SYLLOGISTIC REASONING FROM PETRUS HISPANUS TO MODERN CONCEPTUALIZATIONS

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ABSTRACT. This paper is going to present the syllogistic reasoning beginning with the Middle Ages and comparing the traditional type of this kind of reasoning with some modern conceptualizations (Hodges and Makinson). In this paper is emphasized the view of Petrus Hispanus, a Middle Age philosopher who based his point on Aristotle's and Boethius' logic. There is a difference between their presentations of this reasoning. The conceptual difference between Aristotle's and the traditional one is going to be pointed. The problem of the fourth figure is also going to be considered. The conclusion of this analysis is going to underline the evolution and continuity of the syllogistic reasoning.

Keywords: *syllogism, fourth figure, proposition, Summulae Logicales, reasoning, traditional logic, modern logic*

In traditional logic syllogism is a form of deductive reasoning which has two premises and a conclusion, it also has only three terms, each of them occurs twice and the middle one does not appear in the conclusion. Aristotle defines this kind of reasoning as "a discourse in which, certain things having been supposed, something different from the things supposed results of necessity because these things are so."[2] This definition is too large because it removes the differences between syllogism and other kinds of deductive reasoning.[3] Some additions are made to the first definition: syllogistic reasoning is formed of two premises which are linked by a middle term.

When Aristotle decides which propositions can be premises he denies this right to singular proposition, even if he uses them as examples through his presentation. In traditional logic singular proposition can be understood as universal ones. Affirming or denying something about the object of the singular propositions, the predicate refers to the whole extension of the subject. In Aristotle's logic are

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[2] Aristotel, *Organon*, trans. Mircea Florian, București: Editura IRI, 1997, *Analitica primă*, I, 1, 24b

[3] Rusinoff apud Başkent, „Hypothetical Syllogism in Aristotle and Boethius", 11 sept. 2008, (http://canbaskent.net/logic/early/syllogism.pdf), pp. 1

underlined propositions which have general terms because only this kind of terms are interchangeable and Aristotle wanted to have this propriety for all of the terms used in a syllogism. Singular terms can only be subjects in a categorical proposition.

In Aristotelian logic things are divided in three classes[4]: the first one represents things that cannot be predicate about anything, as proper names, but other can be predicate about these ones, the second class is composed of things about that nothing can be predicate, but that can be predicate about others and the last one contains things that can be both predicates and subjects.

Łukasiewicz denied that things can be predicate about other things, he thought that only predicate as a part of a proposition, as a word with a meaning can have this role. In this case the classification is not about things, it is about terms. Łukasiewicz made a critical analysis for this classification. First of all he considered that it is wrong to say that singular terms cannot be predicate, because even Aristotle used examples with this kind of terms as predicates, for example "That white object is Socrates" or "The one that is coming is Callias" and said about those propositions that represent only an opinion, they are true only contingent.[5] There are examples of propositions which have a singular term as predicate and are not true in a contingent way, for example a tautology which expresses the identity principle and have the same term as subject and predicate – Socrates is Socrates – or use different linguistic expressions to express the same object – Shakespeare is the one that wrote Hamlet.

Even if we accept that Aristotle can be confuted in this part of his doctrine, the need to have interchangeable terms remains. In each figure there is a term which is both a subject and a predicate: in the first figure the middle term, in the second the major one and in the third one the minor. Łukasiewicz saw in this need of interchangeability the only strong reason why Aristotle could exclude the singular terms among the ones that can form a categorical proposition as a premise for the syllogism.[6]

There are some differences between the traditional syllogism and the Aristotelian one. The traditional syllogism represents a deductive reasoning which is formed of three propositions, and the Aristotelian one is a proposition based on an implication, it has the "if – then" form in which the antecedent is the conjunction of the premises, and the consequent is the conclusion.[7] The traditional syllogism can

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[4] Łukasiewicz, *Aristotle's Syllogistic from the Standpoint of Modern Formal Logic*, 2nd ed, London: Oxford University Press, 1955, pp. 5

[5] *Ibidem*, pp. 6

[6] *Ibidem*, pp. 7

[7] Patzig, *Silogistica aristotelică*, trans. Mircea Constantinescu, Niels Offenberger, București: Editura Științifică, 1970 pp. 23

be understood as a rule of reasoning which affirms that based on two propositions of a certain type, a third one – the conclusion – can be obtained using some certain rules.

