

# Epistemic Vindication, Truthlikeness and Credence: Defending the Unified Approach

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**ABSTRACT.** There are two main approaches used to evaluate someone's epistemic accuracy. On one hand, we have the approach that evaluates the truthlikeness of some propositions and orders them by their closeness to truth. On the other hand, we can look at someone's probability assignments and then calculate the accuracy using the Brier score. Recently, Oddie tried to show that these two methods are in tension, because a measure that obeys probabilism will fail to hold Proximity. I argue that the Unified Approach can be saved and, moreover, has some important advantages over the isolated Truthlikeness.

**Keywords:** truthlikeness, probabilism, Brier score, credence, epistemic accuracy

## 1 Introduction

It seems plausible to say that when we can't know the truth, we are epistemically better off sticking close to it. However, it is not always clear what it means to miss the truth but to stay close to it. Narrowly missing the truth can mean, for example, being confident in  $P$  when  $P$  is true instead of being certain that  $P$  is true. Finding out that  $P$  is true when my credence in  $P$  was 0.9 is worse than if my credence were 1, but far better than 0.5. We can also look at other beliefs that are content-wise close to the truth. Believing that the number of planets is 10 (when there are 8 planets) is far better than believing there are 20 planets. This is not all that can be said about closeness to truth. Not all truths are equal, and likewise, not all falsehoods are equal. A stronger truth, namely, one that narrows down the space

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of possibilities, is preferable to a weaker one. Indeed, tautologies are the least valuable truths.<sup>1</sup> For falsehoods it may seem *prima facie* plausible that the stronger falsehoods are worse than weaker ones. Saying, for example, that there are between 9 and 1000 planets is arguably worse than saying there are 9 planets, although both are false.<sup>2</sup> Finally, some falsehoods may be better than some truths.

If Bob's height is 1.75m and I estimate it is 1.751m, it may be closer to the truth than saying it is between 0 and 1000m.

Both these approaches, the credal (probabilistic) and the propositional, contribute to the notion of epistemic accuracy, yet the ways we formalize them are different. Furthermore, probabilistic accuracy is more straightforward, since it's a function of the difference between the estimation of  $X$  and the truth value of  $X$ . Meanwhile, it's much less clear how we should think about truthlikeness from the propositional perspective.

At this stage, there is a rather vast amount of literature around truthlikeness and likewise around credences. However, these two approaches are kept separate in most cases. Attempts to bring them together [1, 6] by finding a formula that would represent an adequate epistemic loss for both probabilistic and propositional inaccuracies have been criticized more recently by Oddie [7]. I aim to show,

firstly, that purely propositional approaches have serious limitations and secondly, that Oddie's recent criticism of the unification doesn't work if we do some small changes to the probabilistic framework. I claim that we have reasons to give up on Proximity, which would render his proof toothless for reasons that Will discuss in the final section. Furthermore, I propose Basicity, which also blocks a step in Oddie's proof.

In the next section, I will talk about the likeness approach to truthlikeness, offering a very short exposition of the main approaches in the literature. Since I want to defend using the Briar score for measuring both probabilistic and content-

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<sup>1</sup> It may seem intuitive, though, that firstly, not all tautologies are equal and secondly, that some tautologies may be more valuable than some truths. For example, the law of non-contradiction can be seen as less valuable than, let's say, the law of large numbers or the fundamental theorem of calculus; likewise, a belief in the external world can be seen as less valuable than believing the fundamental theorem of calculus. Here I will not address this issue. Firstly, I assume logical omniscience for the rational agent. She will be more confident in any (obscure) mathematically proven result than in the existence of the external world. Secondly, my focus is on estimating the truth in an exhaustive set  $S$  of mutually exclusive propositions, where any estimation will be a disjunction of members of  $S$ . Here, the only tautology will be on a par with just asserting  $S$ .

