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INVOLUNTARY UNEMPLOYMENT IN A NEOCLASSICAL MODEL

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Abstract. We show the existence of involuntary unemployment without assuming wage rigidity using a neoclassical model of consumption and production. We consider a case of indivisible labor supply and increasing returns to scale under monopolistic competition. We derive involuntary unemployment by considering utility maximization of consumers and profit maximization of firms in an overlapping generations (OLG) model with two or three generations. In a two-periods OLG model it is possible that a reduction of the nominal wage rate reduces unemployment. However, if we consider a three-periods OLG model including a childhood period, a reduction of the nominal wage rate does not necessarily reduce unemployment.

JEL classification: E12, E24.

Keywords: involuntary unemployment, indivisible labor supply, two or three-periods overlapping generations model, monopolistic competition, increasing returns to scale.

1. Introduction

According to Otaki (2009) the definition of involuntary unemployment consists of two elements, the nominal wage rate is set above the nominal reservation wage rate, and the employment level and economic welfare never improve by lowering the nominal wage rate.

Umada (1997) derived an upward-sloping labor demand curve from mark-up principle for firms under increasing returns to scale technology, and argued that such an upward-sloping labor demand curve leads to the existence of involuntary unemployment without wage rigidity. Lavoie (2001) presented a similar analysis. But his model of firms' behavior is ad-hoc. Otaki (2009) has shown the existence of involuntary unemployment using efficient wage bargaining according to McDonald and Solow (1981). The arguments of this paper, however, do not depend on bargaining.

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In this paper we consider utility maximization of consumers and profit maximization of firms in an overlapping generations (OLG) model with two or three generations under monopolistic competition with increasing returns to scale technology according to Otaki (2007), (2009), (2011) and (2015), and show the existence of involuntary unemployment without assuming wage rigidity. We consider indivisible labor supply. As discussed by Otaki (2015) (Theorem 2.3) and Otaki (2012), if labor supply is divisible and it can be infinitesimally small, there exists no unemployment.

About the indivisible labor supply also please see Hansen (1985). He studies the existence of unemployed workers and fluctuations in the rate of unemployment over the business cycle with indivisible labor supply. To treat an indivisible labor supply in a representative agent model he assumes that people choose lotteries rather than hours worked. Each person chooses a probability of working, then a lottery determines whether or not he actually works. There is a contract between firms and individuals that commits the individual to work the predetermined number of hours with the probability which is chosen by an individual. The contract is being traded, so the individual is paid whether he works or not. The firm provides complete unemployment insurance to the workers.

However, we do not consider a representative consumer. We analyze utility maximization of an employed consumer and that of an unemployed consumer separately.

In this paper similarly to Otaki (2007), we derive a fiscal multiplier (or the Keynesian cross) from the maximization behavior of consumers and firms and market clearing conditions. There are several studies from the standpoint of New Keynesian economics on multipliers (Mankiw (1988), Reinhorn (1998), Startz (1989)). However, as Otaki (2007) says, they commonly emphasize the complementarity between consumer incomes and profits, however as proved by Reinhorn (1998), optimal fiscal expenditure is equal to zero. Thus, expansionary fiscal policy is always harmful. We extend the theory using a dynamic OLG model according to Otaki (2007). It allows the government to use seigniorage to finance its expenditure.

In the next section we analyze the relation between indivisibility of labor supply and the existence of involuntary unemployment in a two-periods OLG model. We show that the real wage rate is increasing with respect to the employment, on the other hand the reservation real wage rate for individuals is constant given the expected inflation rate. Thus, when the real wage rate is larger than the reservation real wage rate, there exists no mechanism to reduce the difference between them.

In a two-periods OLG model it is possible that a reduction of the nominal wage rate reduces unemployment. However, if we consider a three-periods OLG model including a childhood period, a reduction of the nominal wage rate does not necessarily reduce unemployment. Please see Section 3. In Appendix we present details of calculations.

2. Indivisible labor supply and involuntary unemployment

We consider a two-periods (young and old) OLG model under monopolistic competition according to Otaki (2007, 2009, 2011 and 2015). There is one factor of production, labor, and there is a continuum of goods indexed by $z \in [0,1]$. Each good is monopolistically produced by Firm z. Consumers are born at continuous density $[0,1]\times[0,1]$ in each period. They can supply only one unit of labor when they are young (period 1).

2.1. Consumers

We use the following notations.

