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SUPERCONDUCTIVITY IN LOW DIMENSIONAL SYSTEMS WITH DIFFERENT ENERGY DISPERSIONS

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ABSTRACT. We analyzed the possibility for occurrence of superconductivity in low dimensional systems, taking into consideration a linear and a constant dispersion law for the electronic excitations. Using a mean field BCS-like model we calculate the zero temperature energy gap, the critical temperature, and the Gelikman-Kresin ratio. In the case of graphene with a linear dispersion the coupling strength should exceed a critical value. Taking realistic parameters it was shown that the occurrence of the superconducting state is not possible. We find out an opposite situation for a two-dimensional system with a constant dispersion.

Keywords: *Superconductivity, low dimensional systems, graphene, linear dispersion, constant dispersion.*

Superconductivity is a quantum phenomenon of the electron system that manifests at macroscopic scale. This phenomenon is due to an instability of the Fermi liquid state which leads to a new ground state of correlated paired electrons [1]. Here it was shown that this state is stabilized whenever there exist an attractive interaction between electrons. In a common metal such an attraction is always provided by the electron-phonon interaction. This behaviour however is strongly modified in the case of high-temperature

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superconductors [2]. These are layered compounds with possible new pairing mechanism which leads to unconventional superconductivity. It was proposed that superconductivity emerges from antiferromagnetic spin fluctuations in a doped system and in the weak coupling [3, 4]. Other theoretical models proposed the non-Fermi character of these compounds [5]. Here, in contrast to classical Fermi liquid behaviour the superconducting state appear only if the coupling factor exceeds a critical value [6]. Other models consider the Bose-Einstein condensation phenomenon [7, 8]. More recent, after the discovery of graphene [9, 10], experiments on doped graphene reveal that graphene can be driven to the superconducting state [11]. A weak-coupling theory of superconductivity of Dirac electrons in graphene layers was proposed by Kopnin and Sonin [12]. On the other hand, experiments indicate the lack of superconductivity in undoped graphene, even an attractive electron-electron interaction λ could exist. The absence of superconductivity is rather due to the small density of states close to the Dirac point. In order to see this we adopt a simple mean-field model using a self-consistent equation for the order parameter Δ [13, 14]

$$\Delta = 4\lambda \int_0^{k_c} \frac{d^2k}{(2\pi)^2} \cdot \frac{\Delta}{E_k} \cdot \tanh\left(\frac{E_k}{2k_BT}\right) \tag{1}$$

The factor 4 comes from summation over the valley and band indices, k_c is cut-off momentum, $E_k = \sqrt{\varepsilon_k^2 + \Delta^2}$, k_B is the Boltzmann constant, and T the temperature. At the Dirac point $\varepsilon_k^2 = \hbar^2 v_F^2 k^2$, where v_F is the Fermi velocity. First we consider the $T = 0$ K case, when $\Delta \to \Delta_0$, and $\tanh(E_k / 2k_BT) \to 1$. We obtain

$$\Delta_0 = 4\lambda \int_0^{k_c} \frac{2\pi k dk}{(2\pi)^2} \cdot \frac{\Delta_0}{\sqrt{\hbar^2 v_F^2 k^2 + \Delta_0^2}} \tag{2}$$

or

$$1 = \frac{2\lambda}{\pi} \int_0^{k_c} \frac{dk \cdot k}{\sqrt{\hbar^2 v_F^2 k^2 + \Delta_0^2}} \tag{3}$$

Introducing a new variable $x = \hbar^2 v_F^2 k^2$, we get after integration

$$1 = \frac{2\lambda}{\pi\hbar^2 v_F^2}\left[\sqrt{\varepsilon_c^2 + \Delta_0^2} - \Delta_0\right] \qquad (4)$$

where $\varepsilon_c = \hbar v_F k_c$. Define the critical coupling λ_c through

$$\lambda_c = \frac{\pi\hbar^2 v_F^2}{2\varepsilon_c} \qquad (5)$$

and solving eq.(4) for Δ_0 one obtains

$$\Delta_0 = \varepsilon_c \cdot \frac{\lambda^2 - \lambda_c^2}{2\lambda\lambda_c} \qquad (6)$$

Since $\Delta_0 \geq 0$, one conclude that $\lambda \geq \lambda_c$.

