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ROMANIAN ECONOMY PROFILE BASED ON A STOCHASTIC ANALYSIS

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Abstract. The paper captures, based on a simplified, small-size dynamic stochastic general equilibrium model and on Bayesian instruments, the evolution of the Romanian economy since the beginning of this century. The depicted economy is governed by basic representative agents: household, firm, financial organism, embodied by banks, and monetary authority, playing a subsidiary role. Two shocks are considered, the technology real shock and the money stock nominal shock, both of them impacting the economy and changing the trajectory of the relevant economic variables. The steady-state return time of the model variables and the estimates of the parameters allow us to set the pattern of future economic trends and to make pertinent decisions so as to attain sustainable macro-stabilization.

JEL classification: C61, D12, D22

Keywords: stochastic analysis, Bayesian approach, steady-state of variables, technology and money stock impulse, general equilibrium

1. Introduction

The dynamic stochastic general equilibrium models are nowadays largely approached and debated, being used in combination with econometric techniques in order to generate estimations (Smets and Wouters, 2003) or to make predictions (Christoffel et al., 2007) at a macroeconomic level.

Following the classical line, by resorting to the Real Business Cycle model (Kydland and Prescott, 1982), with perfectly competitive markets characterised by flexible prices and wages, or to the Keynesian one, by making use of a New Keynesian model (Rotemberg and Woodford, 1997), considering monopolistic markets with sticky prices (Calvo, 1983) or wages (Erceg, 2000) and, most important, taking into account the rationality of representative economic agents, focussed on the maximisation of their benefits, and the structural shocks hitting the economy, such models became a reference point for economists all over the world.

Considering the most recent such approaches, we could mention several important works, having brought additional knowledge to the existing literature.

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Trying to capture the increasing level of unemployment caused by the economic crisis, Christiano et al. (2010) resort to a DSGE model, ignoring the wage stickiness, while preserving the Calvo price rigidity, creating a mix of the Real Business Cycle and New Keynesian model, the results being suggestive from the economic perspective.

Sargent and Surico (2011) undertake to analyse the relationship between the interest rate and the inflation rate, on one hand, and the increase in the money supply, on the other hand, deducing, via a DSGE model with sticky prices, the instability of the former low frequency regressive parameters.

Lees et al. (2011) use the forecasting feature of a DSGE model, more specifically the conditioned predictive technique, therefore testing in real time, while developing the model, the relevance of successive forecasts.

Smets et al. (2014) also use the predictive function of this type of models, capturing the influence exercised by individual forecasts relating to the interest rate, the inflation rate and the gross domestic product on the overall predictive performance, while Merola (2015) makes appeal to the most recent element considered when approaching the DSGE models, the financial accelerator, revealing the role of the financial mechanisms in translating the dysfunctions of the financial markets onto the real economy.

Destined for academic purposes or theoretical economic analyses, these models quickly expanded, getting into the real economic life as an effective macroeconomic, particularly monetary policy instrument, being at present adopted and developed by the Central Banks of various countries.

Although widely used abroad, their echo delayed at the level of Romania, just a few attempts being visible in this respect. The cornerstone of the national economy will be indubitable the model under construction of the National Bank of Romania, lately started, model meant to help the related authority to make best economic decisions. Until its finalisation, there will be a considerable lack of significant information in this area, reason for which I decided to dedicate myself to their analysis, as in Hudea (2012, 2014, 2015) among many others.

The present approach comes to add value to the information arising from the implementation of the DSGE models to the Romanian economy, not only by providing pertinent outcomes as result of the Bayesian estimation process, but also, especially, by insisting on the impact on the model variables of two highly important shocks that determine structural changes in the short and in the long run.

Given the above-mentioned, we decided to resort in this paper to a brief version of a dynamic stochastic general equilibrium model developed by Schorfheide (2000), a model that assumes the existence of a continuous series of representative economic agents acting on the market: household, firm and commercial bank. The presence of the monetary authority is also considered, it having, however, a lower influence on the real variables of the economy. The latter is impacted by two types of shocks, a real one, the shock on technology and a nominal one, the shock on the stock of money.