$c^i(z)$: consumption of good z at period i, $i=1,2$.
$p^i(z)$: price of good z at period i, $i=1,2$.
X^i: consumption basket at period i, $i=1,2$.

$$X^i=\left\{\int_0^1 c^i(z)^{1-\frac{1}{\eta}}dz\right\}^{\frac{1}{1-\frac{1}{\eta}}},\qquad i=1,2,\qquad \eta>1.$$

β: disutility of labor, $\beta>0$.
W: nominal wage rate.
Π: profits of firms which are equally distributed to each consumer.
L: employment of each firm and the total employment.
L_f: population of labor or employment at the full-employment state.
$y(L)$: labor productivity, which is increasing with respect to the employment, $y(L)\geq 1$.

δ is the definition function. If a consumer is employed, $\delta=1$; if he is not employed, $\delta=0$. The labor productivity is $y(L)$. It is increasing with respect to the employment of a firm. We define the employment elasticity of the labor productivity as follows.

$$\zeta=\frac{y'}{\frac{y(L)}{L}}.$$

We assume $0<\zeta<1$. Increasing returns to scale means $\zeta>0$. η is (the inverse of) the degree of differentiation of the goods. At the limit when $\eta\to+\infty$, the goods are homogeneous. We assume

$$\left(1-\frac{1}{\eta}\right)(1+\zeta)<1$$

so that the profits of firms are positive.

The utility of consumers of one generation over two periods is

$$U(X^1,X^2,\delta,\beta)=u(X^1,X^2)-\delta\beta.$$

We assume that $u(X^1,X^2)$ is homogeneous of degree one (linearly homogeneous). The budget constraint is

$$\int_0^1 p^1(z)c^1(z)dz+\int_0^1 p^2(z)c^2(z)dz=\delta W+\Pi.$$

$p^2(z)$ is the expectation of the price of good z at period 2. The Lagrange function is

$$\mathcal{L}=u(X^1,X^2)-\delta\beta-\lambda\left(\int_0^1 p^1(z)c^1(z)dz+\int_0^1 p^2(z)c^2(z)dz-\delta W-\Pi\right).$$

λ is the Lagrange multiplier. The first order conditions are

$$\frac{\partial u}{\partial X^1}\left(\int_0^1 c^1(z)^{1-\frac{1}{\eta}}dz\right)^{\frac{\frac{1}{\eta}}{1-\frac{1}{\eta}}} c^1(z)^{-\frac{1}{\eta}} = \lambda p^1(z), \tag{1}$$

and

$$\frac{\partial u}{\partial X^2}\left(\int_0^1 c^2(z)^{1-\frac{1}{\eta}}dz\right)^{\frac{\frac{1}{\eta}}{1-\frac{1}{\eta}}} c^2(z)^{-\frac{1}{\eta}} = \lambda p^2(z). \tag{2}$$

They are rewritten as

$$\frac{\partial u}{\partial X^1}X^1\left(\int_0^1 c^1(z)^{1-\frac{1}{\eta}}dz\right)^{-1} c^1(z)^{1-\frac{1}{\eta}} = \lambda p^1(z)c^1(z), \tag{3}$$

$$\frac{\partial u}{\partial X^2}X^2\left(\int_0^1 c^2(z)^{1-\frac{1}{\eta}}dz\right)^{-1} c^2(z)^{1-\frac{1}{\eta}} = \lambda p^2(z)c^2(z). \tag{4}$$

Let

$$P^1 = \left(\int_0^1 p^1(z)^{1-\eta}dz\right)^{\frac{1}{1-\eta}}, P^2 = \left(\int_0^1 p^2(z)^{1-\eta}dz\right)^{\frac{1}{1-\eta}}.$$

They are prices of the consumption baskets in period 1 and period 2. By some calculations we obtain (please see Appendix)

$$u(X^1,X^2) = \lambda\left[\int_0^1 p^1(z)c^1(z)dz + \int_0^1 p^2(z)c^2(z)dz\right] = \lambda(\delta W + \Pi), \tag{5}$$

$$\frac{P^2}{P^1} = \frac{\frac{\partial u}{\partial X^2}}{\frac{\partial u}{\partial X^1}}, \tag{6}$$

$$P^1X^1 + P^2X^2 = \delta W + \Pi. \tag{7}$$

The indirect utility of consumers is written as follows

$$V = \frac{1}{\varphi(P^1,P^2)}(\delta W + \Pi) - \delta\beta. \tag{8}$$

$\varphi(P^1,P^2)$ is a function which is homogeneous of degree one. The reservation nominal wage rate W^R is a solution of the following equation.

$$\frac{1}{\varphi(P^1,P^2)}(W^R + \Pi) - \beta = \frac{1}{\varphi(P^1,P^2)}\Pi.$$

From this

$$W^R = \varphi(P^1,P^2)\beta.$$

The labor supply is indivisible. If $W > W^R$, the total labor supply is L_f. If $W < W^R$, it is zero. If $W = W^R$, employment and unemployment are indifferent for consumers, and there exists no involuntary unemployment even if $L < L_f$.

