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RENORMALIZATION FUNCTION FOR THE ELECTRON-FLEXURAL PHONON INTERACTION

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ABSTRACT. The renormalization function for the electron-phonon interaction is discussed. The system is considered as two-dimensional, and we consider the case of flexural phonons. The flexural phonons have a dispersion which is linear for wave-vectors less than a critical value q_c, and quadratic for $q > q_c$. The renormalization function differs from the standard expression, and leads to modifications of the normal and superconducting properties of materials.

Keywords: *Renormalization function, two-dimensional systems, flexural phonons.*

The electron-phonon interaction is one of the fundamental interactions in condensed matter, and plays, together with the Coulomb interaction, an important role in many physical phenomena. The electronic excitations in a solid are strongly modified due to the Coulomb interaction and to the coupling to the lattice vibrations. These modifications affect the transport and thermodynamic properties of a solid. The electron-phonon interaction plays also a fundamental role in the attractive electron-electron interaction which is the origin of the electron pairing mechanism of the phenomenon of superconductivity [1]. Here the transition temperature T_c is determined by the material dependent quantities, λ - the coupling factor, and μ^* - the Coulomb pseudopotential

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$$T_c \cong \langle\omega\rangle \exp\left\{-\frac{1}{\lambda-\mu^*}\right\} \tag{1}$$

In this model, the adiabatic approximation for the dynamics of electrons and phonons is used. This situation correspond to the validity of the Migdal's theorem [2], because the ratio between the Debye energy ω_D, and the Fermi energy E_F is very small. This is the case of many common metals. However, in high-temperature superconductors, including the fullerene compounds, the Fermi energy is small ($E_F \approx 0.1$ eV) and is of order of the Debye energy. This situation implies a breakdown of Migdal's theorem [3]. More recent the validity of the Migdal's theorem in graphene and Weyl semimetals is analyzed in Ref.[4]. If other excitations (e.g. antiferromagnetic magnons) mediate the electron-electron attractive interaction, the Migdal's theorem seems to be also valid [5].

Here we will analyze the electron-phonon interaction, calculating the standard self-energy to the first order, and the wave-function renormalization term Z [6,7]. The self-energy is given by

$$\Sigma(i\omega_n) = -Tg^2 \sum_{k_s} \sum_{q} \sum_{s} G_0(i\omega_s, \varepsilon_{k_s}) D_0(i\omega_n - i\omega_s, \omega_q) \tag{2}$$

Here: T - is the temperature, g - the electron-phonon coupling strength, G_0 - the usual Green's function in the Matsubara representation

$$G_0(i\omega_s, \varepsilon_{k_s}) = \frac{1}{i\omega_s - \varepsilon_{k_s}} \quad ; \quad \omega_s = 2\pi T\left(s+\frac{1}{2}\right) \tag{3}$$

and D_0 - the Green's function for phonons

$$D_0(i\omega_m, \omega_q) = \frac{-\omega_q^2}{\omega_m^2 + \omega_q^2} \quad ; \quad \omega_m = 2\pi T m \tag{4}$$

For the sum over k_s we use the simplification

$$\sum_{k_s} \to N(0) \int_{-E/2}^{E/2} d\varepsilon \tag{5}$$

where we assumed a constant density of states in the band between $-E/2$ and $E/2$. Using eqs.(3-5) we obtain

$$\Sigma(i\omega_n) = -2iT\lambda \sum_q \sum_s \arctan\left(\frac{E}{2\omega_s}\right) \cdot \frac{\omega_q^2}{(\omega_n - \omega_s)^2 + \omega_q^2} \tag{6}$$

or

$$\Sigma(i\omega_n) = -i\omega_n \lambda T \sum_q \sum_{s>0} \arctan\left(\frac{E}{2\omega_s}\right) \cdot \frac{8\omega_q^2 \omega_s}{[(\omega_s - \omega_n)^2 + \omega_q^2][(\omega_s + \omega_n)^2 + \omega_q^2]} \tag{7}$$