Once the model is implemented in Matlab, the Metropolis-Hastings algorithm is called to generate based on a Bayesian approach, the posterior distribution of the model parameters, allowing us to construe the same. The impulse-response functions are also rendered in order to capture the reaction of the model variables when hit by a structural shock.

2. Model description

The model approached in this paper is a simplified version of the one developed by Schorfheide (2000) and rendered by Griffoli (2007) in his attempt to lay the groundwork of the dynamic stochastic general equilibrium modelling.

Given the relatively high length of such model, we exclusively present hereinafter the first order conditions arising from the maximisation problems, solved considering the model variables at steady state, when the time index is lost, the remainder of the model as well as the significance of model variables and parameters being detailed in Appendix 1.

Thus, the model FOCs are:

- the Euler equation of consumption

$$E_t\left[\frac{\overline{P}_t}{\overline{C}_{t+1}\times\overline{P}_{t+1}\times m_t}\right]=-\beta\times e^{-\alpha\times(\gamma+\varepsilon_{A,t+1})}\times P_{t+1}\times\frac{\alpha\times\overline{K}_t^{\alpha-1}\times N_{t+1}^{1-\alpha}+(1-\delta)}{\overline{C}_{t+2}\times\overline{P}_{t+2}\times m_{t+!}} \tag{1}$$

- the Euler equation of crediting

$$\frac{1}{\overline{C}_t\times\overline{P}_t}=\beta\times\left[(1-\alpha)\times\overline{P}_t\times e^{-\alpha\times(\gamma+\varepsilon_{A,t+1})}\times\frac{\overline{K}_{t-1}^{\alpha}\times N_t^{1-\alpha}}{\overline{W}_t}\right]\times\frac{1}{E_t\left[\overline{L}_t\times m_t\times\overline{C}_{t+1}\times\overline{P}_{t+1}\right]} \tag{2}$$

- the borrow-related constraint of firms

$$\overline{W}_t=\frac{\overline{L}_t}{N_t} \tag{3}$$

- the aggregate resource-related constraint

$$\overline{C}_t+\overline{K}_t=e^{-\alpha\times(\gamma+\varepsilon_{A,t})}\times\overline{K}_{t-1}^{\alpha}\times N_t^{1-\alpha}+(1-\delta)\times e^{-(\gamma+\varepsilon_{A,t})}\times\overline{K}_{t-1} \tag{4}$$

- the production function of firms

$$\overline{Y}_t=e^{-\alpha\times(\gamma+\varepsilon_{A,t})}\times\overline{K}_{t-1}^{\alpha}\times N_t^{1-\alpha} \tag{5}$$

- the inter-temporal labour market equation

$$\frac{\psi}{1-\psi}\times\frac{\overline{C}_t\times\overline{P}_t}{1-N_t}=\frac{\overline{L}_t}{N_t} \tag{6}$$

- the interest rate equilibrium equation

$$R_t=(1-\alpha)\times\overline{P}_t\times e^{-\alpha\times(\gamma+\varepsilon_{A,t+1})}\times\frac{\overline{K}_{t-1}^{\alpha}\times N_t^{-\alpha}}{\overline{W}_t} \tag{7}$$

- the money market equilibrium equation

$$m_t=\overline{C}_t\times\overline{P}_t \tag{8}$$

- the credit market equilibrium equation

$$\overline{L}_t=m_t-1+\overline{D}_t \tag{9}$$

- the money growth specific shock equation

$$\frac{A_t}{A_{t-1}}=e^{(\gamma+\varepsilon_{A,t})} \tag{10}$$

- the equations linking observables to stationary variables

$$\frac{Y_t}{Y_{t-1}}=e^{(\gamma+\varepsilon_{A,t})}\times\frac{\overline{Y}_t}{\overline{Y}_{t-1}} \tag{11}$$

$$\frac{P_t}{P_{t-1}}=\frac{m_{t-1}}{e^{(\gamma+\varepsilon_{A,t})}}\times\frac{\overline{P}_t}{\overline{P}_{t-1}} \tag{12}$$

3. Data, Methodology and Results

Thirteen final equations of the above-rendered model, namely equations (1) - (12), were selected to be implemented in order to obtain the estimates of the model parameters as well as the impulse response functions of the model variables. The programme used for such implementation was Matlab 7.11.0, with its specific tool dedicated to dynamic stochastic general equilibrium modelling, Dynare 4.3.0.