Indivisibility of labor supply may be due to the fact that there exists minimum standard of living even in the advanced economy (please see Otaki (2015)).

Let $\rho = \frac{P^2}{P^1}$. This is the expected inflation rate (plus one). Since $\varphi(P^1, P^2)$ is homogeneous of degree one, the reservation real wage rate is

$$\omega^R = \frac{W^R}{P^1} = \varphi(1, \rho)\beta.$$

If the value of ρ is given, ω^R is constant.

Otaki (2007) assumes that the wage rate is equal to the reservation wage rate at the equilibrium. However, there exists no mechanism to equalize them. We assume that β and ω^R are not so large.

2.2. Firms

Let

$$\alpha = \frac{P^1 X^1}{P^1 X^1 + P^2 X^2} = \frac{X^1}{X^1 + \rho X^2}, 0 < \alpha < 1.$$

From (3) $\sim$ (7),

$$\alpha(\delta W + \Pi)\left(\int_0^1 c^1(z)^{1-\frac{1}{\eta}} dz\right)^{-1} c^1(z)^{-\frac{1}{\eta}} = p^1(z).$$

Since

$$X^1 = \frac{\alpha(\delta W + \Pi)}{P^1},$$

we have

$$(X^1)^{\frac{1}{\eta}-1} = \left(\int_0^1 c^1(z)^{1-\frac{1}{\eta}} dz\right)^{-1} = \left(\frac{\alpha(\delta W + \Pi)}{P^1}\right)^{\frac{1}{\eta}-1}.$$

Therefore,

$$\alpha(\delta W + \Pi)\left(\frac{\alpha(\delta W + \Pi)}{P^1}\right)^{\frac{1}{\eta}-1} c^1(z)^{-\frac{1}{\eta}} = \left(\frac{\alpha(\delta W + \Pi)}{P^1}\right)^{\frac{1}{\eta}} P^1 c^1(z)^{-\frac{1}{\eta}} = p^1(z).$$

Thus,

$$c^1(z)^{\frac{1}{\eta}} = \left(\frac{\alpha(\delta W + \Pi)}{P^1}\right)^{\frac{1}{\eta}} P^1 (p^1(z))^{-1}.$$

Hence,

$$c^1(z) = \frac{\alpha(\delta W + \Pi)}{P^1}\left(\frac{p^1(z)}{P^1}\right)^{-\eta}.$$

This is demand for good z of an individual of younger generation. Similarly, his demand for good z in period 2 is

$$c^2(z) = \frac{(1-\alpha)(\delta W + \Pi)}{P^2}\left(\frac{p^2(z)}{P^2}\right)^{-\eta}.$$

Let M be the total savings of consumers of the older generation carried over from their period 1. It is written as

$$M = (1-\alpha)(\overline{W}\overline{L} + L_f\overline{\Pi}).$$

$\overline{W}$, $\overline{L}$ and $\overline{\Pi}$ are the nominal wage rate, the employment and the profit in the previous period. Then, their demand for good z is

$$\frac{M}{P^1}\left(\frac{p^1(z)}{P^1}\right)^{-\eta}.$$

The government expenditure constitutes the national income as well as consumptions of younger and older generations. The total demand for good z is written as

$$c(z) = \frac{Y}{P^1}\left(\frac{p^1(z)}{P^1}\right)^{-\eta}.$$

Y is the effective demand defined by

$$Y = \alpha(WL + L_f\Pi) + G + M.$$

G is the government expenditure (about this demand function please see Otaki (2007), (2009)). The total employment, the total profits and the total government expenditure are

$$\int_0^1 Ldz = L, \int_0^1 \Pi dz = \Pi, \int_0^1 Gdz = G.$$

We have

$$\frac{\partial c(z)}{\partial p^1(z)} = -\eta\frac{Y}{P^1}\frac{p^1(z)^{-1-\eta}}{(P^1)^{-\eta}} = -\eta\frac{c(z)}{p^1(z)}.$$

From $c(z) = Ly(L)$,

$$\frac{\partial L}{\partial p^1(z)} = \frac{1}{y(L) + Ly'}\frac{\partial c(z)}{\partial p^1(z)}.$$

The profit of Firm z is

$$\pi(z) = p^1(z)c(z) - \frac{W}{y(L)}c(z).$$

P^1 is given for Firm z. $y(L)$ is the productivity of labor, which is increasing with respect to the employment L.

