

A Formal Reconstruction of Interpretive Scarcity

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ABSTRACT. The theory of hermeneutical injustice analyzes the wrongs suffered due to deficiencies in the shared interpretive resources of a society. A hermeneutical injustice is diagnosed when individuals of a social type are persistently and systematically hindered to understand a significant part of their experiences due to such interpretive scarcity. The theory uncovers a number of important phenomena, such as the relationship between symmetry in ignorance and asymmetry in consequence. In this paper, we develop a reconstruction of deficient interpretive resources to capture such phenomena more precisely and investigate novel implications of the existing theoretical account. The formal reconstruction is set in the framework of imprecise probabilities, a generalization of classical probability which represents an additional quality of uncertainty or ignorance. This quality we deploy to reconstruct a kind of interpretive deficiency and analyze it for both normative force and explanatory power. We conclude that (1) symmetrical ignorance is consistent with asymmetric epistemic action even under shared values and (2) insufficiently developed interpretive re-sources, as represented by imprecise probabilities, can induce a dispersion of credal states. The former is a contribution to the established theoretical discourse, whereas the latter is a novel observation; in tandem, these conclusions illustrate the continuity of the reconstruction with existing work as well as its surplus value in aiding discovery.

Keywords: Epistemic Injustice, Hermeneutical Injustice, Imprecise Probabilities, Bayesian Epistemology, Psychiatric Diagnosis

1. Introduction

Human language and the underlying conceptual structures are inevitably ambiguous, vague, and hence require continuous interpretation in their use to make sense of and communicate human experiences. This essential openness

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enables creativity and adaption to ever-changing conditions of human existence; however, it is also the source of misunderstanding, failed communication and prone to abuse by the powerful: They can use their social position, intentionally or as a structural condition, to manipulate the negotiation for meaning in their interest.

In recent discussions, this latter phenomenon is often understood as a form of hermeneutical injustice. The term denotes a gap or shortcoming in the shared interpretive resources disadvantaging members of a social type in their ability to understand and communicate a substantial part of their experiences (Fricker (2007, ch. 7), Medina (2017)).

While the general idea of hermeneutical injustice is compelling enough, the details, and hence the identification, comparison and evaluation of particular cases, remain somewhat open to interpretation – an irony maybe not lost on the scholars engaged in the conversation. As noted, we should not expect any linguistic or conceptual resources to be free of ambiguity, closed for interpretation. Nevertheless, if the theory of hermeneutical injustice is itself meant to provide a powerful interpretive resource, we have to partake in the negotiation of its content.

In this paper, we investigate the utility of the framework of imprecise probabilities as an analytic tool in the theory of hermeneutical injustice. Unlike classical probability, imprecise probabilities allow not only for the uncertainty inherent in a probabilistic reconstruction of belief, but an additional dimension of ignorance about those very probabilistic judgments (see Wheeler, 2022, for an introduction). This more expressive theory extends the conceptual space to leave open cognitive gaps – which, as we will argue, are suitable to reconstruct interpretive gaps described by the theory of hermeneutical injustice.

In particular, we

1. offer a reconstruction of a subset of interpretive deficiencies in terms of imprecise probabilities to then
2. recover the relationship between shared ignorance and divergent epistemic as well as practical consequences and
3. show how resources deficient in this manner can, according to the theory of imprecise probabilities, render evidence inert or even have it disperse further the credal state of its recipient.

Before we turn to this investigation, a word of caution is in order: Formal methods come with a great deal of abstraction and idealization, and imprecise probability theory is not an exception. We should therefore be aware that any such account covers only an aspect of the phenomenon. The purpose is to improve our understanding of this aspect, and to thereby contribute to a broader intellectual

project – not to replace, but to complement. The following analysis ought to be understood not as one of the whole complex phenomenon of hermeneutical injustice, but the particular subset described.

The investigation proceeds as follows: It starts with a very brief introduction to the theory of imprecise probabilities, eschewing some of its more subtle technical detail in service of the application at hand. Then, Section 3.1 introduces the theory of hermeneutical injustice and the underlying notion of deficient interpretive resources and maps it onto the framework of imprecise probabilities. Section 3.2 reconstructs the epistemic symmetry of hermeneutical gaps and shows the conditions under which its asymmetric consequences arise for members of different social types. Section 3.3 follows the implications of the possibility of dilation to a dispersive effect of hermeneutical gaps on credal states. The paper closes with a summary and short outlook in Section 4.

Imprecise Probabilities

2.1 Basic Idea

We can rather intuitively approach imprecise probabilities as a generalization of classical probability. Consider first the following case: An agent holds $p(F) = 0.4$ and $p(E) = 0.3$.¹ However, the agent is ignorant of the probability of the joint event $F \cap E$. From the credal states regarding E and F , the agent can narrow down the possible probabilities for the joint event to $[0, 0.3]$ – any probability within this interval would be consistent with everything the agent knows about these two events.²

Imprecise probabilities generalize this basic idea of ignorance, such that an agent holds a set P of probability distributions instead of a single distribution p .³ An important, if not unique characterization of a credal set is given by its lower⁴

¹ We interpret probabilities as graded beliefs or credences. Unless noted otherwise, we assume that credences obey the axioms of probability and the agents follow conditionalization for learning. See e. g. citetjoyce2011development for a historical introduction to subjective probability or Bovens and Hartmann (2003) for a systematic application to normative epistemology.