As this is a simple model, we had just two observable variables, the gross domestic product, respectively the inflation rate, both related data, analysed quarterly, being taken over from the Romanian National Institute of Statistics. The paper had in view a 12-year period, resulting in 48 observations. The data have been used in logarithm and differentiated, the first observation being eliminated.

The estimation process was preceded by the calibration of several parameters, according to personal results obtained in previous studies, while considering the literature in the matter, with a production elasticity in relation to capital (α) of 0.323, as in Almeida (2009) or a subjective discount factor (β) of

0.992 and a capital depreciation rate (δ) of 0.030, as in Andrés et al. (2006).

The method used for estimation was based on Bayes theorem, this involving the setting of prior probabilities, with various types of distributions, which provide some initial information about the data before their actual observation (see *Table 1*).

The posterior probabilities, rendered in Matlab, are obtained by resorting to the Metropolis-Hastings algorithm, which is nothing else but a Monte Carlo type method that simulates the distribution of parameters by generating Markov chains representing sequences of possible estimates, the most relevant of them being selected after their filtering based on several rules.

The results obtained on the model implementation are rendered below.

Table 1. Estimation results

Parameter	Prior mean	Mode	Posterior mean	Confidence interval	
γ	0.009	0.009	0.008	0.004	0.012
m^*	1.000	1.000	0.999	0.988	1.010
ρ	0.600	0.950	0.947	0.925	0.964
ψ	0.650	0.629	0.624	0.530	0.691
σ_{ε_A}	0.150	5.872	6.065	5.048	7.296
σ_{ε_M}	0.150	1.130	1.205	0.966	1.430

The intercept of the technology shock equation (γ) is identical to its anticipated value, being quite low, therefore reflecting a high persistence of the related structural shock. As expected, the steady-state money stock growth rate (m^*) maintains to the same value, suggesting its stability over time.

The autoregressive parameter of the monetary stock (ρ) amounts to 0.95, indicating a very strong influence of the money growth level registered in the previous period on its current level, in contradiction with what we had set as prior at the beginning of the estimation process. However, given that such value is less than one, even when considering the upper extreme of the confidence interval, this meaning the absence of the unit root, we deduce the existence of other, less powerful, impacting elements as well.

The persistence of the consumers' habits (Ψ) reaches a value of 0.62, in accordance with the one initially established as per Adolfson et al. (2005), even if a little bit lower than the latter. Its level reveals the perpetuation of such habits in time, yet underlining their changing when affected by various influencing factors.

The volatility of the structural shocks considered ($\sigma_{\varepsilon A}$, $\sigma_{\varepsilon M}$) is extremely high, especially as for the technology shock, evidencing serious variation in this regard at the level of the studied country, for the analysed period, fully complying with the real facts observed during the last decade.

Figure 1, rendered below, reflects the graphical representation of the estimation results presented in *Table 1*, including also the estimates of the calibrated parameters, with their Matlab associated notations: α – alpha, β – beta, δ – delta, γ – gamma, m^* - m_star, ρ – rho, ψ – psi, $\sigma_{\varepsilon A}$ – SE_eps_a and $\sigma_{\varepsilon M}$ – SE_eps_m).