<sup>2</sup> It is not very clear, though, if it is the case that weaker falsehoods are always worse. As I will say later, it's not clear whether a weaker false claim is further from the truth than a stronger one, if the minimum distance from the truth is the same for both.

wise truthlikeness, I will keep my attention on likeness because it allows us to compare atomic sentences between themselves. The third section will provide some examples that will test our intuitions about which falsehoods are closer to the truth. What I hope to show is that there is no single criterion for truthlikeness, no general formula that would do justice to all, or even most of the examples. All examples involve quantitative estimates. Cases involving the similarity between atomic sentences that are purely qualitative are outside of the scope of this paper, although I believe a unified (probabilistic and content-wise) solution can be achieved there, too. The fourth section explores some of the purely content-based, non-probabilistic formulas designed to measure the truthlikeness of quantitative atomic sentences. Each of them has important shortcomings that will be revealed. The fifth section will go through the Brier score, which is used for measuring probabilistic errors, and some criteria for the adequacy of such measuring scores. In the sixth section I will present Oddie's proof that using the Brier score for combining the probabilistic and content-based approaches fails because of inconsistencies between the criteria from section 5. I will show how the proof can be blocked by some restrictions over which kind of propositions can enter the Brier score. The last section will present some final remarks and potential changes to section 5's criteria that will further weaken Oddie's proof.

## 2 Truthlikeness: the likeness approach

There are three main ways to approach truthlikeness.

**Content:** Truthlikeness is determined by the content of the proposition. Despite the name, this is a syntactic approach developed by Popper [8], in which a true theory is closer to truth than another true theory when it is logically stronger. The truth content of a false theory is closer to the truth than the whole theory, due to its falsity content. It is a consequence of Popper's account that, given that increasing the truth content of a theory also increases its falsity content, if the falsity content brings a theory further from the truth, then all false theories are equally far from the truth. By comparing only their truth contents, a false theory is closer to the truth than another false theory when its truth content is larger (i.e. contains more true propositions; every theory contains at least some true propositions, namely the tautologies). Finally, another consequence is that no false theory can be closer to the truth than a true theory, no matter how weak the true one is and how much stronger the false one. Thus, any falsity will be farther from the truth than any tautology. There were many refinements of the content approach, but it is limited by not being able to compare the semantic contents of propositions: all we can do is to look at the logical strength and the size of the truth content.

**Consequence:** Because of the trivialization result of Tichy and Miller [11, 3], as I said above, whenever we add a true consequence to a false theory, we also add a false consequence. If we add a truth  $T$  to a false theory  $H$ , and  $H$  is closed under entailment, then we also add the conjunction of  $T$  and a false sentence in  $H$ , which will be false. This means that we can't increase the truth content of a false theory without increasing its falsity content, and by a similar reasoning we can't increase the falsity content without increasing the truth content (for a more detailed exposition, check the proof by Tichy and Miller). The consequence approach tries to pick a class of relevant consequences to avoid the situation in which all false theories are equally far from the truth. Many proposals have been made, but we still remain with the challenge that every false theory will be worse than any tautology. A solution was proposed by Schurz and Weingartner [10] that involves adding weights on relevant truths. Choosing weights is also part of my proposal, with the advantage of integrating probabilistic accuracy.

**Likeness:** This is the account of truthlikeness that I will be focused on from now. The advantage over the content approach is that it allows us to assign distances between propositions. It is also not of a purely syntactic nature: we can order atomic sentences, not just theories (by looking at the number of true and false components), and it is suitable for quantitative estimates of similarity, when the atomic sentences involve numbers. Thus, we can compare theories like 'Bob is 1.75m tall and has written three books' and 'Bob is 1.76m tall and has written four books' not just by the number of true propositions they have, but by how close to the truth each conjunct is (let's say Bob is 1.74m tall and has written two books - both theories are false, each containing two false atomic sentences, but the first one is closer to the truth).

Also, we are not talking here about closeness to 'the' truth (maybe capitalized), but closeness to contextual truths. In this regard we will not be able to compare the truthlikeness of propositions or theories in general, like asking whether the Copenhagen interpretation of quantum mechanics is more truthlike than Darwinian evolution or correctly guessing that Bob's height is 1.75m. The situations I am talking about are framework-dependent. Specifically, we have a set  $S$  of propositions to pick from, where  $S$  is exhaustive (so it contains the truth), and our hypotheses will be disjunctions over  $S$ .<sup>3</sup>

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<sup>3</sup> We may add the condition that only some propositions can be part of  $S$ . For example, we may want to keep  $S$  discrete.