² It is possible to discuss imprecise probabilities in purely mathematical terms, as Pedersen and Wheeler (2014) demonstrate, leaving a plurality of philosophical interpretations. For our purposes, it is most appropriate to adapt the interpretation of indeterminate credal states.

³ In principle, the credal set can be topologically varied, but as much of the literature, we assume P is convex.

⁴ For a more detailed discussion of the technical aspects, see Wheeler (2022); Haenni et al. (2010).

$$\underline{p}(E) = \inf \{p(E) : p \in \mathbb{P}\}$$

and upper

$$\bar{p}(E) = \sup \{p(E) : p \in \mathbb{P}\}$$

probability; we rely on Wheeler (2022) and Haenni et al. (2010) for the technical background.

Upper and lower probabilities will be used below to guide imprecise decision-making.

We consider two possible sources of the kind of ignorance expressed in the imprecise framework. The most obvious case is that of a simple lack of knowledge. To commit oneself to a particular distribution is, in such a case, simply not rationally warranted.

The traditional counterpoint to this idea is objective Bayesianism. As, for example, Joyce (2011) lays out, objective Bayesians argue for the existence of a uniquely rational credal state relative to a set of evidence. This view can be framed primarily in terms of the choice of priors, that is, the initial credal state prior to evidence; but more broadly understood, objective Bayesians are committed both to a uniquely rational prior and position in the space delineated by empirical constraints (Williamson, 2007). The possibility of imprecise probabilities undermines, however, the standard picture. Under an imprecise framework, we are no longer asking which of an infinite number of distributions to choose, and trying to identify a non-arbitrary choice. Instead we can simply retain the whole space of possible distributions admissible under the constraints of our rationality and given evidence.

As a side note, this reconstruction of ignorance is sometimes used to give a probabilistic accounting of suspension of judgment. For example, if an agent has to judge the probability of a coin landing heads, no value seems to express a suspension of judgment. The most natural candidate, $p(H) = 0.5$ is as much a judgment as any other value of p , not a suspension. Imprecise probabilities allow the agent to hold instead the entirely non-committal credal state $p \in [0, 1]$, which seems to be a proper reconstruction of full suspension. Even more, the framework allows the agent's credal state to be a smaller interval, expressing suspension over a narrower field of equipossible probabilities.

A second reason for imprecision has been suggested by Elkin and Wheeler (2018). They look at the problem of peer disagreement assuming individuals hold precise credal states. Peer disagreement is an obstinate problem, as the peer status imposes a requirement to take others' beliefs into account, while the further assumptions of the disagreement are meant to restrict any grounds for doing so.⁵ We will not go

⁵ See Kelly (2011) for a discussion.

too much into the details, as peer disagreement is not much of a concern in hermeneutical injustice, but take an important idea from this particular approach: Elkin & Wheeler suggest to resolve peer disagreement by constructing the convex closure of the individual credal states of the disagreeing parties, and adopt the resulting imprecise probability as the rational credal state of the group. This process can be transferred to scenarios where the agents do not consider each other as peers, potentially leading to different weights of the individual credal states in the aggregation process. But it nevertheless gives us an account of how we end up with imprecision in a social setting: If it is impossible, for instance due to constraints on evidence sharing, to converge on a shared, single credence, the collective credal state is expressible as an imprecise probability. This understanding importantly aligns with the use of imprecise probabilities to reconstruct shared interpretive resources. So, while individual agents may also be more adequately represented as holding indeterminate credal states, the crucial point is that they need to operate on imprecise probabilities representing shared resources when engaging with other epistemic agents.

So, to summarize, imprecise probability gives us a tool to represent credal states which are not fully determinate. With the basic conception set up, we turn to two more particular issues which will be important in the reconstruction of hermeneutical gaps: How agents can make rational decisions under imprecise probabilities, and how evidence may inevitably dilate an agent's credal state when it is already somewhat indeterminate.

2.2 Imprecise Decision-Making

Ultimately, we care about beliefs, concepts and the like because of their practical consequences. A common way of translating a probabilistically reconstructed credal state into action is by means of a betting model. (Leitgeb and Pettigrew, 2010) Let X be a random variable; a bet on X is a function which associates a payoff with every possible value X could take. For instance, consider X as the outcome of a single coin toss, where the coin can only land heads (H) or tails (T). A bet could then take the values $b(H) = 50$ and $b(T) = -50$. This instantiates a bet which costs 50 to acquire, and pays nothing on tails and 100 back on heads.

