STUDIA UNIVERSITATIS BABEȘ-BOLYAI OECONOMICA
VOLUME 67, ISSUE 1, 2022, pp. 11-26
DOI: 10.2478/subboec-2022-0002

INVOLUNTARY UNEMPLOYMENT UNDER ONGOING NOMINAL WAGE RATE DECLINE IN OVERLAPPING GENERATIONS MODEL

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Abstract: We analyze involuntary unemployment based on consumers' utility maximization and firms' profit maximization behavior with ongoing nominal wage rate decline. We consider a three-periods overlapping generations (OLG) model with a childhood period as well as younger and older periods under monopolistic competition with increasing, decreasing or constant returns to scale technology. When there exists involuntary unemploymnet, the nominal wage rate may decline. We examine the existenbce of involuntary unemployment in that model with ongoing mominal wage rate decline (or deflation). Even if the nominal wage rate declines, we have a steady state with involuntary unemployment and constant output and employment. We need budget deficit or budget surplus to maintain the steady state depending on whether real balance effect is positive or negative. Also we examine the possibility to achieve full-employment by fiscal policy.

JEL classifications: E12, E24, E31
Keywords: involuntary unemployment, monopolistic competition, wage rate decline, real balance effect.

1. Introduction

In this paper we examine the existence of involuntary unemployment with ongoing nominal wage rate decline under monopolistic competiton. Involuntary unemployment is a phenomenon that workers are willing to work at the market wage or just below but are prevented by factors beyond their control, mainly, deficiency of aggregate demand. Umada (1997) derived an upward-sloping labor demand curve from the mark-up principle for firms, and argued that such an upward-sloping labor demand curve leads to the existence of involuntary unemployment without wage rigidity (Lavoie (2001) presented a similar analysis based on Kalecki (1944)). But his model of firm behavior is ad-hoc. Otaki (2009) says that there exists involuntary unemployment for two reasons: (i) the nominal wage rate is set above the reservation

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nominal wage rate; and (ii) the employment level and economic welfare never improve by lowering the nominal wage rate. He assume indivisibility (or inelasticity) of individual labor supply, and has shown the existence of involuntary unemployment using efficient wage bargaining according to McDonald and Solow (1981). The arguments of this paper, however, do not depend on bargaining. If labor supply is indivisible, it may be 1 or 0. On the other hand, if it is divisible, it takes a real value between 0 and 1. Tanaka (2020a) and (2020b) analyzed the existence of involuntary unemployment under perfect competiton or monopolistic competition with indivisible labor supply. As discussed by Otaki (2015) (Theorem 2.3) and Otaki (2012), if the labor supply is divisible and very small, no unemployment exists (About indivisible labor supply also please see Hansen (1985)). However, we show that even if labor supply is divisible, unless it is so small, there may exist involuntary unemployment. We consider consumers' utility maximization and firms' profit maximization in an overlapping generations (OLG) model under monopolistic competition according to Otaki (2007, 2009, 2011, 2015, 2016), and demonstrate the existence of involuntary unemployment without the assumption of wage rigidity.

In the next section we show the existence of involuntary unemployment under monopolistic competition with increasing or decreasing or constant returns to scale technology using a three-periods OLG model with a childhood period as well as younger (working) and older (retired) periods. Also we consider pay-as-you go pension system for the older generation. In a simple two-periods OLG model declines in the nominal wage rate and the price of goods may increase consumption and employment by the real balance effect. In such a model consumers have savings for future consumption, but no debt. In a three-periods model with childhood period they consume goods in their childhood period by borrowing money from (employed) consumers of the previous generation and scholarships, and must repay their debts in the next period. Real value of the debt is increased by declines in the nominal wage rate and the price. In addition to this configuration we consider a pay-as-you go pension system for the older generation which may reduce the savings of consumers. Then, consumptions and employment may be decreased by falling of the nominal wage rate. We think that our model is more general and realistic than a simple two-periods OLG model.

When there exists involuntary unemploymnet, the nominal wage rate may decline. In Section 3 we examine the effects of ongoing decline in the nominal wage rate and the price. In our three-periods OLG model with pay-as-you-go pension increases in consumption and employment due to declines in the nominal wage rate and the price of goods might be negative, that is, there may be negative real balance effect. The positive real balance effect is the fact that a decline in the nominal wage rate increases consumption, and the negative real balance effect means that a decline in the nominal wage rate decreases consumption. The real balance effect is positive (or negative) when the difference between the savings of the older generation consumers net of the pay-as-you-go pensions and the debt of the younger generation consumers is positive (or negative). Whether a budget deficit or a budget surplus is needed to maintain a steady state with constant income and employment and ongoing nominal wage rate decline depends on whether the real balance effect is positive or negative. Also we examine the possibility to achieve and maintain full-employment by fiscal policy.