### 3 Truthlikeness intuitions

Now let's look at some examples:

- 1) How many planets are in the solar system? (true answer: 8)
  - 1.1. There are 10 planets in the solar system.
  - 1.2. There are 7 planets in the solar system.
  - 1.3. There are 9 planets in the solar system.
- 2) How old is Bob? (true answer: 30)
  - 2.1. Bob is between 28 and 35.
  - 2.2. Bob is between 29 and 31.
- 3) How many people are in the house? (true answer: 10)
  - 3.1. There are more than 12.
  - 3.2. There are more than 13.
  - 3.3. There are fewer than 8.
  - 3.4. There are fewer than 8 or more than 10.
- 4) How many states are in the US? (true answer: 50)
  - 4.1. There are 49 states.
  - 4.2. There are 48 or 49 states.
  - 4.3. There are 51 states.
- 5) How many dollars did Bob get from his mother? (true answer: 8)
  - 5.1. Bob has 16 dollars.
  - 5.2. Bob has 4 dollars.
  - 5.3. Bob has 9 dollars
- 6) How many dollars did Bob get from his mother? (true answer: 100)
  - 6.1. Bob has 99 dollars.
  - 6.2. Bob has 50 dollars
  - 6.3. Bob has 200 dollars.
- 7) How many legs are there in the house (everyone is able-bodied)? (true answer: 90)
  - 7.1. There are 91 legs.
  - 7.2. There are 94 legs.
- 8) In which year did the event celebrated as Romania's national day occur? (true answer: 1918)
  - 8.1. 1859
  - 8.2. 1929

Now let's find some plausible orderings. For 1), 1.1 is farthest from the truth, while 1.2 and 1.3 are equally truthlike. Here we can say that the absolute distance gives us the ordering: the smaller the difference, the more truthlike the proposed answer is.

For 2), 2.1 is farther from the truth than 2.2. While both of them are true, 2.2 is logically stronger, since it contains fewer disjuncts.

For 3) it is less obvious how we should order the answers. 3.1 and 3.3 look, *prima facie*, equally close to the truth, because their closest values are at the same distance from 10. This would perhaps be the case if the range of answers were infinite in both directions, but we can't have anything less than 0 in this case. 3.3 seems logically stronger than 3.1. 3.4 seems to be the worst: even though it gets as close to 10 as 3.1 and 3.3, it is logically weaker than both of them.

For 4), 4.1 and 4.3 are equally truthlike, being at a distance of 1 from 50. But how do we compare them to 4.2? We may be tempted to say they are closer to truth than 4.2 because they are logically stronger, and 4.2 has no advantage, since its closest value is also at a distance of 1 from 50.<sup>4</sup>

For 5), it may be plausible to say that 5.1 and 5.2 are on par, because they have fractional similarity. '32 dollars', when the true answer is 16, would be as bad as 16 is when the answer is 8. It is not easy to figure out how to compare them to 5.3.

For 6) the interesting question is whether 6.1 is closer to the truth than 5.3 was for 5), given that we are talking about larger numbers in 6). Arguably, it would be more impressive to be right when the numbers are very large (the larger, the more impressive), although it may be due to our psychological difficulty with thinking about larger numbers. Being wrong by 1 looks far more impressive when we are estimating a number with three or four digits than a single-digit number.

For 7), it's clear that the background knowledge that able-bodied humans have two legs makes 7.2 much closer to the truth than 91, even though the absolute difference is smaller for 7.1.

For 8), although 1929 is closer in time to 1918, it is plausible to think that someone would be more accurate in guessing 1859, which is another important year in Romanian history (the union of Wallachia and Moldova happened at January 24th 1859, and could have been picked as the national day - at least, a more plausible proposal than any day in 1929, when nothing historically important, on that scale, happened).

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<sup>4</sup> It is also interesting to ask whether 4.2 would be closer to truth if 4.1 and 4.3 would answer with 48 and 52, respectively.

#### 4 The propositional account

Kuipers [2], in his 2024 paper, gives his own range of examples, then tries to find some distance and similarity measures to accommodate them. My criticism is twofold. Firstly, he sets some limiting conditions, like accepting only quantitative theses with positive real numbers, which by default render some of his measures useless for cases in which  $S$  has propositions positing negative numbers. Secondly, he seems to use the examples as an exhaustive list of cases that have to be accounted for<sup>5</sup>, which doesn't seem to cover a large number of cases.

Kuipers [2] offers some generally accepted constraints on truthlikeness measures.