A few points of clarification: We did not specify a unit for the values of the bet, and generally, it should not be assumed this is equivalent to money. The payoff, whatever realizes it, is such that larger numbers are always proportionally preferred to smaller ones. There are limits to the plausibility of this assumption, e.g. if we were looking at payoffs going to infinity. However, in the model world we are looking at, it seems an innocuous idealization to assume such a unit.

Now we need to relate those bets, which are a stylized representation of risky actions, to credal states. Let us assume an agent who believes the coin is biased in favor of tails, e.g. $p(\text{Tails}) = 0.6$. Such an agent would not buy the bet we constructed above, because it has negative expectation given their credal state. On the other hand, the agent would be happy to sell such a bet, as it has positive expectation for them to do so. There is a credal state where the willingness to buy and sell uniquely meet, in our case at 0.5. Thus, we can relate a particular credal state with a disposition to bet, and thus, engage in action with risky outcomes related to the credal state.

Note, that in order to make this connection, we have to assume an agent rational in a fairly narrow sense: The agent's credal states must satisfy the axioms of probability, and they must act by maximizing expected payoffs. The standard formal device to support these conditions are constructions of combinations of bets which result in sure loss or sure loss in expectation, so called Dutch books (see Vineberg, 2022, for an overview). Violations of the axioms of probability are irrational, according to this line of argumentation, because they expose an agent to sure loss. Similar, though more controversial arguments apply to the prescriptions for evidence assimilation, in our case conditionalization. Optimization itself is in some contexts considered a problematic assumption, and we will not offer a full defense here. The assumption seems admissible in the current context as it drives the agent to believe and act in accordance with their ends, while there is no straightforward reason to settle for a weaker principle such as satisficing in the current context.

We remain agnostic as to whether this understanding of rationality is the best explication of the concept. But it covers an important aspect of rational action in the face of risky options. It ties an agent's dispositions for action to their belief, and whether one assumes rationality reduces to this kind of relationship or not, an agent who pays no mind to how their beliefs relate to their actions either does not hold those beliefs or is irrational.

So far, this only applies to classical probabilities. But we can with relative ease extend it to imprecise ones. Let us assume an agent who holds the credal state $p(\text{Tails}) \in [0.4, 0.6]$. This agent would buy a bet paying 100 if the coin comes up tails for up to 40. If they paid more, their credal state does not entail a non/negative expected outcome. Vice versa, the agent would sell such a bet for at least 60, as once again, if they sold it for less, their expectation could be negative according to their credal state. In general, the selling price for a bet with this structure depends on the upper probability, the buying price on the lower as defined above.

This is a marked difference to the classical case, which represents the decision-theoretic counterpart to suspension of judgment: There is now a subset of risky actions for which the agent refuses to take the certain loss nor wants to exchange

it for the uncertain gain. We will offer an interpretation of this rather technical fact in terms of hermeneutical gaps below.

This decision procedure is also suggested by Elkin and Wheeler (2018) in their approach to peer disagreement. In essence, they suggest a partial suspension of judgment, downstream of which, on the decision model we presented, is a refraining from certain risky actions. The resolution, or maybe better, management or negotiation, of disagreement in a social group can therefore lead to an area of ignorance that translates into an unwillingness or inability to act in certain ways – which we will interpret as a reconstruction of a hermeneutical lacuna. The link to action will then be crucial to determine if a particular gap induces injustice. But before we can turn to our interpretation, we need to lay out a very particular consequence of allowing for indeterminate credal states: Dilation.

2.3 Dilation

In a classical framework of graded belief, i.e. a reconstruction of degrees of belief combined with Bayesian conditionalization, more information is never epistemically detrimental (Pedersen and Wheeler, 2014). Disturbingly to some, this is no longer true in the world of imprecise probabilities. Here, so-called dilation can take place.

Definition 1 (Dilation) *Given two random variables X and Y and credal set \mathbb{P} , X dilates Y if regardless of what value X takes, learning it increases the uncertainty about Y in \mathbb{P} .*

By increasing uncertainty, we here mean that the credal set of Y prior to conditionalization on the outcome of X is contained in the posterior credal set for each value of X .

Putting it a bit more informally, in a case of dilation we actually lose knowledge by gathering evidence. Let us make this concrete, first with a rather artificial, but very clear example borrowed from Pedersen and Wheeler (2014).

Example 1 (Dilating Coin Toss) *An agent holds a credal state on the outcome of two coin tosses. They believe the first coin to be fair, but the second coin to be correlated to the first in some manner – but this correlation could be anything from perfect correlation to complete anti-correlation. So we have $p(T_1) = 0.5$ and $p(T_2|T_1) \in [0, 1]$. Now by learning the result of the second toss, regardless if it is heads or tails, the agent has to revise their credence in $p(T_1)$ to cover the whole unit interval. Hence, regardless of the outcome of the second coin toss, learning it will have the agent lose knowledge.*

Of course, the agent also gains knowledge about the second toss itself – the point is that overall knowledge does not monotonically increase with new information being learned.