Here we introduced the coupling factor $\lambda = g^2 N(0)$. The wave-function remormalization is given by

$$Z(i\omega_n) = 1 - \frac{1}{i\omega_n}\Sigma(i\omega_n) \tag{8}$$

In the following we will consider the static case ($\omega_n = 0$), and we neglect the temperature dependence ($T \to 0$), when we replace

$$T\sum_{s>0} \to \frac{1}{2\pi}\int_0^\infty d\omega \tag{9}$$

We will have

$$Z = 1 + \frac{\lambda}{2\pi}\sum_q \int_0^\infty d\omega \cdot \arctan\left(\frac{E}{2\omega}\right) \cdot \frac{8\omega_q^2 \omega}{\left(\omega^2 + \omega_q^2\right)^2} \tag{10}$$

Now we use the notation: $a = 2\omega_q / E$, and change the variable: $\omega / \omega_q = x$. We obtain

$$Z = 1 + \frac{4\lambda}{\pi}\sum_q \int_0^\infty dx \cdot \arctan\left(\frac{1}{a \cdot x}\right) \cdot \frac{x}{\left(x^2 + 1\right)^2} \tag{11}$$

The integral is easy evaluated and gives: $\pi /[4(1+a)]$. The renormalization function will be

$$Z = 1 + \frac{\lambda E}{2}\sum_q \frac{1}{\omega_q + \frac{E}{2}} \tag{12}$$

In order to find out an analytical form of the renormalization function we will consider a two-dimensional system, and consider the dispersion ω_q that correspond to flexural phonons [8, 9]:

$$\omega_q = \sqrt{\frac{\gamma q^2 + \kappa q^4}{\rho}} \tag{13}$$

Here: γ - is the sample specific coefficient induced by the external strain, κ - is the bending stiffness, and ρ - the mass density. This expression can be approximated as

$$\omega_q \cong q_c \alpha_0 q \tag{14}$$

for $q < q_c$, and

$$\omega_q \cong \alpha_0 q^2 \tag{15}$$

for $q > q_c$, with: $q_c = \sqrt{\gamma / \kappa}$ and : $\alpha_0 = \sqrt{\kappa / \rho}$. The sum over q wave-vectors is evaluated using

$$\sum_q \to \frac{A}{(2\pi)^2} \cdot B \cdot \int d^2 q \tag{16}$$

where we consider the unit area for A, and $B = 3\sqrt{3}/2$ for the case of honeycomb lattice. We split the wave-vector integral in two contributions, accoring to eqs.(14, 15). After evaluating these integrals one obtains

$$Z = 1 + \frac{3\sqrt{3}}{4\pi} \cdot \frac{\lambda q_c^2}{r} \cdot \left\{ 1 - \frac{1}{r} \cdot \ln(1+r) + \frac{1}{2} \cdot \ln\left[\frac{1 + r(q_M / q_c)^2}{1+r} \right] \right\} \tag{17}$$

where: $r = 2\alpha_0 q_c^2 / E$, and q_M is a wave-vector cut-off. In the wide-band approximation (E - large), the renormalization function becomes

$$Z \cong 1 + \frac{3\sqrt{3}}{8\pi} \cdot \lambda \cdot q_M^2 \tag{18}$$

This result differs from the standard expression $Z = 1 + \lambda$, and leads to modifications of many physical properties of solids, due to modifications of density of states and of the coupling g [10-14]. The magnitude of modifications can be estimated if we restore the dimensions by replacing

q_M with aq_M in eq.(18) (a - the lattice constant), and taking $q_M \sim 1/a$. One obtains $Z \approx 1 + 0.2\lambda$, a result which reveal the smallness of the correction factor. The renormalization function Z can also be evaluated for other bosonic excitations that are important for various properties of solids, as magnetic excitations [15-17], and even for the case of non-Fermi systems [18-21]. Here we obtained a simple result which can be used, as a starting point, in future investigations, in order to find out the modifications of the normal and superconducting properties of newly discovered materials. A more realistic result, for the case of graphene, can be obtained taking an energy dependent density of states, instead of simplification used in eq.(5).

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