**Normalized Range:**  $0 \leq d(x, y) \leq 1$  and  $0 \leq s(x, y) \leq 1$ , where  $d(x, y)$  is the distance between  $x$  and  $y$  and  $s(x, y) = 1 - d(x, y)$  is the similarity between  $x$  and  $y$ .

**Triangle Inequality:**  $d(x, z) \leq d(x, y) + d(y, z)$  in order for the distance to be metric.

**Unique Target:**  $TL(x) = 1$  if  $x = t$ , where  $TL(x)$  is the truthlikeness of  $x$ , and  $t$  is the true proposition.

**Symmetry:**  $d(x, y) = d(y, x)$  and  $s(x, y) = s(y, x)$ .

**Conceptual Continuity:**  $TL(x, t) = 1 - d(x, t)$ .

He examines the measures proposed by Tichy-Oddie and Niiniluoto, together with his own:

i) The Tichy-Oddie formula [5, 12]:

$$TL_{TO}(X, t) = 1 - \left( \frac{\sum_{x \in X} d(x, t)}{c(X)} \right) = 1 - d_{av}(X, t) = TL_{av}(X, t, c(X))$$

Where  $c(S)$  is the cardinality of  $S$ . Its main problem is that logical strength and truthlikeness are not positively correlated, because the part that accounts for distance from truth is calculated as the average distance.

ii) Niiniluoto's measure [4]:

$$TL_N(X, t) = 1 - \left[ \gamma d_{\min}(X, t) + \gamma' \left( \frac{\sum_{x \in X} d(x, t)}{\sum_{x \in U} d(x, t)} \right) \right] = 1 - [\gamma d_{\min}(X, t) + \gamma' d_{av}(X, t) s^*(X, U)] d_{av}(U, t)$$

Where  $0 \leq \gamma, \gamma' \leq 1$ . As it can be verified, it lacks Conceptual Continuity, because it can't be reduced to  $1 - d(x, t)$ .

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<sup>5</sup> He doesn't explicitly say that his list of examples, used to provide intuition, is exhaustive, but he evaluates his 5 proposed measures using the examples without providing grounds for more flexibility. For example, none of his measures would give a satisfying answer to the 'number of legs' question.

Kuipers' proposals are:

iii) The minimum distance truthlikeness, which takes the distance from the closest member of  $X$  to  $t$  (which is 0 if  $X$  is true):

$$TL_{min}(X, t) = (1 - d_{min}(X, t)) \times s^*(c(X), c(\{t\})) = \frac{1 - d_{min}(X, t)}{c(X)} = \frac{s_{max}(X, t)}{c(X)}$$

iv) The average distance truthlikeness, which takes the average distance from each member of  $X$  to  $t$ :

$$TL_{av}(X, t) = (1 - d_{av}(X, t)) \times s^*(c(X), c(\{t\})) = \frac{1 - d_{av}(X, t)}{c(X)} = \frac{s_{av}(X, t)}{c(X)}$$

v) The weighted distance truthlikeness:

$$TL_w(X, t) = wTL_{min}(X, t) + (1 - w)TL_{av}(X, t)$$

The minimum distance truthlikeness doesn't distinguish between disjunctions of equal logical strength where the minimum distance is the same for both, but the average or maximum distances are different. So, for example, "Bob's age is 13, 14, or 15" is as close to the truth (16) as "Bob's age is 17, 20, or 30". The average and weighted measures are not sensitive to the size of  $X$ .

Besides these truthlikeness measures Kuipers proposes some distance measures:

$$d_{nad}(x, y) = \frac{|x - y|}{|x - y| + 1}$$

This is the normalized average distance and is invariant on translation, meaning it returns the same distance for the same numerical intervals between numbers, however large. It varies on ratios, meaning it returns different distances for the same ratios (like those between 8 and 16 or 16 and 32).

$$d^*(x, y) = \frac{\max(x, y) - \min(x, y)}{\max(x, y)} f$$

Unlike the 'standard' distance measure above, the fractional distance is invariant on scale and varies on translation, accommodating the intuition that similar scale differences should be equally truthlike, while similar absolute differences between small or large numbers should receive different distance scores.

As I said above, these distance measure are not adequate for sets containing negative numbers and don't address peculiar cases like in my examples 7) and 9).