In the opposite limit, when $T \to T_c$, T_c being the critical temperature, the order parameter $\Delta \to 0$. We will have

$$1 = 4\lambda \int_0^{k_c} \frac{d^2 k}{(2\pi)^2} \cdot \frac{1}{\varepsilon_k} \cdot \tanh\left(\frac{\varepsilon_k}{2k_B T_c}\right) \qquad (7)$$

or

$$1 = \frac{2\lambda}{\pi\hbar v_F} \int_0^{k_c} dk \cdot \tanh\left(\frac{\hbar v_F k}{2k_B T_c}\right) \qquad (8)$$

With the variable $y = \hbar v_F k / 2k_B T_c$, one obtains

$$1 = \frac{2\lambda}{\lambda_c} \cdot \frac{k_B T_c}{\varepsilon_c} \int_0^{\varepsilon_c / 2k_B T_c} dy \cdot \tanh(y) \qquad (9)$$

After evaluating the integral, and define $b = \varepsilon_c / 2k_B T_c$, we have

$$\frac{\lambda_c}{\lambda} \cdot b = \ln\left[\cosh(b)\right] \qquad (10)$$

Assuming $\varepsilon_c >> T_c$, ($b >> 1$), eq.(10) reduces to the approximate expression

$$\ln\left(\frac{1}{2}\right) \cong b \cdot \frac{\lambda - \lambda_c}{\lambda} \qquad (11)$$

From here we obtain the critical temperature

$$k_B T_c \cong \frac{\varepsilon_c}{2\cdot\ln 2}\cdot\frac{\lambda-\lambda_c}{\lambda} \qquad (12)$$

Again, in order to have a critical temperature λ should exceed λ_c. Finally, the Gelikman-Kresin ratio will be

$$\frac{2\Delta_0}{k_B T_c} \cong 2\cdot\ln 2\cdot\left(1+\frac{\lambda}{\lambda_c}\right) \qquad (13)$$

The results given by eqs.(6) and (12) should be carefully analyzed. In the case of pristine graphene, taking realistic parameters [14], one conclude that λ_c exceeds λ, and $\lambda_c \approx 20\cdot\lambda$. In this case the superconducting phase is absent even in the presence of a finite electron-phonon coupling strength. This conclusion remains valid for the case of twisted graphene bilayer far away from the magic angle, when the two graphene layers are almost uncoupled. The situation changes if one consider the case of linear dispersion in the presence of doping (characterized by finite chemical potential μ). In this case it was shown [14] that the superconducting state exist, and the critical temperature is of order of $T_c \approx 10$ K.

In the following we will consider a model of a two-dimensional system, with spin and valley degeneration, and with a constant dispersion $\varepsilon_k^2 = \Delta_g^2$, in order to find out if the superconducting state exist. At $T = 0$ K the equation for the energy gap becomes

$$\Delta_0 = 4\lambda\int_0^{k_c}\frac{2\pi k dk}{(2\pi)^2}\cdot\frac{\Delta_0}{\sqrt{\Delta_g^2+\Delta_0^2}} \qquad (14)$$

with $k_c = \varepsilon_c/\hbar v_F$. Using eq.(5), the momentum cut-off will be $k_c = \pi\hbar v_F/2\lambda_c$. The energy gap is given by

$$\Delta_0 = \sqrt{\left(\frac{\lambda}{\lambda_c}\cdot\frac{\varepsilon_c}{2}\right)^2-\Delta_g^2} \qquad (15)$$

which shows the possibility of occurrence of the superconducting state if $\Delta_g < \lambda\varepsilon_c/2\lambda_c$. Assuming that $\lambda << \lambda_c$, one has $\Delta_g << \varepsilon_c$. The critical temperature is obtained from the equation

$$1 = 4\lambda \int_0^{k_c} \frac{2\pi k dk}{(2\pi)^2} \cdot \frac{1}{\Delta_g} \cdot \tanh\left(\frac{\Delta_g}{2k_B T_c}\right) \quad (16)$$

