

*Dedicated to Professor Dr. Sorin Dan Anghel on His 65<sup>th</sup> Anniversary*

## SPIN SUSCEPTIBILITY IN TWO DIMENSIONS

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**ABSTRACT.** The static spin susceptibility of a two dimensional electron system is calculated, for two distinct situations. The first system is one with a parabolic dispersion law, and the second system is one with a linear dispersion law, corresponding to graphene systems. The temperature dependence of the static spin susceptibility is analyzed up to room temperatures for both systems.

**Keywords:** *Static spin susceptibility, two-dimensional systems, parabolic dispersion law, graphene density of states*

### INTRODUCTION

The importance of two dimensional electron gas appear in developments in semiconductors through the achievement of structures in which the electronic behavior is two-dimensional. This means that the carriers are confined in a potential such that their motion in one direction is restricted, leaving only a two-dimensional motion in a plane normal to the confining potential. In this respect the important systems with two-dimensional behavior are MOS structures, quantum wells and superlattices [1]. Other important low dimensional systems are high-temperature superconductors [2]. A more recent two-dimensional system discovered is graphene [3,4], with very interesting properties due to its band structure and linear dispersion law. Among these properties are those connected to the magnetic response of the system to an external field [5]. In this paper we will analyze the temperature

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dependence of the static spin susceptibility of a two-dimensional system for two different situations, the parabolic and the linear dispersion. The first case correspond to a classical electron gas, and the second correspond to the electronic system of graphene.

## MODEL

The static spin susceptibility can be determined using the relation [6,7] :

$$\chi = g_L^2 \mu_B^2 \int dE \frac{\rho(E)}{4k_B T \cosh^2\left(\frac{E-\mu}{2k_B T}\right)} \quad (1)$$

Here:  $g_L$  - is the Lande electron g-factor,  $\mu_B$  - is the Bohr magneton, and  $\mu$  - is the chemical potential. In the following we will analyze the static spin susceptibility in the case of two-dimensional systems.

The first system we analyze is the electron system with a parabolic dispersion law:  $E_k = k^2 / 2m$ , where  $m$  - is the electron mass. For a unit area, and for both spin orientations ( $\hbar = 1$ ), the electron density of states is [8]:

$$\rho = \frac{m}{\pi} \quad (2)$$

which does not depend on energy. In this case, and in the wide band limit, the static spin susceptibility reduces to:

$$\chi = g_L^2 \mu_B^2 \frac{m}{2\pi} \int_{-\infty}^{\infty} \frac{dx}{\cosh^2 x} = \frac{g_L^2 \mu_B^2 m}{\pi} \quad (3)$$

which is a constant in the entire temperatures interval. In the case of a finite band,  $-E_c < E < E_c$ , and the spin susceptibility is given by the following formula:

$$\chi = \frac{g_L^2 \mu_B^2 m}{2\pi} \left[ \tanh\left(\frac{E_c - \mu}{2k_B T}\right) + \tanh\left(\frac{E_c + \mu}{2k_B T}\right) \right] \quad (4)$$

Here there is an analytical temperature dependence. Additionally, the chemical potential is a function of temperature too. For the parabolic dispersion law, the chemical potential is given by [9]:

$$\mu \equiv \mu(T) = k_B T \ln(e^{E_F/k_B T} - 1) \quad (5)$$

where:  $E_F = n\pi/m$  is the Fermi energy, and  $n$  - the electron density. Then the spin susceptibility of a two-dimensional system with a finite energy band becomes:

$$\chi = \frac{g_L^2 \mu_B^2 m}{2\pi} \left\{ \tanh \left[ \frac{E_c}{2k_B T} - \frac{1}{2} \ln(e^{E_F/k_B T} - 1) \right] + \tanh \left[ \frac{E_c}{2k_B T} + \frac{1}{2} \ln(e^{E_F/k_B T} - 1) \right] \right\} \quad (6)$$

In order to estimate this temperature dependence, up to room temperature, we will consider the following parameters:  $E_c = 5$  eV, and  $E_F = 2$  eV, which are typical values for metallic densities in two dimensions. Up to room temperature ( $T \propto 0.03$  eV) the spin susceptibility is not sensitive to temperature. The temperature influence becomes important for temperatures close to the Fermi temperature.