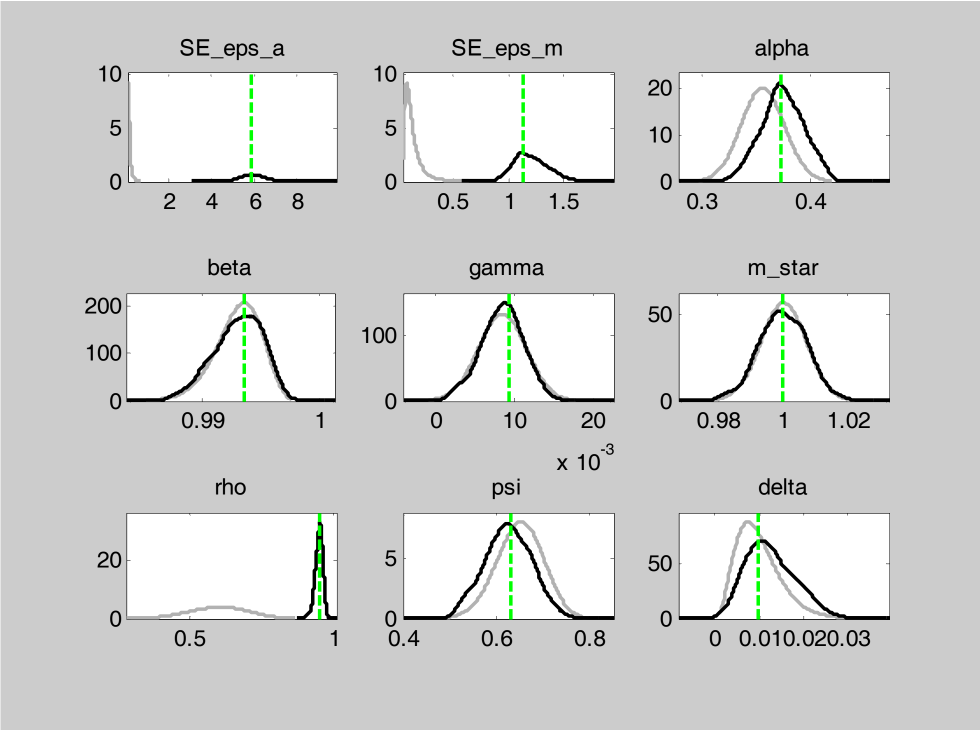

Figure 1. Estimation representation

We remark the similarity existing between the mode arising from the posterior kernel maximization process (the discontinuous line) and the one generated by the Metropolis-Hastings algorithm while determining the posterior distribution of parameters.

The estimation representation graph also reveals some differences, save for the calibrated and m^* parameters, between the prior and the posterior distribution, therefore standing for the relevance of the additional information provided by the evidence.

Figures 2 and *3* below, displaying the output corresponding to the impulse-response function analysis, reflect the reaction of the model variables to the occurrence of the technology shock and of the money stock shock, all notations corresponding to the ones mentioned in Appendix 1, with the specification that small letters indicate their steady state level and that y_bar and p_bar denote $\bar{y}$, respectively $\bar{p}$.