The employment elasticity of the labor productivity is

$$\zeta = \frac{y'}{\frac{y(L)}{L}}.$$

The condition for profit maximization with respect to $p^1(z)$ is

$$c(z) + \left[p^1(z) - \frac{y(L) - c(z)y'\frac{1}{y(L)+Ly'}}{y(L)^2}W\right]\frac{\partial c(z)}{\partial p^1(z)}$$

$$= c(z) + \left[p^1(z) - \frac{1 - Ly'\frac{1}{y(L)+Ly'}}{y(L)}W\right]\frac{\partial c(z)}{\partial p^1(z)}$$

$$= c(z) + \left[p^1(z) - \frac{W}{y(L) + Ly'}\right]\frac{\partial c(z)}{\partial p^1(z)} = 0.$$

From this

$$p^1(z) = \frac{W}{y(L) + Ly'} - \frac{c(z)}{\frac{\partial c(z)}{\partial p^1(z)}} = \frac{W}{(1+\zeta)y(L)} + \frac{1}{\eta}p^1(z).$$

Therefore, we obtain

$$p^1(z) = \frac{W}{\left(1 - \frac{1}{\eta}\right)(1+\zeta)y(L)}.$$

With increasing returns to scale, since $\zeta > 0$, $p^1(z)$ is lower than that in a case without increasing returns to scale given the value of W.

2.3. Involuntary unemployment

Since the model is symmetric, the prices of all goods are equal. Then,

$$P^1 = p^1(z).$$

Hence

$$P^1 = \frac{W}{\left(1-\frac{1}{\eta}\right)(1+\zeta)y(L)}. \quad (9)$$

The real wage rate is

$$\omega = \frac{W}{P^1} = \left(1 - \frac{1}{\eta}\right)(1+\zeta)y(L).$$

If ζ is constant, this is increasing with respect to L.
The aggregate supply of the goods is equal to

$$WL + L_f\Pi = P^1Ly(L).$$

The aggregate demand is

$$\alpha(WL + L_f\Pi) + G + M = \alpha P^1Ly(L) + G + M.$$

Since they are equal,

$$P^1Ly(L) = \alpha P^1Ly(L) + G + M, \quad (10)$$

or

$$P^1Ly(L) = \frac{G+M}{1-\alpha}. \quad (11)$$

In real terms

$$Ly(L) = \frac{1}{1-\alpha}(g+m), \quad (12)$$

where

$$g = \frac{G}{P^1}, \qquad m = \frac{M}{P^1}.$$

$\frac{1}{1-\alpha}$ is a multiplier. (12) means that the employment L is determined by $g + m$. It can not be larger than L_f. However, it may be strictly smaller than L_f $(L < L_f)$. Then, there exists *involuntary umemployment*. Since the real wage rate $\omega = \left(1 - \frac{1}{\eta}\right)(1 + \zeta)y(L)$ is increasing with respect to L, and the reservation real wage rate ω^R is constant, if $\omega > \omega^R$ there exists no mechanism to reduce the difference between them without increasing unemployment.

Figure 1 depicts the relation between the real wage rate and the employment, where L is obtained by

$$L = \frac{1}{(1-\alpha)y(L)}(g + m).$$

E is the equilibrium point.

If we consider the following budget constraint for the government with a lump-sum tax T on the younger generation consumers,