In Aristotle's language, propositions are formulated differently. Instead of using a copula, Aristotle used expressions as "to belong" or "to be contained about", in this way the predicate is the one which almost all the time is the first one. Therefore, the four categorical propositions have a different linguistic form in traditional logic and in Aristotelian one. In traditional logic, a universal affirmative proposition is "S is P" and in Aristotle's language it becomes "A belongs to all/every B". The problem which rises is why did Aristotle use those expressions. Alexander of Afrodisia founded three possible answers:

(1) the conclusion is more obvious;

(2) the role of each term is clearer;

(3) the predicate, which is more general by its nature, is the first one[8].

The first point can be applied to the first figure, because it would maintain the evidence of the conclusion, the other two points underline the fact that the predicate would be in nominative and the subject in dative or genitive.

Petrus Hispanus defines syllogism as an *oratio*, a discourse, the definition is alike with the Aristotelian one: "a syllogism is a discourse in which, once some things have been posited, something else necessarily happens through what have been posited"[9]. The example offered by Petrus is:

> "Every animal is a substance
> Every man is an animal
> So, every man is a substance."[10]

Petrus adds that every syllogism is composed of three terms and two propositions: the major one which has the major term (P) and the minor one which has the minor term (S). Because two proposition cannot be composed of three terms, one of them must occur twice, that is the middle (M). The terminology is borrowed from Aristotle, who used it based on the first figure. The middle is the term which has the middle position, more exactly, it is contained by the major and it contains the minor. Aristotle considered that if A is stated about all B and B about all C, it is required that A to be stated about all C, the same is in the case of the negative propositions. In traditional logic those terms are defined based on their

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[8] Alexander of Afrodisia apud. Patzig, *op.cit.*, pp. 33
[9] Petrus Hispanus, *Tractatus*, Introduction by L.M.De Rijk, Assen: Van Gorcum & Comp. B.V., 1972, pp. 43
[10] Omne animal est substantia. Omnis homo est animal. Ergo omnis homo est substantia. (*Ibidem*, pp. 43)

position in the conclusion, but in Aristotle's and Petrus' examples of the syllogism often do not even have conclusion, therefore, the definition of the minor and major is independent of their role in conclusion. It can be said that their position in conclusion is a convention, in the first figure this positioning is the base for the perfect modes, but in the other figures it is an arbitrary decision.[11]

Hence, the major is almost all the time the largest extensionally, the next one is the middle term and the species, represented by the minor term. There are some exceptions, for example if *salamander* is taken as major, *animal* as minor and *mammal* as the middle.[12]

The figure consists in the positioning the three terms based on their role as subject and predicate. Petrus, as Aristotle, considered that this positioning can be made in three ways. Every figure is defined based on the position of the middle, therefore[13]: the first figure is when the term which is subject in the first proposition is predicate in the second one, the second figure is when the same term is predicate in both premises and the third figure is when the same term is subject in both premises.

In traditional logic are four figures, even if it is not expressed directly by Aristotle, as Petrus, offered some examples of valid syllogism of the fourth figure, but there are considered indirect modes of the first figure, which can be reducible at those valid modes of the first figure. The fourth figure was added to the other ones by Aristotle's students of Peripatetic School. This idea is expressed in Dumitriu and Kneale and Kneale. It seems that Theoprastus was the first one who added the five valid modes of the fourth figure.[14]

Rose analyses the problem of the fourth figure and he concludes that the manner of representation of those figures represents the base for the number of the figures discovered. The linear manner of representation used by Aristotle made the representation of the fourth figure impossible. Often Aristotle represents the categorical propositions using a form as AB and does not indicate the quality, nor the quantity or the function of the terms. In Aristotelian language AB means that A is the predicate and B the subject.[15]

In the first figure the middle term has the middle position, the major is predicate about the middle and the middle is predicate about the minor, this figure can be symbolised as PMS, where PM represents the major premise, MS is the minor one and PS is the conclusion. In the second figure the middle is predicate about both other terms, this case could be represented in the linear manner by MPS where MP

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[11] Patzig, *op. cit.,* pp. 152

[12] Kneale and Kneale, *Dezvoltarea logicii*, trad. Cornel Popa, Cluj – Napoca: Editura Dacia, 1974, pp. 79

