 2.** Graphic representation of the ideal and damped oscillations for a system with elastic constant  $k$ , mass  $m$  and attenuation coefficient  $\alpha$ .

The second element of the set-up is the software Scope 1.4.1., offered freely on the internet by Zeinitz [6]. This software allows the use of any PC with sound card as oscilloscope with 2 channels in the range 20 – 20000 Hz. It provides also a signal generator in the same range of frequencies. The entry of scope is set to be connected to the microphone or line IN of the PC, and will receive the signal induced into the coils  $L_1$  and  $L_2$ . The output of the signal generator is set to be

connected to the line output or headphone output of the sound card of PC. The connections and all the parameters of the experiment, the frequency, the base time, the gain and the amplitude of the generated signal, can be set from the software. We set the frequency at 824 Hz and the amplitude of generated signal 0.5 V. When the ferrite bar is situated in the middle of the transformer, the tensions  $u_1$  and  $u_2$  induced in the coils  $L_1$  and  $L_2$  of the transformer are identically, but in opposite phase, so that the tension available on the ends 1 and 1' of the transformer is zero. If the ferrite bar is displaced from its equilibrium position, the tensions induced in the coils  $L_1$  and  $L_2$  are not equals, and the resulting tensions on the ends 1-1',  $u_{11'} = u_1 - u_2 \neq 0$  will be different from zero. The difference is proportional with the displacement of the ferrite bar from its equilibrium position. If the ferrite is oscillating, the tension  $u_{11'}$  will have the amplitude depending on the elongation of the oscillation. We obtain an oscillation with frequency  $\nu_0 = 824\text{Hz}$  modulated in amplitude by the mechanical oscillation.

## RESULTS AND DISCUSSIONS

Low frequency damped oscillations can appear in polymeric systems, especially in the elastomers, above the glass transition temperature. The attenuation of oscillations is very high, so that the duration of the oscillating motions involves only few oscillations. Depending on the elastic modulus of the polymer, and the mass of the oscillating system, the frequency of the oscillations can be of few hertz. Such oscillations cannot be seen directly with the oscilloscope as continuously trace curve due to the refresh rate of the screen, which is much greater than the frequency of the oscillation. Only a fraction of oscillation

$\frac{T_{osc}}{T_{refresh}} = f$  will be displayed on the screen of the oscilloscope. The oscilloscope

can easily displays signals with higher frequencies, [9]. The idea is to modulate a relatively high frequency signal, the carrier frequency, with the low frequency of mechanical oscillation. The oscilloscope is set to display stable image of the carrier signal. The stability of image is not affected by the modulation. As presented in the experimental section this purpose is achieved with an unbalanced transformer. The modulation is realized by the mechanical oscillation.

An elastomer is a viscoelastic system which can be represented by an ideal spring connected in parallel with a shock absorber. The oscillating behavior of the spring is described by Hook's law,  $\vec{F} = -k \cdot \vec{y}$ , where the  $k$  represents the elastic

constant of the polymer and  $y$  represents the elongation of the spring. The shock absorber is described by Newton's law,  $F = \eta \cdot \frac{dy}{dt}$ , where  $\eta$  represents the viscosity of the material and  $\frac{dy}{dt}$  represents the rate of deformation [7]. The motion of such oscillating system is described by the equation:

$$m \frac{d^2 y}{dt^2} = -ky - \eta \frac{dy}{dt} \quad (1)$$

The solution of this equation is a damped oscillation:

$$y = A \cdot \exp(-\alpha t) \cdot \sin(\omega t) \quad (2)$$

$A$  represents the amplitude,  $\alpha$  the attenuation coefficient and  $\omega$  the pulsation of the damped oscillation. The relation existing between the parameters  $\alpha$ ,  $\omega$ ,  $m$  and  $k$  are described by the equations:

$$\alpha = \frac{\eta}{2m}, \quad \omega^2 = \frac{k}{m} - \frac{\eta^2}{4m^2} \quad (3)$$

The period of the damped oscillation  $T = \frac{2\pi}{\omega}$  is greater than the period of the ideal oscillations  $T_0 = \frac{2\pi}{\omega_0}$ , where  $\omega_0^2 = \frac{k}{m}$  and  $T_0 = \sqrt{\frac{m}{k}}$ , [8]. The graphic representation of ideal and damped oscillation is presented in figure 2. Experimentally we can measure the period and the amplitude of the damped oscillations. The amplitude is affected only by the attenuation coefficient  $\alpha$  following an exponential law. Measuring the amplitude of damped oscillations at the moments  $T_i$ , we can calculate the attenuation coefficient  $\alpha$  from the representation:

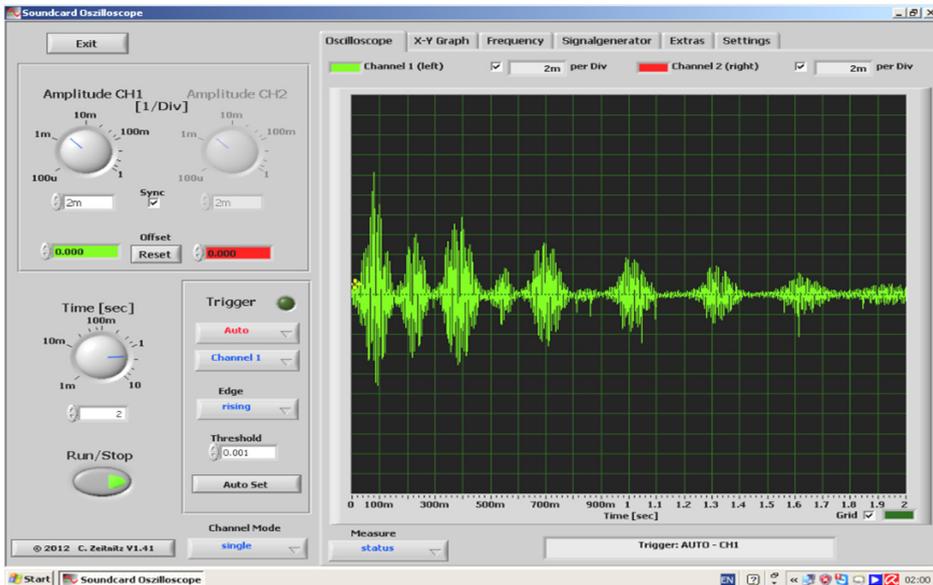
$$A(T_i) = A_0 \exp(-\alpha T_i) \quad (4)$$

Another way is the use of logarithmic decrement of the oscillations,  $A = \frac{A_2}{A_1}$ . To do this we must measure the amplitudes  $A_1$ ,  $A_2$  of two successive

oscillations separated by a time interval equal with a period.  $\Lambda$  represents the rate of loss of mechanical energy  $W$  during a period. Taking into account the attenuation of the amplitude, we can find a direct relation between  $\alpha$  and  $\Lambda$ .

$$\frac{W_2}{W_1} = \exp(-2\alpha T) = \Lambda^2 \quad \alpha = -\frac{\ln \Lambda}{T} \quad (5)$$

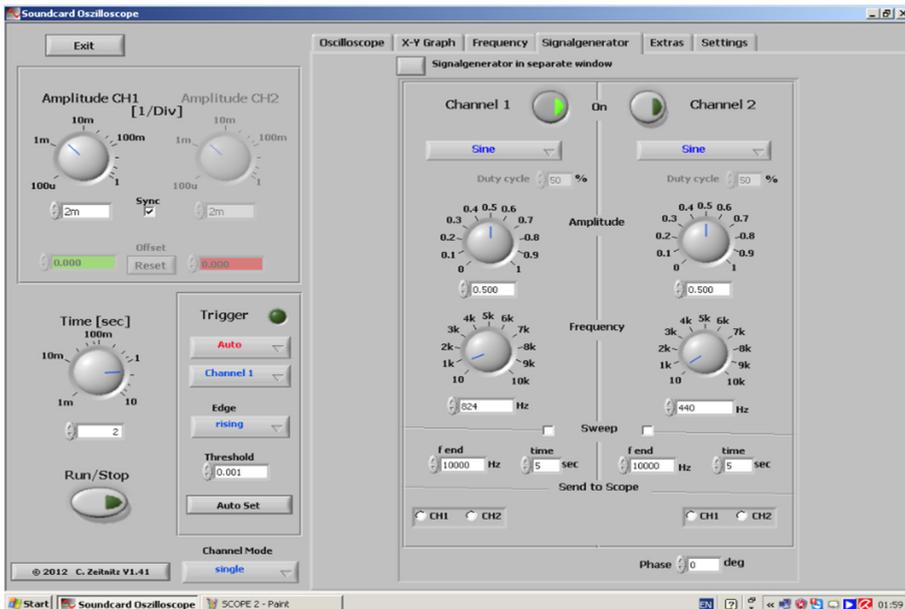
Knowing  $\alpha$ ,  $\eta$ ,  $m$ ,  $T$  we can calculate the viscosity coefficient  $\eta$ . This result is very important, allowing the calculation of the viscoelastic coefficient for polymeric materials with very low flow, as the elastomers. For such materials the classic measurements with viscometers are not possible because the materials are in solid state.