## 5 The credal account

We can see probabilities, or degrees of belief, as estimations of truth values. Ideally, we would be able to fully assent to all true propositions, deny all falsities, and also suspend judgment where there is no fact of the matter. However, the world is far from ideal. In many cases, we don't know which propositions are true, so we can only take more or less educated guesses. In such a scenario, risking to assign only categorical truth values can lead us to fully accept falsities and fully deny truths. If we want to avoid this scenario we can use probabilities to estimate the truth of a proposition.

Here is the formula of the Brier Score:

$$\text{Brier}(\mathbf{X}, \mathbf{P}) = \sum_{i=1}^n \lambda_i (X_i - P_i)^2 \quad (1)$$

where  $\mathbf{X}$  is a partition of a set of exhaustive and mutually exclusive propositions,  $\mathbf{P}$  is a distribution of real numbers, and  $\lambda$  is a set of weights. We observe that when the sum of the squared errors is larger, in order to minimize the Brier score, our estimations need to be as close as possible to the actual truth values.

One of the best features of the Brier score is that it supports Dutch Book arguments. That means that if we want to lower the expected value of the Brier score as much as possible, we must comply with the probability laws. Otherwise, if we choose a probability assignment that violates the probability laws, there will always be another assignment that obeys them and will return a lower score (that is, a lower loss), no matter which proposition is true. A score that satisfies these criteria is called a proper score. So, the Brier scoring is proper [9].

The Brier score has some properties that make up the 'Core' of probabilism according to Oddie. Then, he argues, any measure that adequately captures probabilism and, thus, satisfies the 'Core' (such as Brier), will violate some important properties of truthlikeness, rendering the combination of the two approaches unfruitful. The properties of the 'Core' are the following:

**Truth Directedness:** For any distance function  $d(v, p)$ , where  $p$  is a probability and  $v \in \{0, 1\}$ ,  $d$  is strictly increasing if  $v = 0$  and strictly decreasing if  $v = 1$ . That is, the distance increases as the probability diverges from the truth value.

**Symmetry:** for any distance function  $d(v, p)$ ,  $d(1, p) = d(0, 1 - p)$ . This means that misidentifying a truth by a degree is punished the same way as misidentifying a falsehood by the same degree.

**Boundedness:** For any distance function  $d$ ,  $d = 0$  if and only if  $|v - p| = 0$ , and  $d = 1$  if and only if  $|v - p| = 1$ .

**Additivity:**  $Loss(P^i) = \sum_j \lambda_j d(X_j, P_j)$ , meaning that the total loss is a weighted sum of the local losses. Note that Brier is just a particular type of Loss.

**Admissibility:** For all  $j$ ,  $\lambda_j \geq 0$ ,  $\sum_i \lambda_i = 1$  and  $\lambda_{Taut} < 1$ . The sum of the weights has to be 1, all weights have to be non-negative and the weight of the tautology has to be smaller than 1. As I will show later, this last criterion will be dropped.

**Strong Propriety:** For any probability assignments P and Q,  $E(Loss_P(P)) < E(Loss_P(Q))$ , where  $Loss_P(Q) = \sum_i P_i d(X_i, Q_i)$ . That is, P's estimation of the loss associated with Q is the formula that results from P giving the weights of the loss associated with Q. Also, P is its own best estimator, minimizing its expected loss. Additionally, to the 'Core', we have Weak Proximity, Strict Proximity, and Substitution. These are the criteria of the propositional approach to accuracy that can be transferred to the credal approach. Let  $X_i$  be the true proposition in a partition X, such that we can order the members of X by their proximity to the truth.

**Weak Proximity:** Let  $X_C \subseteq X$  be the set of the propositions in X closest to the Truth.  $X_C^{-T} \subseteq X^{-T}$  denote the subset of the false propositions in X that are closest to the truth. If  $X_k >^i X_j$ , then  $X_k \geq^i X_k \vee X_j$ . This is saying that restricting a disjunction to the members closest to the truth cannot increase the inaccuracy.

**Substitution:** Let D be a disjunction formed out of members of X, and  $D^{k/j}$  is the same disjunction where  $X_j$  is replaced by  $X_k$ . Then if  $X_k >^i X_j$ , then  $D^{k/j} >^i D$  and if  $X_k =^i X_j$ , then  $D^{k/j} =^i D$ . This means that if we replace a member of a disjunction with another member closer to the truth, we decrease the inaccuracy of the disjunction.