[13] Petrus Hispanus, *op.cit.*, pp. 44

[14] Başkent, *op.cit.*, pp. 3

[15] Rose, „Aristotle's Syllogistic and the Fourth Figure", in *Mind*, vol. 74, No. 295 (Jul., 1965), pp. 382-383

and MS are the premises and PS is the conclusion. The last figure has the middle term as subject in both premises, in this case it could be represented as PSM, where PM and SM are the premises and PS is the conclusion. This linear manner of representation for syllogistic figures is alike with the manner of representation in antique manuscripts.[16]

Rose proves that the number of possible figures is based on the way those figures are represented. If it is used the Aristotelian manner there are only three figures, the fourth is impossible to be pictured using a linear manner. A syllogism is possible only when a term is a link for the other two, this can be made in three ways: stating A about C and C about B, or C about both (A and B) or both about C.[17] Those are the three possible figures. The fourth one where M is predicated about P and is predicated by S it is impossible to represent linear, because it would need a premise as MP and another one as SM. If Petrus Hispanus considers the valid syllogisms of the fourth figure as indirect modes of the first ones, Rose considers that Aristotle does not deny the validity of those modes, or the fact that they are realised using a syllogistic form, but they are not syllogisms per se (in a strict Aristotelian manner) because they have to be formulated using one of the figures accepted by Aristotle.[18]

Aristotle considers that a syllogism that has a conclusion which can be converted, can also have as conclusion the converse, because of this all valid modes of the fourth figure can be considered modes of the first figure. Patzig thinks that Aristotle defines the three terms of the first figure by their extension and after this figure, he offers definitions based on their function in the premises. For a syllogism to be perfect it must have the major as predicate of the conclusion, this case is generalised for all the figures, but because of this there are some valid syllogism denied.[19] Therefore, a definition for the fourth figure using the same concepts as Aristotle used assumes a major reorganization of the definitions of the Aristotelian system. In this case the modes of the fourth figure are handled as modes of the first one even if the relations between the terms do not satisfy their definitions of the first figure. The fact that there were added five modes to the first figure destroyed the homogeneity of the system, because the other two figures were not completed. Patzig considers that the absence of the fourth figure is not a mistake and does not reflect that Aristotle did not believe that there exist some valid modes in another form than the three ones already known, the fourth figure is missing in Aristotelian doctrine because it could not be defined in his system.[20]

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[16] Kneale, *op.cit.*, pp. 83
[17] Aristotel, *op.cit.*, I, 23, 41 a (where A / P, B / S și C / M)
[18] Rose, *op.cit.*, pp. 386
[19] Patzig, *op.cit.*, pp. 181
[20] *Ibidem*, pp. 182

Therefore, there are different opinions about the fourth figure which is missing not only in Aristotle's work, but in Petrus' too. Starting with the idea that the valid modes of the fourth figure are indirect ones of the first and can be reduced of this one by a simple conversion of the conclusion, and continuing with the idea that this figure cannot be represented in a linear manner, nor defined in Aristotelian system.

Returning to Petrus Hispanus, the mode represents the ordering of the premises by quantity and quality, in fact, it means that there are inserted the four types of categorical proposition in the structure of the figures. His presentation continues with the general rules of syllogism. On the base of these rules the valid and invalid syllogisms can be determined which syllogism is valid or not. There are some differences between those rules and the ones from traditional logic. First of all, the rule that assumes that from a universal and a particular premise, the conclusion must be particular, is clearer in Petrus' *Tractatus*, because it underlines that from two universal premises the conclusion can also be particular, in his words: "if a premise is particular, the conclusion must be particular, but not the converse."[21]

Aristotle accepts particular conclusions from universal premises, but he introduces a restriction, he compels that the terms used to form the syllogism cannot be exhausted, they must have at least one element in their extension, because if not the conclusion can be false even the premises are true. Universal propositions do not have existential import, therefore the conclusion, which has one, is not supported by the premises without Aristotle's requirement. On the other hand Boole does not accept a particular conclusion from universal premises in a deductive way. Any particular proposition supports the existence of the subject's class, if we affirm that *Some people have blond hair* there must exists at least one person. The general propositions do not imply the existence of the subject's class. If we take, for example, the proposition *All humans are mortal* we only affirm that if there is something as a human, it must be mortal. Therefore, particular propositions have the existential import, and the universal ones do not, and if we have universal premises and a particular conclusion, the reasoning may be valid but not deductive. If a reasoning is deductive, the conclusion cannot include something that premises do not already stipulate. The demonstration can be made using Venn diagrams or predicate logic. For example, in predicate logic the mode aai-3 can be written:

$$\forall x \, (Mx \supset Px)$$
$$\forall x \, (Mx \supset Sx)$$
$$\therefore \exists x \, (Sx \wedge Px)$$