**Fig. 3.** The set-up control panel of the oscilloscope and the recorded damped oscillations

The experimental oscillations obtained by us are presented in figure 3 and the set-up of the signal generator in figure 4. The data, on digital form, were collected directly and stored with the Scope 1.4.1. software and then processed with Kaleidagraph software. The data, as recorded, contain both the carrier frequency and the modulator signal, Fig. 3. In the first stage of analyze, the carrier

frequency is eliminated, Fig. 5. From this data we calculated the period  $T$  of the damped oscillations and we read the amplitudes  $A(T_i)$  of the oscillations. We found the value  $T = 0.25$  s. Using the equation  $A(T_i) = A_0 \exp(-\alpha T_i)$  we calculated the attenuation coefficient  $\alpha$ . We found the value  $\alpha = 6.4$  s<sup>-1</sup>. The value of  $\alpha$  was also calculated from the logarithmic decrement of the oscillations. We took two successively values of amplitudes  $A_1 = 1.86$  and  $A_2 = 0.37$ . We used the equation 5 for calculation. We obtained the value  $\alpha = 6.46$  s<sup>-1</sup>. The values determined by both methods are in good agreement. Using this value of  $\alpha$ , the measured values of  $T$  and  $m$ , we calculated the viscosity coefficient  $\eta$  with the equation  $\eta = 2m \cdot \alpha$ . We found the value  $\eta = 0.5$  Ns/m. We calculated also the elastic constant  $k$  with the equation (3). We found the value  $k = 25$  N/m. To verify our result, we used the values  $\alpha$ ,  $\eta$ ,  $m$  and  $T$  to fit the experimental data with the equation (2). The result of the fit is shown in figure 6. We can see a good agreement between experimental data and simulation.



**Fig. 4.** The set-up control panel of the signal generator

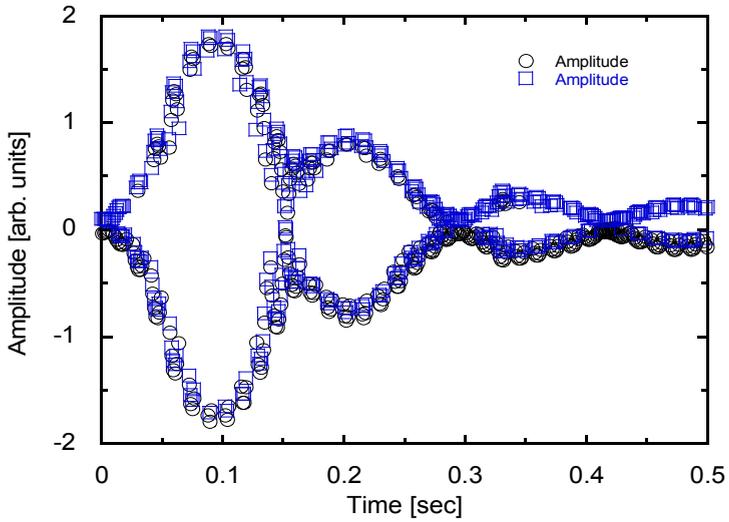


Fig. 5. The modulator signal after elimination of the carrier signal

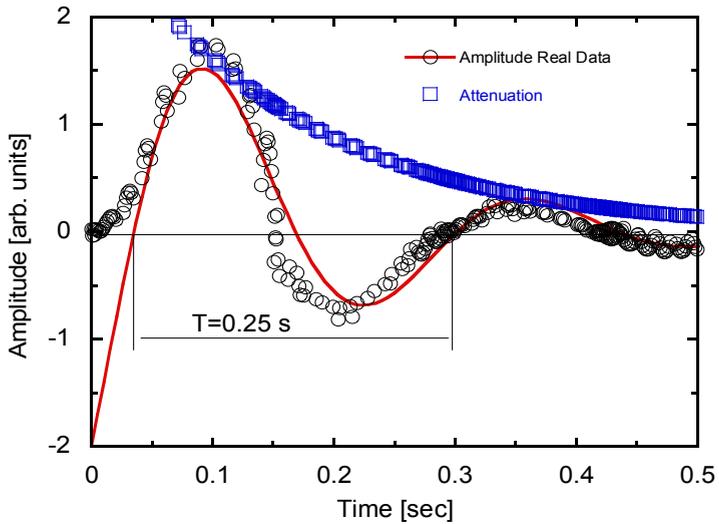


Fig. 6. The fit of the modulator signal with the equation 2

## CONCLUSION

A simple method for digital data recording of low frequency mechanical oscillations was presented. The method can be used for the investigation of the elastomers, allowing quantitative measurement of some parameters, like the elastic constant, attenuation and viscosity coefficients. Such data can be difficult to obtain by other ways for such materials. The method is based on the use of very simple equipment, which can be "home made", and free software available on the internet. Apart from the scientific character and the usefulness for the characterization of the elastomers, the method has a pronounced didactic character, allowing the user to better understand the principle of digital recording of low frequency mechanical oscillations.

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