**Strong Proximity:** results from Weak Proximity and Substitution. Now let  $X_k \geq^i X_k \vee X_j$ . If  $X_j >^i X_h$ , then by Substitution  $(X_k \vee X_j) > (X_k \vee X_h)$ . By Substitution and Weak Proximity,  $X_k >^i X_k \vee X_h$ .

We have to add that if  $D^k \geq^i D^j$ , then  $Loss(D^k, P^k) \leq Loss(D^j, P^j)$ , where  $P^i$  is the probability assignment that gives equal positive probabilities to the members of  $D^i$  and 0 to the others (or, as Oddie prefers, it assigns equal positive probabilities to propositions compatible with  $D^i$  and 0 to the others; These are not equivalent, as I will show in a short time).

Now, an important question is how we choose the weights. If we keep the weights uniform, we will be unable to account for truthlikeness. It is possible for two agents to assign the same probabilities to the true proposition and different probabilities to the false ones, but following the same distribution. Consider the example of Bob's age. Let P be the following assignment: [28: 0.1, 29: 0.2, 30: 0.4, 31: 0.2, 32: 0.1] and let Q be [28: 0.2, 29: 0.1, 30: 0.4, 31: 0.1, 32: 0.2].

Since Bob is 30, if the weights are uniform, P and Q will return equal losses. If we want to incorporate truthlikeness into our account we have to explain why an agent choosing P will be better vindicated than the one choosing Q. One natural way of differentiating between them is to use non-uniform weightings. One way of doing that is choosing world-independent weights, namely a set of weights that applies across all scenarios, no matter which proposition is true. However, this wouldn't help us, since it wouldn't be clear how to assign the weights based on closeness. Indeed, closeness to truth can only be taken into account if the truth is determined. Since the Loss score is construed as the agent's accuracy assessment, its value is determined at the Vindication Time, namely only after the truth is known. Only then can the agent judge how closely she probabilistically estimated the truth. I claim this also applies to truthlikeness. Only after the truth is known, can the agent see which propositions were contentwise closer to the truth. Thus, I propose the criterion of Vindication for world-dependent weights:

**Vindication:** The weights  $\lambda$  are determined in the light of the discovered truth. Thus, for every  $X_i$  in  $X$ , we will have a weighting  $\lambda_i$  that will also minimize the Loss. Note that there are two different minimizations, the probabilistic and the content-centered one.

## 6 Oddie's Incompatibility Proof

Oddie also proposes a criterion of admissibility for world-dependent weights: if  $X_k =^i X_j$ , then  $\lambda_k =^i \lambda_j$  and if  $X_k >^i X_j$ , then  $\lambda_k >^i \lambda_j$ . The motivation given by Oddie is that drastically underestimating the probability of a proposition closer to the truth should be punished more severely than underestimating the probability of a proposition farther from the truth. However, we can also say that overestimating the probability of a proposition further from the truth should be punished more severely than overestimating the probability of a proposition closer to the truth. And given symmetry, overestimation and underestimation are both represented as distances, where no one type of error necessarily brings a larger cost. Hence I only accept the first part of Oddie's admissibility criterion: if  $X_k =^i X_j$ , then  $\lambda_k =^i \lambda_j$ .

Oddie proves that given world-dependent weighting, if we accept Propriety then we are forced to reject Strict Proximity. He proposes an example where  $X_1$  is true,  $X_2$  and  $X_3$  are equally close to  $X_1$  and  $X_4$  is farther from  $X_1$  than both  $X_2$  and  $X_3$ <sup>6</sup>. He then builds composed propositions:

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<sup>6</sup> He offers the example where there are two propositions, h (hot) and r (rainy), and four conjunctions, X1: h & r, X2: h & -r, X3: -h & r and X4: -h & -r. The distance between any two conjunctions is given by the number of common conjuncts. We can also construct X5, which is X1 V X2 and so on.

$$\begin{aligned}
X_5 &= X_1 \vee X_2 \\
X_6 &= X_1 \vee X_3 \\
X_7 &= X_1 \vee X_4 \\
X_8 &= X_2 \vee X_3 \\
X_9 &= X_2 \vee X_4 \\
X_{10} &= X_3 \vee X_4 \\
X_{11} &= X_1 \vee X_2 \vee X_3 \\
X_{12} &= X_1 \vee X_2 \vee X_4 \\
X_{13} &= X_1 \vee X_3 \vee X_4 \\
X_{14} &= X_2 \vee X_3 \vee X_4 \\
X_{15} &= X_1 \vee X_2 \vee X_3 \vee X_4
\end{aligned}$$