From this we can construct an example similar in structure but a bit more evocative than coin tosses. This example also sets the stage for our application to psychiatric diagnosis later on.

Example 2 *Competing Theories* Let S, D be two events. S represents an observable phenomenon, D an underlying theoretical construct. There happen to be two competing theories on the relation between S and D , one suggesting that S is highly indicative of D , whereas the other suggests S to be irrelevant to D . The disagreement between the two theories cannot be resolved, and hence, the scientific community ends up with an indeterminate credal state regarding the conditional probabilities linking S and D . However, for independent reasons, there is a precise credence in S . In this configuration, an agent learning about D dilates their credence in S by virtue of having to take into account the two competing theories.

The case need not be so extreme; for instance, the dilation may not be from a unique distribution to every possible one, but just from $p(E) = 0.5$ to $p(E) \in [0.4, 0.6]$. The dilation effect need also not be the same across all outcomes of the dilating variable. As long as the inclusion condition holds for every one of them, the effect on the credal state for the dilated variable may vary.

We will use this possibility of indeterminate credal states to reconstruct a particularly tricky class of hermeneutical gaps. The possibility of dilation raises some alarms about the rational admissibility of such doxastic states. For our purposes, this is not a concern, as we do not rely on an assumption of full rationality on part of our agents for our argument. The interpretation as a hermeneutical gap may provide a different point of view: Namely, that some indeterminate credal states are rationally inadmissible precisely because they correspond to unjust hermeneutical gaps. This conjecture depends on the relation between epistemic justice and rationality, which to establish goes beyond the scope of the current paper.

But before we can turn to our deployment of the theoretical construct of dilation in the formal reconstruction, we need to lay out what the mapping of deficient interpretive resources onto the formal framework looks like.

3 Reconstructing Interpretive Scarcity

3.1 Hermeneutical Gaps as Imprecise Probabilities

With the formal framework established, we can turn to constructing a model of deficient interpretive resources. To illuminate the rather abstract process of model construction, we will use the interpretive resource of psychiatric diagnosis as a running example.

The case of personality disorders illuminates how centrally importance psychiatric diagnosis can be as an interpretive resource:

Definition 2 Personality Disorder *A personality disorder is an enduring pattern of inner experience and behavior that deviates markedly from the norms and expectations of the individual's culture, is pervasive and inflexible, has an onset in adolescence or early adulthood, is stable over time, and leads to distress or impairment. (See American Psychiatric Association et al., 2013, p. 734)*

This covers a variety of particular personality disorders. We do not need to go into the details of those particular disorders, but a potential example of a hermeneutically unjust diagnostic construct can be identified in the discussion of borderline personality disorder by Dorfman and Reynolds (2023), even though they frame the problem only in terms of testimonial injustice.⁶ For our purposes, a psychiatric diagnosis links a set of symptoms with a theoretical construct, where the symptoms are highly relevant to the patient and the diagnosis is supposed to cover a wide range of experiences across their life. In other words, deficiencies in the diagnostic construct would be systematic and persistent in their negative effect on making sense of significant experiences in the life of the diagnosed person. As will become important in the next section, a diagnosis is also practically linked to practices of treating, or in the case of personality disorders more accurately, managing the disorder.

There is some concern that the category of personality disorders is generally invalid.⁷ For the current discussion, it need not be decided if this is the case. It suffices to observe the breadth and width of experiences which is intended to be correlated, explained, understood and coped with by reference to this type of construct. If it works or not, it is clearly offered as an interpretive resource which is applied in a systematic and persistent manner to someone's experiences.

⁶ A case for the hermeneutical injustice of a diagnostic construct has also been made in the case of autism spectrum disorder Chapman and Carel (2022). This is not in the class of personality disorders, but similarly is taken to cover much of the diagnosed person's experiences.

⁷ See, for instance, Dorfman and Reynolds (2023) for a discussion.

With our running example set up, we turn to the construction of a model. We operate with the following general definition of hermeneutical injustice:

Definition 3 (Hermeneutical Injustice) *A social group exhibits hermeneutical injustice if members of a social type are persistently and systematically hindered in understanding and communicating a significant part of their experiences due to deficiencies in the shared interpretive resources.*⁸

Before we move to the reconstruction of the central notion of shared interpretive resources, it is worth pointing out a few features of this definition. First, it is intentionally written in the passive voice, as we agree with Fricker (cf. 2007, p. 154) that this form of injustice is structural rather than agentic. This does not imply it is not sometimes manifested or even reinforced by individual agents, but it is ultimately located in a collective space, not any particular agent's mind. Second, the definition implies the possibility of deficient interpretive resources which are not unjust. This would be the case, for instance, if the failure in understanding is randomly distributed across social types, or if the deficiency cannot be resolved at all at a given stage of epistemic progress. Especially the latter category is suspiciously vague, but we might think of a lack of understanding of a type of mental illness despite a society's best efforts – but nothing hinges, for our purpose, on this issue. Finally, the definition excludes incidental hermeneutical injustices. Paradigmatic cases of hermeneutical injustice are persistent and systematic, as the most significant interpretive resources themselves are not constructed ad hoc. As with the question of agentic hermeneutical injustice, ad hoc constructions of deficient interpretive resources are possible, but gain their larger meaning as instances of injustice in the wider context. This does not preclude incidental cases from occurring, and those ought to be recognized for the epistemic harm they cause.