Introducing the notation $a = 2\Delta_g \lambda_c / \varepsilon_c \lambda$, the critical temperature will be

$$k_B T_c = \frac{\Delta_g}{\ln\left(\frac{1+a}{1-a}\right)} \quad (17)$$

with $a \leq 1$. With this notation eq.(15) can be rewritten as

$$\Delta_0 = \varepsilon_c \cdot \frac{\lambda}{2\lambda_c} \cdot \sqrt{1-a^2} \quad (18)$$

The Gelikman-Kresin ratio, that measures the departure from the BCS (3.53) result [15, 16, 17], will be

$$\frac{2\Delta_0}{k_B T_c} = \frac{2}{a} \cdot \sqrt{1-a^2} \cdot \ln\left(\frac{1+a}{1-a}\right) \quad (19)$$

Taking the following parameters: $\varepsilon_c \approx 200$ meV, $\lambda / \lambda_c \approx 1/20$, and $\Delta_g = 4$ meV, the energy-gap at zero temperature will be $\Delta_0 \cong 3$ meV, the critical temperature $T_c \approx 20$ K, and the Gelikman-Kresin ratio $2\Delta_0 / k_B T_c \cong 3.3$.

One can conclude that a BCS-mean field model with a weak attractive interaction, due to the electron-phonon coupling, can explain the occurrence of the superconducting state, in low dimensional systems, if a constant dispersion law is considered. Such a dispersion can occur in highly doped two dimensional systems.

REFERENCES

1. J.Bardeen, L.N.Cooper, J.R.Schrieffer, *Phys. Rev.,* 108, 1175 (1957)
2. J.G.Bednorz, K.A.Müller*, Z. Phys. B,* 64, 189 (1986)
3. P.Monthoux, A.Balatski, D.Pines, *Phys. Rev. B,* 46, 14803 (1992)
4. I.Grosu, M.Crisan, *Phys.Rev. B,* 49, 1269 (1994)
5. C.M.Varma, Z.Nussinov, W.von Saarloos, *Phys. Repts*., 361, 267 (2002)
6. I.Grosu, I.Tifrea, M.Crisan*, J. Supercond*., 11, 339 (1998)

7. S.Fujita, S.Godoy, "*Quantum Statistical Theory of Superconductivity*", Plenum Press, New York (1996)
8. M.Crisan, I.Tifrea, D.Bodea, I.Grosu, *Phys. Rev. B,* 72, 184414 (2005)
9. A.K.Geim, K.S.Novoselov, *Nature Mater*, 6, 183 (2007)
10. A.H.Castro Neto, F.Guinea, N.M.R.Peres, K.S.Novoselov, A.K.Geim, *Rev. Mod. Phys*., 81, 109 (2009)
11. B.M.Ludbrook et al., *PNAS* 112, 11795 (2015)
12. N.B.Kopnin, E.B.Sonin, *Phys.Rev.Lett*., 100, 246808 (2008)
13. A.L.Fetter, J.D.Walecka, "*Quantum Theory of Many-Particle Systems*", McGraw-Hill, New York (1971)
14. T.J.Peltonen, R.Ojajärvi, T.T.Heikkilä, *arXiv*: 1805.01039
15. B.Krunavakarn, P.Udomsamuthirun, S.Yoksan, I.Grosu, M.Crisan, *J.Supercond*., 11, 271 (1998)
16. P.B.Allen, B.Mitrovic, "*Solid State Physics*", edited by H.Ehrenreich, F.Seitz, D.Turnbull, Academic, New York (1982)
17. D.C.Mattis, M.Molina, *Phys. Rev. B,* 44, 12565 (1991)