Oddie includes, in the Loss formulas, the local losses of each disjunction that can be formed (not just the basic  $X_1$ ,  $X_2$ ,  $X_3$ , and  $X_4$ ). Thus, he computes the losses in the following manner:

$$\begin{aligned}
\text{Loss}(X_2) &= \lambda_1 + \lambda_2 + \lambda_6 + \lambda_7 + \lambda_8 + \lambda_9 + \lambda_{13} + \lambda_{14} \\
\text{Loss}(X_8) &= \lambda_1 + \dot{\lambda}_2 + \dot{\lambda}_3 + \dot{\lambda}_5 + \dot{\lambda}_6 + \dot{\lambda}_7 + \dot{\lambda}_8 + \dot{\lambda}_9 + \dot{\lambda}_{10} + \dot{\lambda}_{12} + \dot{\lambda}_{13} + \lambda_{14}
\end{aligned}$$

It can be proven that if  $p = 0.5$ , then the distance  $i = d(v, p) < 0.5$  for any proper score that satisfies the 'Core'[7]. Then, by the admissibility of world-dependent weighting,  $\lambda_2 = \lambda_3$ ,  $\lambda_5 = \lambda_6$ ,  $\lambda_9 = \lambda_{10}$ , and  $\lambda_{12} = \lambda_{13}$ . By Weak Proximity, since  $X_2 >^1 (X_2 \vee X_3)$ , which is equivalent to  $X_2 >^1 X_8$ , then  $\text{Loss}(P^2) \leq \text{Loss}(P^8)$ . But this gives the following result:  $(1 - 2i)\lambda_2 + (1 - 2i)\lambda_5 + (1 - 2i)\lambda_9 + (1 - 2i)\lambda_{12} \leq 0$ , which is possible only when  $\lambda_2 = \lambda_3 = \lambda_5 = \lambda_6 = \lambda_9 = \lambda_{10} = \lambda_{12} = \lambda_{13} = 0$ , because  $i < 0.5$  and all weights are non-negative. But by Substitution,  $(X_1 \vee X_2) >^1 (X_2 \vee X_4)$ , so  $X^6 >^1 X^9$ . But, by world-dependent admissibility (as Oddie defined it),  $\lambda_5 > \lambda_9$ , which contradicts the previous result.

I think the proof can be blocked if we relax the world-dependent admissibility criterion and if we restrict what propositions can enter the Loss formula. I propose Basicity:

**Basicity:**  $\sum_i v(X_i) = 1$ . This means that the members of the sum have to be disjoint.

As we can see, Basicity blocks the last step of the proof. Moreover, if we choose a truthlikeness ordering we can impose restrictions on the weights. By checking the restrictions imposed by every ordering we want to accept, we can check if they are mutually compatible.

So, by accepting Basicity, the losses are computed as follows:

$$\begin{aligned}
 \text{Loss}(X_1) &= 0 \\
 \text{Loss}(X_2) &= \lambda_1 + \lambda_2 \\
 \text{Loss}(X_3) &= \lambda_1 + \lambda_3 \\
 \text{Loss}(X_4) &= \lambda_1 + \lambda_4 \\
 \text{Loss}(X_5) &= i\lambda_1 + i\lambda_2 \\
 \text{Loss}(X_6) &= i\lambda_1 + i\lambda_3 \\
 \text{Loss}(X_7) &= i\lambda_1 + i\lambda_4 \\
 \text{Loss}(X_8) &= \lambda_1 + i\lambda_2 + i\lambda_3 \\
 \text{Loss}(X_9) &= \lambda_1 + i\lambda_2 + i\lambda_4 \\
 \text{Loss}(X_{10}) &= \lambda_1 + i\lambda_3 + i\lambda_4 \\
 \text{Loss}(X_{11}) &= j\lambda_1 + k\lambda_2 + k\lambda_3 \\
 \text{Loss}(X_{12}) &= j\lambda_1 + k\lambda_2 + k\lambda_4 \\
 \text{Loss}(X_{13}) &= j\lambda_1 + k\lambda_3 + k\lambda_4 \\
 \text{Loss}(X_{14}) &= \lambda_1 + k\lambda_2 + k\lambda_3 + k\lambda_4 \\
 \text{Loss}(X_{15}) &= i\lambda_1 + m\lambda_2 + m\lambda_3 + m\lambda_4
 \end{aligned}$$