The second system we analyze here is the electron system of graphene, which is a two-dimensional carbon atom based material synthesized about a decade ago [10]. Here the carbon atoms are disposed on a hexagonal lattice. Graphene has unusual properties due to its band structure. The conduction and valence bands touch at six points, two of these being non-equivalent. Around these points the quasiparticle excitations follow a linear Dirac-like energy dispersion:

$$E_{k,\lambda} = \lambda v_F k \quad (7)$$

where:  $\lambda = +1$ , and:  $\lambda = -1$  correspond to the conduction and valence bands respectively, and  $v_F$  is the Fermi velocity of graphene. Using eq.(7), the density of states of the graphene is :

$$\rho(E) = \frac{2}{\pi v_F^2} |E| \theta(|E| - E_c) \quad (8)$$

where  $E_c$  is the band energy cutoff, and  $\theta(x)$  is the Heaviside step function. Using eq.(1) the static susceptibility becomes:

$$\chi = \chi_0 \left\{ \tanh\left(\frac{E_c + \mu}{2k_B T}\right) + \tanh\left(\frac{E_c - \mu}{2k_B T}\right) - \frac{2k_B T}{E_c} \ln \left[ \frac{\cosh\left(\frac{E_c + \mu}{2k_B T}\right) \cosh\left(\frac{E_c - \mu}{2k_B T}\right)}{\cosh^2\left(\frac{\mu}{2k_B T}\right)} \right] \right\} \quad (9)$$

with:

$$\chi_0 = \frac{g_L^2 \mu_B^2 E_c}{\pi v_F^2} \quad (10)$$

Here  $\mu$  is the graphene chemical potential determined using the conservation of the total particle density. For:  $T \ll T_F$ , where  $T_F$  is the Fermi temperature, the approximate value of the chemical potential is given by [11] :

$$\mu \cong E_F \left[ 1 - \frac{\pi^2}{6} \left( \frac{k_B T}{E_F} \right)^2 \right] \quad (11)$$

The static spin susceptibility in the zero temperature limit, for  $E_c > E_F$ , reduces to the following result:

$$\chi \equiv \chi(T = 0) = \frac{2g_L^2 \mu_B^2 E_F}{\pi v_F^2} \quad (12)$$

If we are interested in the temperature dependence of the spin susceptibility, in the temperature range up to room temperature, and for extrinsic case (when  $T_F \propto 1000$  K), we have to approximate eq.(9) to obtain:

$$\chi \cong \chi(T = 0) \left[ 1 - \frac{\pi^2}{6} \left( \frac{k_B T}{E_F} \right)^2 + \frac{2k_B T}{\mu} \cdot \exp\left(-\frac{\mu}{k_B T}\right) \right] \quad (13)$$

This is a slight decreasing function on temperature. For example if:  $E_F = 0.1$  eV, and  $T = 0.03$  eV, we obtain:  $\chi(T = 0.03) \cong 0.893 \cdot \chi(T = 0)$ .

## CONCLUSIONS

We have calculated the temperature dependence of the static spin susceptibility for two-dimensional electronic systems taking into account the cases of parabolic and linear dispersion law. In the first case the spin susceptibility is not sensitive to temperature, up to room temperature. In the second case, for temperatures up to room temperature, there is a slight temperature dependence. However, these simple results can be drastically affected if one take into consideration other important effects in two-dimensional systems. Among these effects are the presence of disorder [12,13], and the presence of an energy gap in the graphene quasiparticle spectrum [14-17]. These effects affect the temperature dependence of the spin susceptibility [18], and other important physical properties [19] of two-dimensional systems.

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