The definition suggests the centrality of the notion of an interpretive resource, and an explication of what it means for it to be deficient. The notion of an interpretive resource does not have an explicit definition in the literature. But it is unquestionably a broad class (Medina, 2012). As such, a uniform formalization of the concept is implausible, and hence we focus on a particular kind of resource, that we consider important, but certainly not all-encompassing.

⁸ This definition is based on Fricker (see 2007, p. 154), but is adapted; we exchange the notion of obscuring by hindering to understand and communicate because we find the original term insufficiently clear. We also left out the notion of epistemic marginalization, as we consider it already contained in the rest of the definition. Other changes are merely aesthetic.

The kind of resource is constituted by a concept as it stands in inferential relations with experiences. Inference may not be the first one thinks of when analyzing understanding, but inferences play a major role in understanding.

Our focus is on inferences between a set of observations and a higher-level concept invoked to correlate, explain or predict those observations. In terms of our running example, a set of symptoms and a diagnosis. The probabilistic relation between symptoms and diagnosis is structurally similar to a common cause explanation (Spohn, 1994), and we draw on this structure to reflect the interpretive power of a diagnosis in its aspect as a tool for inference. This is captured in the following equations:⁹

Definition 4 Common Interpretation *Let S_1 , S_2 and D be three events. We call D a common interpretation of S_1 and S_2 if the following conditions hold:*

$$\begin{aligned} p(S_1 \cap S_2|D) &= p(S_1|D)p(S_2|D) \\ p(S_1 \cap S_2|\bar{D}) &= p(S_1|\bar{D})p(S_2|\bar{D}) \\ p(S_1|D) &> p(S_1|\bar{D}) \\ p(S_2|D) &> p(S_2|\bar{D}) \end{aligned}$$

Under these conditions, D provides an interpretation of both S_1 and S_2 . Vice versa, higher credence in D implies higher credences in S_1 and S_2 . In the following, we assume this describes an important aspect of the interpretive function of diagnosis (D) in the understanding of symptoms (S_1, S_2).

This model enables several important modes of understanding. We will focus on two such modes:

1: It allows one to make sense of the apparently random correlation of a set of events. Within our example, it offers an interpretation of a patient's inordinate number and extent of negative experiences. As the definition indicates, if the experiences support the diagnosis of a personality disorder, the patient can read their experiences through a new lens. This allows to understand one's experiences as less random, and hence more meaningful. Indeed, the struggle against randomness appears to be one of the central elements of sense-making.

2: The second mode runs in the reverse direction of inference: Rather than reasoning towards a higher-level concept or theory, an agent reasons downwards in the direction of particular experiences. This enables one to uncover experiences as meaningful, or even predict such experiences. The latter is maybe not straightforwardly

⁹ Note, that we do not assume the temporal order necessary for a causal relation. Further-more, as a categorical matter, a diagnosis cannot be the cause of symptoms.

taken as understanding. But our example illustrates how it is: For the patient, it is an element of their understanding of themselves to predict how they would experience certain events. They might even seek out such events to test the interpretative resource suggested to them in the form of a diagnosis.

Thus, we can see how inferences are constitutive of certain kinds of understanding, and how an important type of inferential relation can be captured in a simple probabilistic model.

But the attentive reader may have noticed that so far, we have not really used the imprecise part of the imprecise probability framework. It will be used to express what is deficient about a given interpretive resource. But before we formally do that, a word of clarification is in order. Much can be wrong with an interpretive resource in the sense of the inferential model laid out above. The credences may not be probabilistically coherent, they may not be consistent with further evidence in the wider network of belief or they might simply fail to cover in their underlying algebra the actually relevant events. We do not want to exclude the possibility that any or all of these problems also instantiate deficiencies relevant to the theory of hermeneutical injustice. But we will not take a position on this question, as we only purport to show that imprecision reconstructs an important aspect of the phenomenon and, to show the utility of the model, derive a novel observation. If the same can be done for those other defects, they were to be included in a more general formal reconstruction of defective interpretation.

An interpretive resource represented as a set of credal states allowing the interpretive inferences stated is thus deficient if and to the degree that the credences are imprecise. The immediate reason to identify deficiency with imprecision lies in the consequence it has for interpretive inferences as laid out above. The less precise the credences which represent an interpretive resource are, the more imprecise will any interpretations turn out. Here is a simple example of this phenomenon:

Example 3 Imprecise Diagnosis *As before, let S_1 , S_2 and D be three events representing symptoms and a diagnosis. But now, the following holds:*

$$\begin{aligned} p(S_1|D) &\in [a, b] \\ p(S_2|D) &\in [c, d] \end{aligned}$$

where $a, b, c, d \in [0, 1]$ and $b > a, d > c$. Analogously, the conditional probabilities on \bar{D} are also allowed to be imprecise. We can still assume that the upper probabilities c, d or even the lower a, b satisfy the four conditions stated in the previous example. Even in this most benevolent case, imprecision implies that learning about an event is less informative than in the precise case.