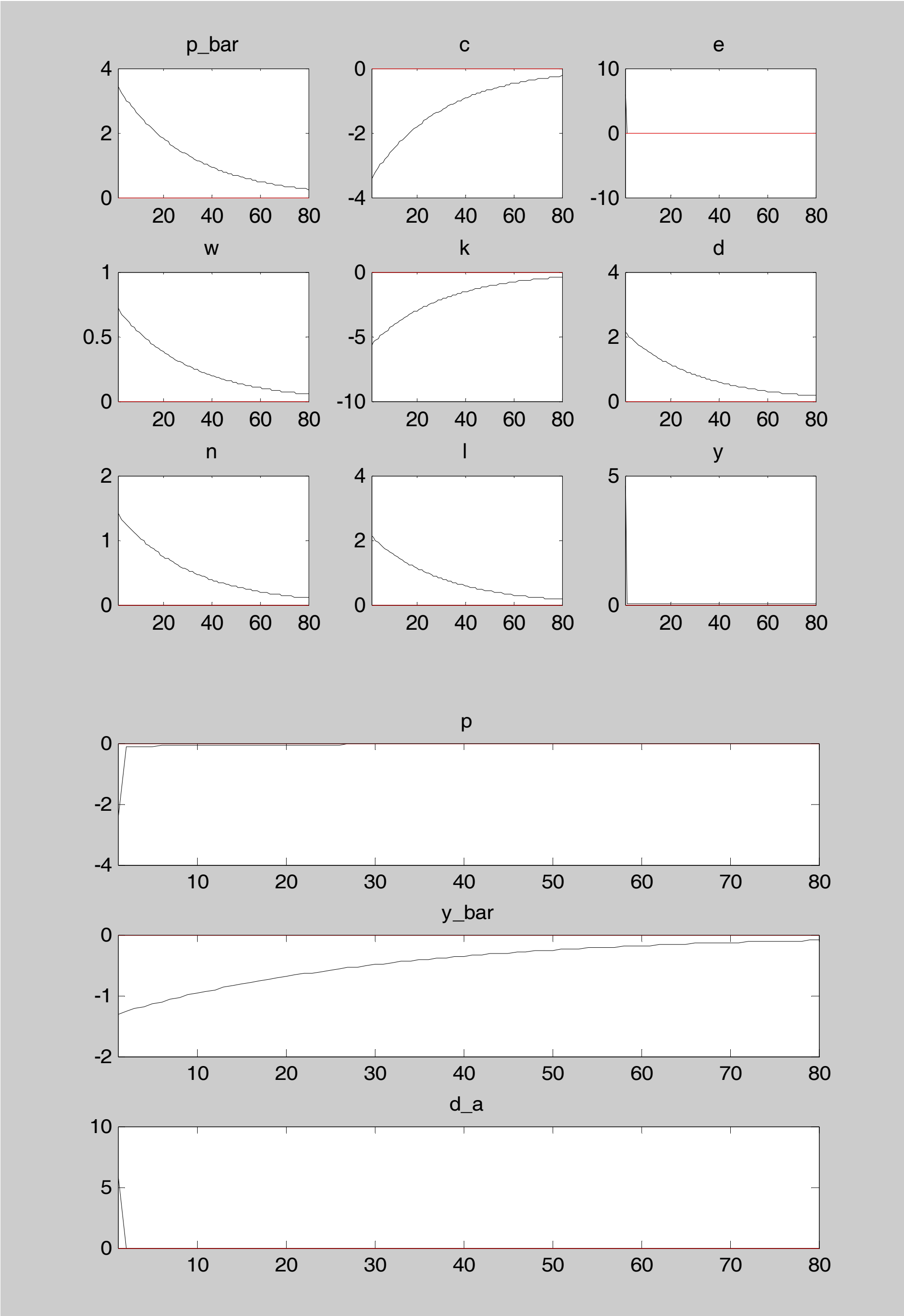

Figure 2. Response of the model variables to a technology impulse

As revealed by *Figure 2*, a positive shock on technology causes a quick drop of the capital stock level, as this one is turned into investments. This can be translated by the orientation of the resources existing in economy rather towards investments, which seem to be more profitable in the long-run, than towards consumption, therefore generating an immediate increase of the first one in the detriment of the latter.

The reaction of the gross domestic product is predictable, the diminishing of consumption being so high that it cannot be fully covered by the augmentation of investments, therefore generating, on short term, an overall lowering of the output.

The significant number of loans granted by the commercial banks to firms, as well as the important number of deposits made by households to commercial banks reflect the exclusive interest of firms in involving in such investments, individual consumers preferring making savings over investing or consuming.

At the bank level, as showed by the graph above, the equilibrium condition is reached, the level of deposits equalising the level of loans. On the other side, higher investments inevitable lead to higher employment and, given the superior technology, to less capital needed for carrying out the production-related activity.

At the same time, a more valuable technology involves more qualified people, the average wage going upward, despite the increased number of employees. Last, but not least, higher wages generate an increase of the price level, therefore inducing inflation.

Figure 2 above also suggests, with some exceptions, a general recovery by the model variables of their initial position after more that 80 periods since the occurrence of the related shock.

As indicated by *Figure 3*, a positive shock on the money stock causes an increase of the level of prices, irrespective of their nature, be they prices of goods and services, of crediting products or the labour force.

A higher inflation rate, on the other hand, inhibits not only deposits, as expected, but also consumption, the capital stock level going upwards.

Given that the volume of capital exceeds its previous level and the number of employees decreases, we assist to an augmentation of wages.

Following the same pattern as when hit by a technology shock, on the occurrence of a money stock shock, the level of the gross domestic product lowers on the short term, the quantum of investments being insufficient for filling the gap left by the decrease of consumption.