$$G = T,$$

the aggregate demand is

$$\alpha(WL + L_f \Pi - G) + G + M = \alpha(P^1 L_y - G) + G + M.$$

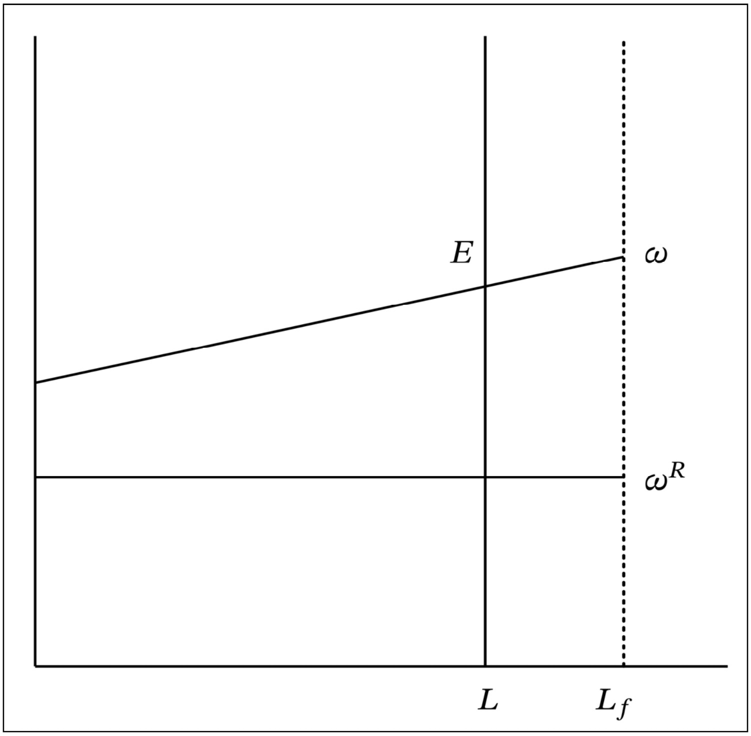

Figure 1: Relation between ω and L

Then, we get

$$L = \frac{1}{(1-\alpha)y}[(1-\alpha)g + m]. \tag{13}$$

This equation means that the balanced budget multiplier is 1.

2.4. Summary of discussions

The real wage rate is determined by a parameter of product differentiation, the labor productivity and its elasticity with respect to the employment. The real aggregate demand and the employment level are determined by the value of $g + m$. It does not depend on the real wage rate. The employment may be smaller than the population of labor, then there exists involuntary unemployment. As mentioned in Introduction Otaki (2009) has shown the existence of involuntary unemployment using efficient wage bargaining according to McDonald and Solow (1981). The arguments of this paper, however, do not depend on bargaining.

From (12) we derive a fiscal multiplier by the government expenditure without tax. It is larger than one. Also, we showed that the balanced budget multiplier is one. These results are obtained using OLG model. It is like the results in Otaki (2007) and is contrast to the results in Mankiw (1988), Startz (1989) and Reinhorn (1998).

The real wage rate is increasing with respect to the employment and the reservation real wage rate is constant. Then, if the real wage rate is larger than the reservation real wage rate, there exists no mechanism to reduce the difference between them without increasing unemployment.

The firms maximize their profits given the demand fuunctions of their goods. Thus, they are happy. Employed consumers determine their consumptions to maximize their utility, and so they are happy. Unemployed consumers are leaved as the only unhappy party.They are willing to work at less than the prevailing real wage, but firms are not hiring because they already maximize their profits[1].

2.5. On a reduction of the nominal wage rate

A reduction of the nominal wage rate induces a proportionate reduction of the prices even when there exists involuntary unemployment (please see (9)) and the employment and the outputs do not change[2]. It does not rescue involuntary unemployment. In Proposition 2.1 of Otaki (2016), it is stated as follows: "Suppose that the nominal wage sags. Then, as far as its indirect effects on the aggregate demand are negligible, this only results in causing a proportionate reduction of the price level. In other words, a reduction of the nominal wage never rescues workers who are involuntarily unemployed."

There may exist *indirect effects* on the aggregate demand of a reduction of the nominal wage rate. If the prices of the goods fall, the real value of the consumption of the older generation may increase, then unemployment may be reduced. This effect is similar to the so-called real balance effect (or Pigou effect). However, if we

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[1] These descriptions are like those in p.142 of Harvey (2016).

[2] By increasing returns to scale, if the employment and the output increase (decrease), the reduction rate of the prices is larger (smaller) than the reduction rate of the nominal wage rate.

consider a three-periods OLG model including a childhood period, a reduction of the nominal wage rate does not necessarily reduce unemployment, and may increase unemployment. Please see Section 3.

In the model of this section no mechanism determines the nominal wage rate. When the nominal value of $G+M$ increases, the nominal aggregate demand and supply increase. If the nominal wage rate rises (for example, by monetary or fiscal policy), the prices also rise. When the rate of an increase in the nominal wage rate is smaller than the rate of an increase in $G+M$, the real aggregate supply and the employment increase. Partition of the effects by an increase in $G+M$ into a rise in the nominal wage rate (and the prices) and an increase in the employment may be determined by bargaining between labor and firm.

2.6. Full-employment case

If $L=L_f$, full-employment is realized. Then, (12) is written as

$$L_f y(L_f)=\frac{1}{1-\alpha}(g+m). \tag{14}$$

Since L_f is constant, this is an identity not an equation. On the other hand, (12) is an equation not an identity. (14) should be written as

$$\frac{1}{1-\alpha}(g+m)\equiv L_f y(L_f).$$

From this we have

$$P^1=\frac{1}{(1-\alpha)L_f y(L_f)}(G+M),$$

where

$$g=\frac{G}{P^1},\qquad m=\frac{M}{P^1}.$$

Therefore, the price level P^1 is determined by $G+M$, which is the sum of nominal values of government expenditure and consumption of the older generation. Also the nominal wage rate is determined by

$$W=\left(1-\frac{1}{\eta}\right)(1+\zeta)y(L_f)P^1.$$

2.7. Steady state

Consider a steady state with $\rho=1$. Let T be the tax revenue for the government expenditure, G, then (10) is written as

$$\alpha(P^1Ly(L)-T)+G+M=P^1Ly(L).$$

The savings of the consumers of the younger generation is

$$(1-\alpha)(P^1Ly(L)-T)=G-T+M.$$

Since at the steady state this is equal to M, which is the consumption of the older generation, we need $G=T$.Thus, we require the balanced budget for the steady state.