We will see in the following two sections how this imprecision is turned into more concrete problems. It bears repeating, though, that not every interpretive resource defective in this sense translates into an injustice. Again, the ignorance might be inevitable relative to the epistemic capacities in a given society, or the experiences could be so randomly scattered across social types that none of them is especially disadvantaged. Or, furthermore, the experiences may simply be not particularly significant. But if the other conditions of our definition are satisfied, the degree of imprecision is read as a measure of the degree of hermeneutical injustice.¹⁰

For our running example, the immediate consequence of imprecision is a wide range in the resulting credal state regarding D upon learning S_1 , S_2 or both. If the inference from a potential symptom to the underlying condition is highly imprecise, there will be a vast range of rationally admissible credences as to the status of the patient. As we will work out in the following section with the help of imprecise decision theory, this leads to downstream problems with treatment and support. But it is also in a more immediate sense reducing the value of the interpretive resource for the patient. Even if certainty cannot be expected, a fairly precise estimate is desirable to enable interpretation in inferential terms. But if the diagnosis is given with a wide interval of possible probabilities, what is one supposed to do with that? The interval may span the range from the diagnosis being highly unlikely to more likely than not. This is indeed a defect in the capacity of a diagnostic construct as a means of making sense of one's experiences. But this is only the most immediate problem. So let us turn to action.

3.2 From Symmetric Deficiency to Asymmetric Disadvantage

Deficiencies in the interpretive resources of a society, as Fricker describes them, affect both the marginalized and the non-marginalized. In her analysis of the development of the term and concept of sexual harassment, neither the harasser nor the harassed are able to make sense of what is happening (cf. Fricker, 2007, pp. 149ff.). But the diagnosis of an injustice is clearly asymmetric, which led Beeby (2011) to criticize the account as not genuinely a case of epistemic injustice. She argues that in purely epistemic terms, both parties are equally harmed by the hermeneutical gap, while the apparent injustice is a consequence of the social

¹⁰ This is a somewhat technical detail, but we do not assume a global measure over imprecise credal sets to determine which is, all things considered, more imprecise. While such a measure could be mathematically defined, it would miss out on the difference in the significance of subsets of credences. Therefore, we only allow the comparison of two credences defined on the same algebra as follows: Credal state A is more imprecise than B if for any element of the algebra, the credal set implied by B is contained in that by A .

background conditions in which the hermeneutical failure is situated. The injustice, then, would not properly be located in the epistemic domain.¹¹

This criticism is significant, as it points out the need to explain in which sense the basic epistemic symmetry results also in an epistemic asymmetry which can provide grounds to diagnose an injustice. As we will reconstruct in our model, what Beeby overlooks is that different epistemic roles and hence, likely epistemic actions, may be asymmetric even though underlying deficiencies are perfectly shared. To see this, consider again our previous example of a patient encountering a clinician looking for diagnosis.

Example 4 Betting on a Diagnosis *Let S_1, S_2, D and a, b, c, d be as in the pre-vious example. Consider two agents deciding to act on $p(D|S_1)$ which according to Bayes theorem is equal to*

$$\frac{p(S_1|D)}{p(S_1)} p(D)$$

The imprecise version is, under our assumptions, calculated by applying Bayes theorem to the prior upper and lower probabilities.¹² Interpreting the mathematical structure, let one agent be a patient being diagnosed, the other a clinician performing the diagnosis. We assume further that the agents agree on utility assignments for the various outcomes, e.g. a pharmacological treatment given a particular diagnosis. Still, they differ in their epistemic positioning vis-a-vis D . The clinician offers an interpretation of S_1 in terms of D when providing a diagnosis, or denies it in the form of \bar{D} . The patient chooses from the opposite perspective, having to decide whether to pick up this interpretation or reject it. In model terms, the clinician giving a diagnosis offers a bet to the patient, who is in the position of the buyer. But as $b > a$, there is a range of values for the diagnosis for which seller and buyer, clinician and patient, will not agree to an exchange, they fail to negotiate an interpretation.¹³

¹¹ There are also important concerns with the idea that hermeneutical gaps have to hold a whole society in their grip for epistemic injustice to occur. As Wylie (2013) points out, it is possible that marginalized groups indeed have the linguistic and conceptual structures to make sense of their experiences while also having to deal in those of the non-marginalized, but are still in important ways hindered by the shared – or rather, non-sharing of interpretive resources in parsing and communicating their experiences. But also, as Mills (2013) points out, it is possible that the ignorance of the non-marginalized is re-imported into particularized resources of marginalized groups.

¹² In general, the posterior distribution for the whole credal set has to be calculated and upper and lower probability arise according to their definition above.