As we will state in the concluding remarks, the main limitation of this paper is that the goods are produced by only labor and there exists no capital and investment of firms. A study of the problem of involuntary unemployment in such a situation is the theme of future research.

Schultz (1992) showed that there does not exist involuntary unemployment in an overlapping generations model. His arguments depends on positive real balance effect on consumption of the older generation consumers. We consider a three-periods overlapping generations model with pay-as-you go pension to explore the possibility of negative real balance effect.

2. Existence of involuntary unemployment

2.1 Consumers

We consider a three-periods (0: childhood, 1: younger or working, and 2: older or retired) OLG model under monopolistic competition. It is a re-arrangement and an extension of the model put forth by Otaki (2007), (2009), (2012). The structure of our model is as follows.

1. There is one factor of production, labor, and there is a continuum of perishable goods indexed by $z \in [0,1]$. Good z is monopolistically produced by firm z with increasing or decreasing or constant returns to scale technology.

2. Consumers consume the goods during the childhood period (Period 0). This consumption is covered by borrowing money from (employed) consumers of the younger generation and/or scholarships. They must repay these debts in their Period 1. However, unemployed consumers cannot repay their own debts. Therefore, we assume that unemployed consumers receive unemployment benefits from the government, which are covered by taxes on employed consumers of the younger generation.

3. During Period 1, consumers supply l units of labor, repay the debts and save money for their consumption in Period 2. They also pay taxes for the pay-as-you go pension system for the older generation.

4. During Period 2, consumers consume the goods using their savings carried over from their Period 1 earnings and the pay-as-you go pension, which is a lump-sum payment. It is covered by taxes on employed consumers of the younger generation.

5. Consumers determine their consumptions in Periods 1 and 2 and the labor supply at the beginning of Period 1. We assume that their consumption during the childhood period is constant.

We use the following notation.

C_i^e: consumption basket of an employed consumer in Period $i,\ i = 1,2$.
C_i^u: consumption basket of an unemployed consumer in Period $i,\ i = 1,2$.
$c_i^e(z)$: consumption of good z of an employed consumer in Period $i,\ i = 1,2$.
$c_i^u(z)$: consumption of good z of an unemployed consumer in Period $i,\ i = 1,2$.
D: consumption basket of an individual in the childhood period, which is constant.
P_i: the price of consumption basket in Period $i,\ i = 1,2$.
$p_i(z)$: the price of good z in Period $i,\ i = 1,2$.

$\rho = \frac{P_2}{P_1}$: (expected) inflation rate (plus one).
W: nominal wage rate.
R: unemployment benefit for an unemployed consumer. $R = D$.
$\widehat{D}$: consumption basket in the childhood period of a next generation consumer.
Q: pay-as-you-go pension for a consumer of the older generation.
Θ: tax payment by an employed consumer for the unemployment benefit.
$\hat{Q}$: pay-as-you-go pension for a consumer of the younger generation when he retires.
Ψ: tax payment by an employed consumer for the pay-as-you-go pension.
Π: profits of firms which are equally distributed to each consumer.
l: labor supply of an individual.
$\Gamma(l)$: disutility function of labor, which is increasing and convex.
L: total employment.
L_f: population of labor or employment in the full-employment state.
$y(Ll)$: labor productivity, which is increasing or decreasing or constant with respect to "employment $\times$ labor supply" (Ll).

We assume that the population L_f is constant. In our model there is no capital, and the interest rate is zero.

We consider a two-step method to solve utility maximization of consumers such that:

1. Employed and unemployed consumers maximize their utility by determining consumption baskets in Periods 1 and 2 given their income over two periods:
2. Then, they maximize their consumption baskets given the expenditure in each period.

We define the elasticity of the labor productivity with respect to "employment $\times$ labor supply" as follows,

$$\zeta = \frac{y'}{\frac{y(Ll)}{Ll}}.$$

We assume that $-1 < \zeta < 1$, and ζ is constant. Increasing (decreasing or constant) returns to scale means $\zeta > 0$ ($\zeta < 0$ or $\zeta = 0$).