2.8. Money demand and supply at the steady state

The demand for money is the sum of
1. savings of the younger generation,
2. tax payment,

The supply of money is the sum of
1. consumption of the older generation,
2. government expenditure,

At the steady state where the price of the good is constant, we have

savings of the younger generation=consumption of the older generation,
tax payment=government expenditure.

Therefore, the demand for money is equal to the supply of money.

3.Three-periods overlapping generations (OLG) model

3.1. Analyses of involuntary unemployment

We add a childhood period (period 0) to a OLG model with two periods, younger period (period 1, working period) and older period (period 2, retired period). In a childhood period people consume the goods by borrowing money from their parents generation (the younger generation) and repay the debts in the next period. Savings of the younger generation may be insufficient for the consumption of the childhood generation. Thus, we assume that the childhood generation consumers can borrow student loan (or scholarship which needs to be paid back) from the government. They must repay the student loan in their period 1 (when they belong to the younger generation). Therefore, in period 1 the consumers of the younger generation have to save money for their consumptions in period 2 (when they belong to the older generation) and repay their debts and student loan. Since the consumers make their consumption plans at the beginning of period 1 (working period), their consumptions in the childhood period are constant. We consider the following utility function of a consumer who is employed

$$u(X^1, X^2, D) - \delta\beta,$$

where

$$D = \left\{\int_0^1 \hat{c}(z)^{1-\frac{1}{\eta}} dz\right\}^{\frac{1}{1-\frac{1}{\eta}}}.$$

$\hat{c}(z)$ is consumption of good z in the childhood period. It is constant. Thus, D is constant.

If a consumer is not employed in his period 1, he can not repay his debt. Therefore, we assume that unemployed consumers receive unemployment benefits from the government. They are covered by taxes on employed consumers of the younger generation. Let R be the unemployment benefit, Θ be the tax for the unemployment benefit. Then, the budget constraint for an employed consumer is

$$\int_0^1 p^1(z)c^1(z)dz + \int_0^1 p^2(z)c^2(z)dz = W - D - \Theta + \Pi.$$

$D + \Theta$ is the sum his own debt repayment and the tax for repayment of the debt of unemployed consumers. Since Θ satisfies

$$D(L_f - L) = L\Theta,$$

we have

$$D + \Theta = \frac{L_f D}{L}.$$

The value of the right-hand side of this equation is given for an employed consumer. The budget constraint of an unemployed consumer is

$$\int_0^1 p^1(z)c^1(z)dz + \int_0^1 p^2(z)c^2(z)dz = R - D + \Pi = \Pi.$$

R is not used for consumption of an unemployed consumer in period 1. If the government aids consumptions of unemployed consumers, it is another policy.

Analyses of consumptions in the younger generation and the older generation are similar to those in the previous case (two-periods OLG model). Let

$$\alpha = \frac{P^1X^1}{P^1X^1 + P^2X^2}.$$

Denote the savings of the older generation by M. Then, the effective demand is

$$Y = \alpha[(W - D - \Theta)L + L_f\Pi] + L_f D' + G + M. \quad (15)$$

D' is the consumption in the childhood period of consumers of the next generation. It is constant. The difference between the two-periods OLG model and the three-periods OLG model exists in the effective demand.

Profit maximization of firms implies

$$P^1 = \frac{W}{\left(1 - \frac{1}{\eta}\right)(1 + \zeta)y(L)}.$$

Using the above effective demand and this condition we can analyze involuntary unemployment. Let us compare (15) with the effective demand in a two-periods OLG model,

$$Y = \alpha(WL + L_f\Pi) + G + M.$$

The difference between them is

$$L_f D' - \alpha(D + \Theta)L.$$

In the case of three-periods OLG model (10), (11) and (12) are written as

$$P^1Ly(L) = \alpha P^1Ly(L) - \alpha(D + \Theta)L + L_f D' + G + M \quad (16)$$

$$= \alpha P^1Ly(L) - \alpha L_f D + L_f D' + G + M,$$

$$P^1 Ly(L) = \frac{L_f D' + G + M - \alpha L_f D}{1 - \alpha},$$

and

$$Ly(L) = \frac{L_f d' + g + m - \alpha L_f d}{1 - \alpha},$$

where

$$g = \frac{G}{P^1}, m = \frac{M}{P^1}, d' = \frac{D'}{P^1}, d = \frac{D}{P^1}.$$

If the value of L obtained from this equation is smaller than L_f, there exists involuntary unemployment.