Figure 3 shows a quicker return to the steady state of the model variables, in average, after 60 periods subsequent to the production of the money stock shock.

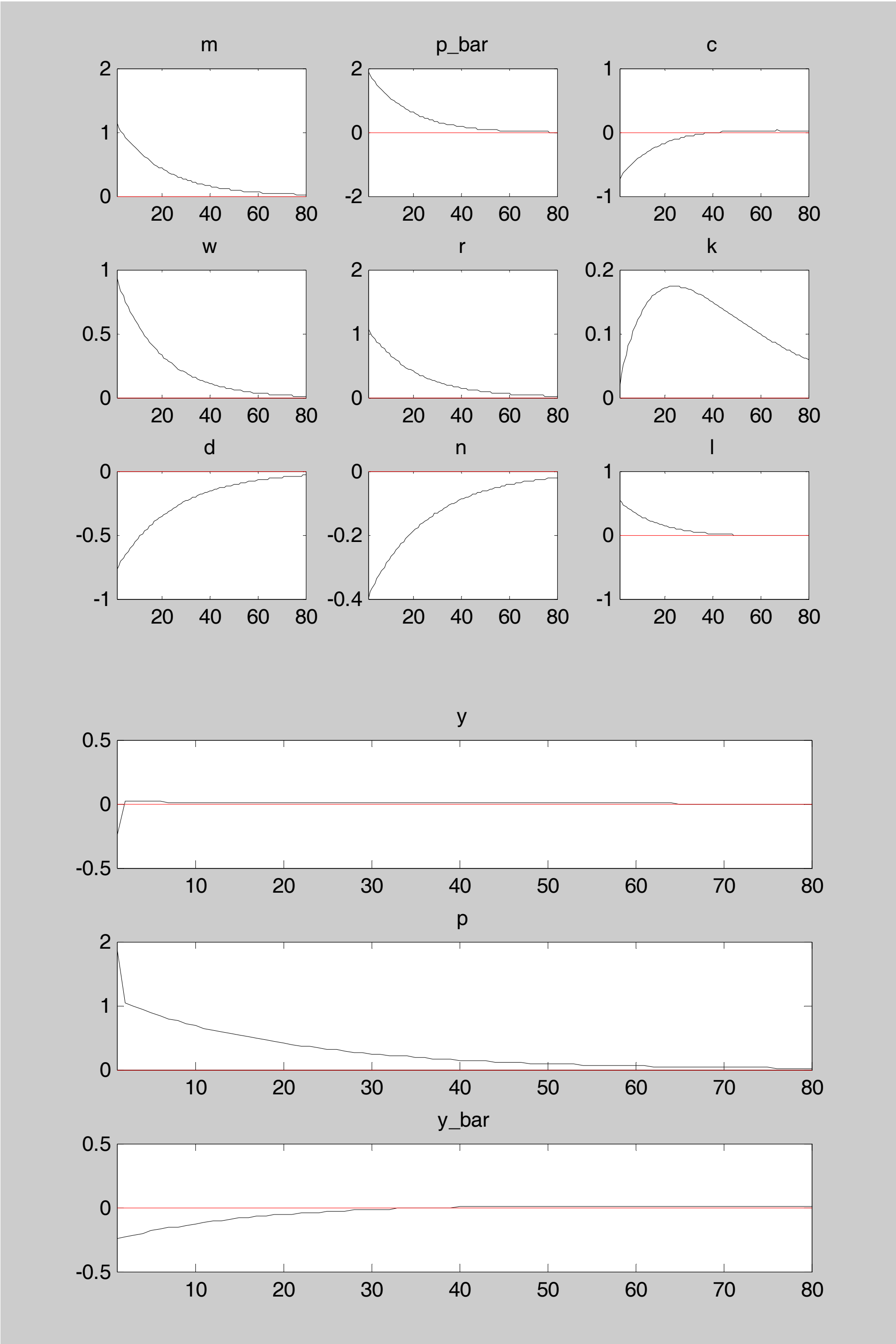

Figure 3. Response of the model variables to a money stock impulse

4. Conclusions

The purpose of the present paper was to diagnose the Romanian economy by means of a simplified version of a small dynamic stochastic general equilibrium model, resorting, in this respect, both to the estimation of the model parameters, by using the Bayes theorem, and to the analysis of the reaction of the model variables to real and nominal structural shocks, by appealing the specific impulse-response functions.