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[21] „si aliqua premissarum est particularis, conclusio debet esse particularis, et non econverso." (Petrus Hispanus, *op.cit.,* pp. 45

In this case, the conclusion cannot be demonstrated if another assumption is not introduced: $\exists x (Sx)$.

The rule that assumes that from two affirmative premises, the conclusion must be affirmative is omitted, but this is common in traditional logic too. The rules about distribution are also omitted.

In Aristotelian syllogistic the concept of distribution does not appear, nor in Petrus' even if Petrus Hispanus defines the concept, but not in the part where he details syllogism. In traditional logic a term is distributed if it is taken/considered in its whole sphere. The definition of this concept given by Petrus in his last tractate (*De distributionibus*) in *Summulae Logicales* is alike with the one from traditional logic:

> "Distribution is the multiplication of a common term effected by a universal marker. In ‚every man', the term ‚man' is distributed or diffused for any of its inferiors[22] by the marker ‚every', and that is the way multiplication of a common term occurs. I say ‚of a common term' because a singular term cannot be distributed. That is why ‚every Socrates', ‚every Plato' and their like are incongruous."[23]

Therefore, if the proposition says something about every instance of the term,[24] it is distributed. In categorical proposition, the subject is distributed in universal ones and the predicate in negative. Between syllogistic rules there are two about distribution of terms:

Rule 1: The middle term must be distributed in at least one premise.

Rule 2: Terms (S, P) cannot be distributed in conclusion unless they are distributed in premises.

Makinson considers that even if the traditional formulation for this concept is vague, it is coherent and meaningful. There is a syntactic approach and a theoretic deduction one. For the syntactic one the categorical proposition are transformed based on the language of predicate logic[25]:

All P are Q becomes $\forall x (Px \supset \exists y (Qy \wedge x = y))$
Some P are Q becomes $\exists x (Px \wedge \exists y (Qy \wedge x = y))$
Some P are not Q becomes $\exists x (Px \wedge \forall y (Qy \supset x \neq y))$
No P are Q becomes $\forall x (Px \supset \forall y (Qy \supset x \neq y))$

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[22] Subset of the term

[23] Hispanus, *op.cit.*, pp. 209 – „Distributio est multiplicatio termini communis per signum universale facta. Ut cum dicitur *'omnis homo'*, iste terminus '*homo*' distribuitur sive confunditur pro quolibet suo inferiori per hoc signum '*omnis*'; et/sic est ibi multiplication termini communis. Dico autem '*termini communis'*, quia terminus singularis non potest distribui. Unde iste sunt incongrue: *'omnis Sortes', 'omnis Plato',* et sic de aliis."

[24] Makinson, „Remarks on the Concept of Distribution in Traditional Logic", in Noûs, Vol. 3, No. 1 (Feb. 1969), pp. 103

[25] *Ibidem*, pp. 104

In this case a term is distributed if in the translation of the proposition its variable has the universal quantifier; if it has an existential one, the term is not distributed. The main idea of the traditional concept is maintained and it becomes clearer.

In the syntactic approach the concept of distribution has some restrictions/ limitations. In the traditional approach the concept was related to the proposition's logical capacity but not to its linguistic form which it expresses. Syntactical approach offers an exact meaning for this concept for the categorical propositions, but terms that appear in more complex propositions are harder to analyse using this approach. Therefore, the concept can hardly pass the limits of traditional logic.[26]

The traditional concept presumes that a term is distributed if every time the proposition affirms or denies something about that term, also made it about every term included in the first one. Embracing Russell's form to represent categorical propositions we have[27]:

All P are Q becomes $\forall x\, (\, Px \supset Qx)$
Some P are Q becomes $\exists x\, (Px \wedge Qx)$
Some P are not Q becomes $\exists x\, (Px \wedge \neg Qx)$
No P is Q becomes $\forall x\, (\, Px \supset \neg Qx)$

In this case a term τ is distributed if and only if:

$\alpha - \forall x\, (\tau`x \supset \tau x) \supset \alpha\, [\tau/\tau`]$ where $\tau`$ is a new letter for a predicate that does not occur in $\alpha$ yet and expression $\alpha\, [\tau/\tau`]$ is obtained by substituting τ with $\tau`$ in α.[28] This approach respects the initial definition; the predicate is still distributed in negatives and the subject in universals.