Let  $\text{Loss}_i$  designate  $\text{Loss}(X_i)$ . By my lights, presupposing a translation-invariant truthlikeness metric and accepting the 'Core', we want these orderings to hold:

$$\begin{aligned}
 \text{Loss}_1 < \text{Loss}_2 = \text{Loss}_3 < \text{Loss}_4 & \quad \text{Loss}_1 < \text{Loss}_5 = \text{Loss}_6 < \text{Loss}_7 & \quad \text{Loss}_2 < \\
 \text{Loss}_8 < \text{Loss}_9 = \text{Loss}_{10} & \quad \text{Loss}_{11} < \text{Loss}_{12} = \text{Loss}_{13} < \text{Loss}_{14}
 \end{aligned}$$

As we can easily check, these relations hold as long as  $\lambda_2 = \lambda_3 = 0 < \lambda_4$ , given that for any  $X_j$  with  $j \geq 5$ , the probabilities are divided equally between the members of the disjunction. Thus, if  $\text{Loss} = \text{Brier}$ ,  $i = 0.25$ .

## 7 Final Remarks

The Unified Approach has some obvious advantages. In addition to allowing credences to inform our assessment of someone's epistemic accuracy, it offers a more flexible account of truthlikeness. By using the weights to retroactively evaluate our closeness to the truth we can satisfy more easily the examples that I gave earlier. If we only try to find some unique measure to capture all our intuitions about truthlikeness, we are doomed to fail. The other solution would be to find many separate measures, for every different intuition we form each time we see new examples. The more elegant solution, in my opinion, is to approach these

differences by changing the weights in the Loss score (Brier, let's say). We can tweak the weights to privilege any similarity ordering we want and, furthermore, we know how to do that once we establish the desired ordering, as I've shown in the previous section.

Oddie's proof also suffers from one additional problem, although I acknowledge that this one heavily depends on our intuitions about particular cases. For example, it's not always obvious that seemingly plausible constraints like Strict Proximity hold. Let's return to the 'number of legs' case, where there are 90 legs in the building. Now let's imagine that all persons in the building but one have two legs and the remaining person either has two legs or has just one leg. In this case, we can say that  $X_{91} >^{90} X_{92}$ , so  $(X_{90} \vee X_{91}) >^{90} (X_{90} \vee X_{92})$ . On the other hand, both guesses imply that there are 45 persons with two legs and an additional person which has one leg according to Guess 1 and two legs according to Guess

2. Given the scenario, it's not clear whether the simple absolute distance metric should imply Strict Proximity here.

We can now ask whether the introduction of Basicity is not too restrictive. I think it's not. We can't introduce into the Brier score propositions whose probabilities sum to a number higher than one. This merely implies that if I think  $X_1$  is true, or has a higher probability, I will not also put a probability on a disjunction containing  $X_1$ . But this doesn't mean I can't compare the claims  $X_1$  or a disjunction containing it. Let's say I think  $X_1$  is true, when in fact  $X_2$  is true. I can put a bet on  $X_1$  and introduce it into the Brier score with the probability 0.9. Then the score will compute my loss. I can then put another bet on  $X_1 \vee X_2$ , which means I put 0.45 on  $X_1$  and 0.45 on  $X_2$ , and then compute the loss, thus seeing which returns a smaller loss. Interesting cases are those of the type 'There are more than 8 planets in the Solar System', which are infinite disjunctions. A bet on such a disjunction can assume unequal probabilities on its members, to obey the probability laws (additivity!). This may not represent someone's belief, who is indifferent as to how exactly many planets there are, and who only believes there are more than eight. Such a person would presumably have probabilistically incoherent beliefs and would be the victim of a Dutch Book argument (or, in de pragmatized terms, their epistemic loss is guaranteed). However, these difficulties and others similar also affect the measures I presented in the fourth section, so the unified approach keeps its advantage. Finally, its main appeal resides in the intuition that precision can be seen on different axes: content-similarity and probabilistic closeness are a core component by which we appreciate how close to the truth a belief is.

Future work on this topic would involve finding adequate metrics for non-numerical truthlikeness and, on the other hand, should cover non-discrete probability assignments for non-discrete scenarios.

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