The study reveals, for the covered period of twelve years, a high persistence of the consumers' habits and of the technology shock, a constant money growth rate, the current level of the money growth being strongly influenced by its previous value, as well as a high volatility of the technology and money stock shocks. Also, the discrepancies between the prior and the posterior distributions testify the informative role of data in estimating the model, the setting of priors not being decisive in this regard.

As for the response of the model variables to various impulses, we ascertain an obvious connection between the increase of the technological level and the orientation of the economy resources to investments, the consumption lowering so much that the gross domestic product registers, in the short-run, an overall decrease. The benefits are clear, considering the diminish of unemployment and the higher wages, the newly created conditions requiring more qualified people and involving an adequate remuneration of the same. The shock on the money stock sets the premises for the augmentation of the prices of goods and services, credits and labour force, the generated inflation restraining people from making deposits and from consuming, the general effect being, as in the previously depicted shock, the decrease of the gross domestic product in the immediately following period.

Such type of study is useful both for outlining the image of an economy, with its specific characteristics, and for making pertinent decisions at microeconomic and macroeconomic level, the latter being mainly related to the construction of efficient fiscal and monetary policies.

In our case study, the Government should take all necessary steps to stimulate consumption, this involving, mainly, investments in infrastructure, with long term positive effects on employment and, implicitly, on increased revenues, as well as reductions of taxes of all kind. The monetary organism, on the other side, should inject more money in the economy, therefore determining the lowering of the interest rate and thus decreasing the volume of deposits in favour of consumption. Higher consumption, with increased investments, based on crediting due to a tempting reduced interest rate, will generate a superior value of the gross domestic product, with all its subsequent economic benefits.

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Appendix 1

The model considers a continuous series of households *j*, with $j \in (0,1)$, aiming at maximising their satisfaction, transposed into the solution to the following optimum problem:

$$\max_{C_{j,t}, N_{j,t}, M_{j,t+1}, D_{j,t}} \mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t \times \left[\ln C_{j,t} \times (1-\psi) + \ln\left(1 - N_{j,t}\right) \times \psi \right] \tag{13}$$

where E_0 is the present value expected by household *j*, β^t, the subjective discount factor, $C_{j,t}$, the consumption of household *j*, $N_{j,t}$, the labour time of household *j* and ψ, the consumers' habits

given the cash in advance constraint:

$$P_t \times C_{j,t} \le M_{j,t} + W_{j,t} \times N_{j,t} - D_{j,t} \tag{14}$$

where P_t is the average price of consumption goods, $M_{j,t}$, the money held in cash by household *j*, $W_{j,t}$, the wage of household *j* and $D_{j,t}$, the bank deposits of household *j*

with $$D_{j,t} \ge 0$$

and the inter-temporal budget constraint:

$$M_{j,t+1} = \left(M_{j,t} + W_{j,t} \times N_{j,t} - P_t \times C_{j,t} - D_{j,t}\right) + R_{j,t} \times D_{j,t} + F_{j,t} + B_{j,t} \tag{15}$$

where $R_{j,t}$ is the gross interest rate received for bank deposits by household *j*, $F_{j,t}$, the firm dividends of household *j*, $B_{j,t}$, the bank dividends of household *j*

There is also a continuous series of firms *i*, with $i \in (0,1)$, fighting to maximise their profit by finding the optimum solution to the following problem:

$$\max_{F_{i,t}, K_{i,t+1}, N_{i,t}, L_{i,t}} \mathrm{E}_0 \sum_{t=0}^{\infty} \beta^{t+1} \times \left[\frac{F_{i,t}}{C_{j,t+1} \times P_{t+1}} \right] \tag{16}$$

where $K_{i,t}$ is the capital stock of firm *i*, $N_{i,t}$, the labour demand of firm *i*, $F_{i,t}$, the dividends of firm *i*, and $L_{i,t}$, the loans of firm *i*