¹³ The language of the model makes the interaction sound very competitive or even antagonistic. As can be gleaned from Dorfman and Reynolds (2023), psychiatric diagnosis is at least in some cases correctly characterized as non-cooperative, this is not an inherent feature of the process and the

Abstracting from the particulars of the example, the account suggested by the reconstruction can be laid out as follows: Two agents may have the same credal set regarding a set of events as well as the same valuations for outcomes, but differ due to their positioning as sellers and buyers of bets. This works well when we consider the communication of experiences subject to hermeneutical shortcomings, as one type of person typically has that experience, while another is typically the recipient of attempts to communicate it.¹⁴ The diagnosis example also illustrates how this manifests in sense-making itself: The clinician does not have the same stake as the patient in applying a particular diagnosis, and as described, their positions relative to the experience may also be on different ends of the spectrum.¹⁵

If we take seriously the criticism put forth by Beeby, it is important to point out that the injustice here is not purely due to unjust non-epistemic social background conditions; those are also likely to be in place, e.g. sexist prejudice towards female patients or classism common in the discipline of psychology (Rickett, 2020). Within the model, this can be represented by allowing for different valuations. Such differences would further exacerbate the divergence of the two agents in the model, so leaving out this feature does not threaten our argument. But there is also a genuinely epistemic asymmetry: One epistemic agent is in the position to make sense of *their* experience, while the other is asked to do this for someone else's. The experience, and hence the credal states associated with its interpretation, do not have the same relevance for agents of different social types. But relevance is an epistemic category, not exclusively a practical one. Therefore, our reconstruction maintains that there is indeed a genuine epistemic injustice resulting from a symmetrical hermeneutical gap.

3.3 Dispersion Through Dilation

In the previous section, we reconstructed an agreed-upon feature of the theory of hermeneutical injustice and applied it to discuss a line of criticism. The purpose of this exercise, beyond its subject-matter argument, was to show the ability of the formal framework to capture features of the informal theory in a useful way. Now, we want to derive a conjecture for a novel aspect of the theory: Dispersive interpretive deficiencies.

model does not materially imply such an understanding. It is perfectly compatible with viewing the failure as an unfortunate consequence of poor resources insufficient to enable cooperation between two well-meaning parties.

¹⁴ Note, that this does not generally turn the scenario into a case of testimonial injustice, as no deflating of credibility needs to take place.

¹⁵ Note that there are also cases where the stakes are reversed, because the patient has an interest not to accept a particular diagnosis as an interpretation of their experience.

Dilation, as laid out above, denotes a set of credal states such that learning the outcome for a random variable X renders an agent's credal state on another random variable Y less precise. In terms of the theory, an interpretive deficiency may have an agent become more ignorant on receiving evidence and processing it in a way otherwise considered rational, i. e. by conditionalization.¹⁶

Example 5 Dispersive Interpretation Assume, again, the two events S and D . Suppose further that a clinician has a precise estimate of the likelihood of S in a prospective patient due to available population-level information. But the relation between symptom and diagnosis is such that learning if D or $\neg D$ induces dilation in $p(S)$, in numbers e.g.

$$\begin{aligned} p(S) &= 0.1 \\ p(S|D) &\in \{0.1, 0.15\} \\ p(S|\bar{D}) &\in \{0.01, 0.1\} \end{aligned}$$

Thus, learning the diagnosis a prospective patient received would make the clinicians credal state about them experiencing a particular symptom less precise. This dispersion of the credal state renders the diagnosis a hindrance in the reasoning about and hence interpretation of the patient's symptom. With respect to S , it would be better not to know about D . Presumably, there are other good reasons to try to learn about D , but given the relation to S , this has the side effect of making it more difficult to make sense of and communicate S for those who suffer it.

The example should make it clear why we refer to this type of interpretive deficiency as dispersive: It widens the credal state of the epistemic agent, i.e. it disperses the indeterminacy over a wider range of credal states. We do not make, at this point, any claim as to how common this type of problem actually is. It is indeed rather difficult to determine in a given case if this is happening, as it requires a rather detailed knowledge of both actual and counterfactual beliefs. Even if we observe, for instance, that an agent's credal state became more indeterminate upon receiving evidence of a particular outcome, while it is easy to theoretically construct a comprehensive credal state, it is a well known problem to measure what someone would have believed if the events unfolded differently.

¹⁶ As we noted, we will not take a stand on the question if imprecise probabilities are themselves irrational. The point is that an agent, operating as rational as possible given they already hold an indeterminate credal state, is subject to the phenomenon of dilation and hence dispersive interpretive deficiency.

The example directs our attention further to the question if the indeterminate credal states are located in an individual mind, or rather reside in the collective space. For an individual, it seems difficult, if not unintelligible, to hold such starkly opposed views about the relation between *S* and *D*. The best way to make sense of such an individually indeterminate state is as the agent entertaining different theories and acting thereupon, rather than as straightforward indeterminate belief.