3.2. Steady state

Consider a steady state with $\rho = 1$. Let T be the tax revenue for the government expenditure, G, then (16) is written as

$$\alpha(P^1 Ly(L) - T - L_f D) + L_f D' + G + M = P^1 Ly(L).$$

G does not include student loan. Since at the steady state where $\rho = 1$ we have $D = D'$, the savings of the consumers of the younger generation is

$$(1 - \alpha)(P^1 Ly(L) - T - L_f D) = G - T + L_f D' - L_f D + M = G - T + M.$$

this is equal to M, which is the consumption of the older generation, at the steady state, we need $G = T$.

3.3. Money demand and supply at the steady state

The demand for money is the sum of
1. savings of the younger generation,
2. tax payment,
3. repayment of student loan,
4. repayment of other debt.

The supply of money is the sum of
1. lending of the younger generation,
2. consumption of the older generation,
3. government expenditure,
4. supply of student loan

At the steady state where the price of the good is constant, we have

savings of the younger generation=consumption of the older generation,
repayment of debt other than student loan=lending of the younger generation,
repayment of student loan=supply of student loan,
tax payment=government expenditure.

Therefore, the demand for money is equal to the supply of money. The taxes for repayment of the debts of unemployed consumers are included in the repayment of student loan and the repayment of debts other than student loan, not "the tax revenue".

3.4. On a reduction of the nominal wage rate

If the nominal wage rate reduces, the prices of the goods reduce. Without any special policy even if the prices of the goods reduce, we can consider that the real values of the government expenditure, g, and the consumption in the childhood period of the next generation, d', are maintained. On the other hand, the nominal values of the consumption of the older generation, M, the debt (including the student loan) of the younger generation, D, and the tax for repayment of the debt, Θ, are maintained even if the prices of the good reduce. Therefore, a reduction of the nominal wage rate increases or decreases the effective demand and employment whether

$$M - \alpha(D + \Theta)L = M - \alpha L_f D$$

is positive or negative. Thus, there may exist posive and negative real balance effects.

Since at the steady state

$$M = (1 - \alpha)(P^1 Ly - T - L_f D),$$

we obtain

$$M - \alpha L_f D = (1 - \alpha)(P^1 Ly - T) - L_f D. \quad (17)$$

Whether this is positive or negative is not clear. It depends on whether savings for the retirement stage is larger, or consumption in the childhood stage is large. In the former case (17) is likely to be positive, and in the latter case it is likely to be negative. The relation between L and L_f, that is, whether the situation is close to full employment or not, or L is large or not affects the sign of (17). In the former case it is likely to be positive, and in the latter case it is likely to be negative. Also if α, which is the marginal propensity to consume of the younger generation, is large, (17) is likely to be negative. Thus, a reduction of the nominal wage rate does not necessarily reduces involuntary unemployment. If the existence of involuntary unemployment induces a reduction of the nominal wage rate and $M - \alpha L_f D < 0$, involuntary unemployment increases and the state goes away from the full-employment state. The discussion in this section is from the different perspectives of the real balance effect for which the argument was fought by Pigou (1943) and Kalecki (1944).

4. Concluding Remarks

In this paper we have examined the existence of involuntary umemployment under indivisibility of labor supply using a monopolistic competition model with increasing returns to scale. Mainly, we have shown the following results.

1). We have derived involuntary unemployment from indivisibility of labor supply. We think that although the labor supply must not be infinitely divisible, it need not be infinitely indivisible.

2). We have shown that a fiscal multiplier by the government expenditure without tax is larger than one and the balanced budget multiplier is one.

How the so-called the first postulate (relation between the price and the marginal cost based on profit maximization) and the second postulate (relation between the real wage rate and labor supply) of classical economics by Keynes are treated in this paper? Since we determine the prices of the goods according to profit maximization behavior of firms, we accept the monopolistic competition version of the first postulate. Because consumers determine consumptions and labor supply (one or zero) to maximize their utility, we accept the second postulate for employed consumers. Unemployed consumers determine their consumptions given their unemployment situation. Thus, they also choose their optimal behaviors. However, their utility is apparently lower than that when they are employed. They want to be employed. Therefore, for unemployed consumers we do not accept the second postulate.

In a model of consumption and labor supply it is usually assumed that there exists only one consumer, or situations of all consumers are the same. Then, involuntary unemployment means that working hours of consumers are shorter than those they require. But, unemployment essentially means that a consumer is not employed by any firm. Therefore, unemployed consumers should be treated in distinction from employed consumers.