with $$L_{i,t} \ge W_{i,t} \times N_{i,t}$$

under the constraint:

$$F_{i,t} \le L_{i,t} + P_t \times \left\lfloor K_{i,t}^{\alpha} \times \left(A_{i,t} \times N_{i,t}\right)^{1-\alpha} - K_{i,t+1} + (1-\delta) \times K_{i,t} \right\rfloor - W_{i,t} \times N_{i,t} - L_{i,t} \times R_{i,t} \tag{17}$$

where α is the elasticity of production in relation to capital, $A_{i,t}$, the technological level of firm *i*, δ, the depreciation rate, and $R_{i,t}$, the credit gross interest rate paid to bank by firm *i* with $$K_{i,t}^{\alpha} \times \left(A_{i,t} \times N_{i,t}\right)^{1-\alpha} = Y_{i,t}$$

$$K_{i,t+1} - (1-\delta) \times K_{i,t} = I_{i,t}$$

where, $Y_{,t}$ is the production of firm *i* and $I_{i,t}$, the investments of firm *i*

At the same time, we encounter a continuous series of commercial banks *n*, with $n \in (0,1)$, which, by undertaking, as the above-mentioned firms, to maximise their benefits, solve the following optimum problem:

$$\max_{B_{n,t}, L_{n,t}, D_{n,t}} \mathrm{E}_0 \sum_{t=0}^{\infty} \beta^{t+1} \times \left[\frac{B_{n,t}}{C_{j,t+1} \times P_{t+1}}\right] \tag{18}$$

where $B_{n,t}$ represents the dividends provided to households by bank *n*, $L_{n,t}$, the loans granted to firms by bank *n*, $D_{n,t}$, the deposits made by households at bank *n*, and $X_{n,t}$, the money injected by the Central Bank in bank *n*

with $$L_{n,t} \le D_{n,t} \times X_{n,t}$$

under the constraint:

$$B_{n,t} = D_{n,t} + R_{i,t} \times L_{n,t} - R_{j,t} \times D_{n,t} - L_{n,t} + X_{n,t} \tag{19}$$

By aggregating the microeconomic elements at macroeconomic level, we get the following equilibrium equations, the model variables being denoted hereinafter without their related index:

$$\sum_j W_{j,t} = \sum_i W_{i,t}$$

$$\sum_j N_{j,t} = \sum_i N_{i,t}$$

$$\sum_j R_{j,t} = \sum_i R_{i,t}$$

$$\sum_j F_{j,t} = \sum_i F_{i,t}$$

$$\sum_j D_{j,t} = \sum_n D_{n,t}$$

$$\sum_j B_{j,t} = \sum_n B_{n,t}$$

$$\sum_i L_{i,t} = \sum_n L_{n,t}$$

to which should be added the equations of equilibrium on the market of goods and services and on the money market, as follows:

$$Y_t = C_t + I_t \tag{20}$$

$$P_t \times C_t = M_t + X_t \tag{21}$$

There are two stochastic processes rendered by the shocks affecting the economic variables, the shock on technology, following an AR(1) process, with intercept, and the shock on the money stock growth rate, determined as the average between its steady-state value and its previous period value:

$$\ln A_t = \gamma + \ln A_{t-1} + \varepsilon_{A,t} \tag{22}$$

$$\ln m_t = (1-\rho) \times \ln m^* + \rho \times \ln m_{t-1} + \varepsilon_{M,t} \tag{23}$$

with $\varepsilon_{A,t}$ and $\varepsilon_{M,t}$ normally distributed with zero mean and constant variance.

The model variables are rendered stationary by dividing the same, depending on the case, to the technological level (for the real variables), to the money stock (for the nominal variables) and to the technological level – money stock ratio (for the price variable):

$$\overline{U}_t = \frac{U_t}{A_t}$$

where $$U_t = \{Y_t, C_t, I_t, K_t\}$$

$$\overline{V}_t = \frac{V_t}{M_t}$$

where $$V_t = \{D_t, L_t, W_t\}$$

$$\overline{P}_t = P_t \times \frac{A_t}{M_t}$$