This teoretical deduction approach makes possible the generalization of the concept to more complex propositions. The first approach is syntactical, therefore to determine if a term is distributed or not, the quantifiers matter. The second one is semantic. To determine if τ is distributed in a proposition it must be checked if something is implied by something, and this leads to a circularity: it cannot be checked if a reasoning is valid without knowing which terms are distributed and on the other hand it cannot be analysed the distribution of the terms without knowing what propositions are implied. Makinson gives an example for Barbara in which case we cannot find if the subject is distributed without knowing if the mode is valid.[29] Hodges defines the concept of distributed term so: a term X is said to be distributed in an atomic sentence φ of S^{30} if X occurs in φ and there is a first – order sentence φ' which is

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[26] *Ibidem*, pp. 105
[27] Russell apud Makinson, *op.cit.*, pp. 105
[28] Makinson, *op.cit.*, pp. 105
[29] *Ibidem*, pp. 106
[30] *S* is the language of the syllogism where the four categorical propositions are transformed using the language of first order logic.

equivalent to φ, in which X does not occur positively; likewise X is said to be undistributed in φ. For example, if the proposition A(P,Q) is taken, this one is equivalent[31] with:

$$\forall x \, (\neg Px \lor Qx)^{32}$$

Therefore, P is distributed and Q undistributed in a universal affirmative proposition.

Hodges considers that if the distribution concept is identified with the presence of the universal quantifier, there are some cases of distributed terms which cannot be found using this method, for example the predicate in a particular negative proposition.

The authors from Port – Royal consider that what φ states about X it states about X's subsets, and because of this X is distributed in E (X, Y) and in O (X, Y) also, based on the fact that if the gender is denied, so are the species.[33] It is not enough that the undistributed term to be defined using the negation of the distributed one, as the authors of Port – Royal did, it must be introduced a new definition for the second term. Through introduction of the terms *downward monotone* and *upward monotone* it can be concluded that a term is distributed if it is downward monotone and undistributed if it is upward monotone. Hodges offers the following definitions for those terms[34]:

Definition 1: a term X in a formula φ is downward monotone in φ if for every structure M in which φ is true, φ is also true in N which differs of M only in one interpretation, more accurate, X^N is a proper subset of X^M.

Definition 2: A term X is upward monotone in φ if for every structure M in which φ is true, φ is also true in N which differs from M only in one interpretation, more accurate, X^M is a proper subset of X^N.

Therefore, even if are used different approaches of redefining the concept of distribution, Makinson considers that using the theoretical deduction approach the distribution of a term is based on the logical force of the proposition not on its formula, in this case the concept is not syntactic. Even if the meaning of the concept passes beyond the limits of traditional logic, its utility is limited to the reasoning of this kind of logic.[35]

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[31] Using Russell's method of transformation already presented above.

[32] Hodges, „The Laws of Distribution for Syllogisms", în Notre Dame Journal of Formal Logic, vol. 39, no. 2 (spring 1998), pp. 224

[33] *Ibidem*, pp. 226

[34] *Ibidem*, pp. 226-227

[35] Makinson, *op.cit.*, pp. 108

In conclusion, the syllogistic reasoning in its traditional form is different of the Aristotle's syllogism. Petrus Hispanus is placed as a middle between those two perspectives. He follows Aristotelian conception but based on Boethius's translation, so a modified one. Boethius kept Aristotle's definitions and concepts, but changed his language, especially his natural one. Petrus, as Boethius, relinquishes the strict formulation of the categorical propositions from Aristotle's language and uses the traditional logic's formulation. The number of the figures, the valid modes and rules follow the Aristotelian doctrine. In this case, we may conclude that through Petrus' work, which represented one of the most well-known logic textbooks of Middle Ages, is realised one of the first transitions to what we call today traditional syllogism.

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