The limit of this paper is that there exists no capiital and investment of firms, and the good is produced by only by labor. The analysis of involuntary unemployment in the case where the good is produced by labor and capital is a theme of future research. In the future research we also want to consider the effects of fiscal policies in a state with involuntary unemployment.

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Appendix: Derivations of (5), (6), (7) and (8)

From (3) and (4)

$$\frac{\partial u}{\partial X^1}X^1\left(\int_0^1 c^1(z)^{1-\frac{1}{\eta}}dz\right)^{-1}\int_0^1 c^1(z)^{1-\frac{1}{\eta}}dz=\frac{\partial u}{\partial X^1}X^1=\lambda\int_0^1 p^1(z)c^1(z)dz,$$

$$\frac{\partial u}{\partial X^2}X^2\left(\int_0^1 c^2(z)^{1-\frac{1}{\eta}}dz\right)^{-1}\int_0^1 c^2(z)^{1-\frac{1}{\eta}}dz=\frac{\partial u}{\partial X^2}X^2=\lambda\int_0^1 p^2(z)c^2(z)dz.$$

Since $u(X^1,X^2)$ is homogeneous of degree one,

$$u(X^1,X^2)=\frac{\partial u}{\partial X^1}X^1+\frac{\partial u}{\partial X^2}X^2.$$

Thus, we obtain

$$\frac{\int_0^1 p^1(z)c^1(z)dz}{\int_0^1 p^2(z)c^2(z)dz}=\frac{\frac{\partial u}{\partial X^1}X^1}{\frac{\partial u}{\partial X^2}X^2},$$

and

$$u(X^1, X^2) = \lambda\left[\int_0^1 p^1(z)c^1(z)dz + \int_0^1 p^2(z)c^2(z)dz\right] = \lambda(\delta W + \Pi).$$

From (1) and (2), we have

$$\left(\frac{\partial u}{\partial X^1}\right)^{1-\eta}\left(\int_0^1 c^1(z)^{1-\frac{1}{\eta}}dz\right)^{-1} c^1(z)^{1-\frac{1}{\eta}} = \lambda^{1-\eta}p^1(z)^{1-\eta},$$

and

$$\left(\frac{\partial u}{\partial X^2}\right)^{1-\eta}\left(\int_0^1 c^2(z)^{1-\frac{1}{\eta}}dz\right)^{-1} c^2(z)^{1-\frac{1}{\eta}} = \lambda^{1-\eta}p^2(z)^{1-\eta}.$$

They mean

$$\left(\frac{\partial u}{\partial X^1}\right)^{1-\eta}\left(\int_0^1 c^1(z)^{1-\frac{1}{\eta}}dz\right)^{-1} \int_0^1 c^1(z)^{1-\frac{1}{\eta}}dz = \lambda^{1-\eta}\int_0^1 p^1(z)^{1-\eta}dz,$$

and

$$\left(\frac{\partial u}{\partial X^2}\right)^{1-\eta}\left(\int_0^1 c^2(z)^{1-\frac{1}{\eta}}dz\right)^{-1} \int_0^1 c^2(z)^{1-\frac{1}{\eta}}dz = \lambda^{1-\eta}\int_0^1 p^2(z)^{1-\eta}dz.$$

Then, we obtain

$$\frac{\partial u}{\partial X^1} = \lambda\left(\int_0^1 p^1(z)^{1-\eta}dz\right)^{\frac{1}{1-\eta}} = \lambda P^1,$$

and

$$\frac{\partial u}{\partial X^2} = \lambda\left(\int_0^1 p^2(z)^{1-\eta}dz\right)^{\frac{1}{1-\eta}} = \lambda P^2.$$

From them we get

$$u(X^1, X^2) = \lambda(P^1X^1 + P^2X^2),$$

$$\frac{P^2}{P^1} = \frac{\frac{\partial u}{\partial X^2}}{\frac{\partial u}{\partial X^1}}, \tag{6}$$

and

$$P^1X^1 + P^2X^2 = \delta W + \Pi. \tag{7}$$

Since $u(X^1, X^2)$ is homogeneous of degree one, λ is a function of P^1 and P^2, and $\frac{1}{\lambda}$ is homogeneous of degree one because proportional increases in P^1 and P^2 reduce X^1 and X^2 at the same rate given $\delta W + \Pi$. We obtain the following indirect utility function.

$$V = \frac{1}{\varphi(P^1, P^2)}(\delta W + \Pi) - \delta\beta. \tag{8}$$

$\varphi(P^1, P^2)$ is a function which is homogenous of degree one.

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