But within the context of shared interpretive resources, the obvious interpretation of the problematic imprecise probabilities understands them as the product of a social process of negotiating the positions of different agents into one shared resource. Under this interpretation, the individuals hold straight-forward credences, but in social interactions, have to draw upon the shared interpretive resources. But those, in accordance with the resolution of disagreement into a collective imprecise credence may without any further assumptions on individual credence, lead to the scenario described in the example.

The example also brings out a particular connection to testimonial injustice. If the patient testifies and, due to dispersive interpretive deficiency, this leaves their clinician more uncertain about a particular symptom, the interaction is easily mistaken for a case of testimonial injustice. However, it is distinct in two ways: First, *D* need not always be something to be learned from testimony. In the case of diagnosis, the clinician may construct it themselves from other symptoms. In different circumstances, the agent may learn the dilating information in any way evidence can be acquired. Second, the description does not entail a discounting of credibility, and hence, no paradigmatic testimonial injustice is committed.

But dispersive interpretive resources may still be a contributing factor to testimonial injustice. As we pointed out before, dilation implies that a piece of information may indeed be detrimental in purely epistemic terms. This means that an epistemic agent, faced with dilation, is incentivized not to learn the dilating information. Applied to testimony in the example, this provides a reason for the clinician to dismiss the patient's testimony about *D* to avoid the dilation of their credal state regarding *S*. Vice versa, a patient may have an incentive to withhold testimony not to dilate their audience's credal states.¹⁷ Thus, we can see dispersive interpretive resources as one of a number of possible mechanisms translating hermeneutical into testimonial injustice.

¹⁷ This phenomenon is close to what Dotson (2011) refers to as testimonial smothering.

4 Conclusion

In this paper, we reconstructed a type of interpretive deficiency in the framework of imprecise probabilities. We recovered the common observation that symmetric epistemic problems can induce asymmetrical downstream effects, even if the valuations of different subjects align. This is a consequence of different social positioning leading to different social types taking up distinct roles in attempts of sense-making and communication. Our result refutes the concerns that hermeneutical injustice may not be a genuine case of epistemic injustice. We used a decision-theoretic model with imprecise probabilities to analyze this phenomenon, and thereby also established a potential connection to testimonial injustice.

Our second main result is a theoretical accounting of a kind of hermeneutical injustice which, to our knowledge, has not been explicitly discussed. By following the implications of the theory of imprecise probabilities, we investigate a phenomenon we call interpretive dispersion. On the level of the model, imprecise probabilities allow for dilation, meaning that evidence on one variable may, regardless of the outcome, widen the uncertainty about a second variable. For the target of the effects of interpretive deficiencies on credal states, we understand this as a faulty interpretive resource rendering evidence on an issue inert or even epistemically detrimental in the sense of dispersing further an uncertain credal state rather than reducing uncertainty. We apply this to a case of testimony, where it may undermine a speaker without discounting their credibility. We conjecture that some prejudices establish an interpretive structure which is well described by dispersal: Whatever the subject of the prejudice says, this only raises more doubt on the content of their testimony. On a more philosophically basic level, this demonstrates the limits of the mere provision of further evidence. When the interpretive resources used to integrate new evidence are deficient in the sense we describe, this evidence cannot by itself remedy the faulty epistemic state. Fighting the injustice requires the deeper work of reforming or revolutionizing conceptual and theoretical structures.

To conclude, we want to once again be clear about the limitations of our investigation and sketch future directions. It should be clear that our account does not cover all forms of hermeneutical injustice noted in the literature. Some of them may not reduce to propositional content, others may, in the terms of our probabilistic approach, be properly located in the underlying algebra, rather than the credal states, and hence can only be properly analyzed in reference to a different algebra of events. Furthermore, we relied on the modest assumption that inference and correlation of experiences is at least an important part of understanding. However, these cognitive tasks are not a full reconstruction of the notion of understanding underlying the theory of hermeneutical injustice. In this further sense, we can only cover an aspect of the phenomenon.

For the future, we briefly discuss two directions we find particularly interesting. First, we believe it would be quite fruitful to embed the model in an agent-based model to simulate a more genuinely social dynamic. Current ABM of belief formation are built on classical probability models (Merdes et al., 2021; Holman and Bruner, 2015; Zollman, 2007; Weatherall et al., 2020), and would also benefit from an expansion of their expressive power. Second, reconstructing understanding in probabilistic terms deserves more attention. One could attempt, for instance, to combine an account of understanding in terms of grasping causal explanations with a probabilistic account of causal explanation to construct a set of credal states to capture a notion of understanding. We believe this notion of understanding to be too narrow for the purpose of hermeneutical justice, and hence would suggest a broader exploration what can and cannot be expressed in a probabilistic framework, imprecise or classical.

We hope that our analysis helps to encourage an optimistic outlook on the application of formal methods in the theory of epistemic justice. While great care has to be taken to not lose key insights from the informal discourse, formal reconstructions can clarify and aid theoretical speculation.

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