Since the taxes for unemployed consumers' debts are paid by employed consumers of the same generation, D and Θ satisfy the following relationship.

$$D(L_f - L) = L\Theta.$$

This means

$$L(D + \Theta) = L_f D.$$

The price of the consumption basket in Period 0 is assumed to be 1. Thus, D is the real value of the consumption in the childhood period of consumers.

Since the taxes for the pay-as-you-go pension system are paid by employed consumers of younger generation, Q and Ψ satisfy the following relationship:

$$L\Psi = L_f Q.$$

The utility function of employed consumers of one generation over three periods is written as

$$u(C_1^e, C_2^e, D) - \Gamma(l).$$

We assume that $u(\cdot)$ is a homothetic utility function. The utility function of unemployed consumers is

$$u(C_1^u, C_2^u, D).$$

The consumption baskets of employed and unemployed consumers in Period i are

$$C_i^e = \left(\int_0^1 c_i^e(z)^{\frac{\sigma-1}{\sigma}} dz\right)^{\frac{\sigma}{\sigma-1}}, \qquad i = 1,2,$$

and

$$C_i^u = \left(\int_0^1 c_i^u(z)^{\frac{\sigma-1}{\sigma}} dz\right)^{\frac{\sigma}{\sigma-1}}, \qquad i = 1,2.$$

σ is the elasticity of substitution among the goods, and $\sigma > 1$.

The price of consumption basket in Period i is

$$P_i = \left(\int_0^1 p_i(z)^{1-\sigma} dz\right)^{\frac{1}{1-\sigma}}, \qquad i = 1,2.$$

The budget constraint for en employed consumer is

$$P_1 C_1^e + P_2 C_2^e = Wl + \Pi - D - \Theta + \hat{Q} - \Psi.$$

Employed consumers of the younger generation lend money to consumers in the childhood period of the next generation. It is repaid in the next period. The budget constraint for en unemployed consumer is

$$P_1 C_1^u + P_2 C_2^u = \Pi - D + R + \hat{Q}$$

Since $R = D$,

$$P_1 C_1^u + P_2 C_2^u = \Pi + \hat{Q}.$$

Let

$$\alpha = \frac{P_1 C_1^e}{P_1 C_1^e + P_2 C_2^e}, \ 1 - \alpha = \frac{P_2 C_2^e}{P_1 C_1^e + P_2 C_2^e}. \qquad (1)$$

Since the utility functions $u(C_1^e, C_2^e, D)$ and $u(C_1^u, C_2^u, D)$ are homothetic, α is determined by the relative price $\frac{P_2}{P_1}$, and do not depend on the income of the consumers. Therefore, we have

$$\alpha = \frac{P_1 C_1^e}{P_1 C_1^e + P_2 C_2^e} = \frac{P_1 C_1^u}{P_1 C_1^u + P_2 C_2^u},$$

$$1-\alpha = \frac{P_2 C_2^e}{P_1 C_1^e + P_2 C_2^e} = \frac{P_2 C_2^u}{P_1 C_1^u + P_2 C_2^u},$$

From the budget constraints for employed and unemployed consumers we obtain the following demand functions for consumption baskets.

$$C_1^e = \alpha \frac{Wl + \Pi - D - \Theta + \hat{Q} - \Psi}{P_1},\ C_2^e = (1-\alpha)\frac{Wl + \Pi - D - \Theta + \hat{Q} - \Psi}{P_2},$$

and

$$C_1^u = \alpha \frac{\Pi + \hat{Q}}{P_1},\ C_2^u = (1-\alpha)\frac{\Pi + \hat{Q}}{P_2}.$$

Lagrange functions in the second step for employed and unemployed consumers are

$$\mathcal{L}_1^e = \left(\int_0^1 c_1^e(z)^{\frac{\sigma-1}{\sigma}} dz\right)^{\frac{\sigma}{\sigma-1}} - \lambda_1^e \left[\int_0^1 p_1(z) c_1^e(z) dz - \alpha(Wl + \Pi - D - \Theta + \hat{Q} - \Psi)\right], \quad (2)$$

$$\mathcal{L}_2^e = \left(\int_0^1 c_2^e(z)^{\frac{\sigma-1}{\sigma}} dz\right)^{\frac{\sigma}{\sigma-1}} - \lambda_2^e \left[\int_0^1 p_2(z) c_2^e(z) dz - (1-\alpha)(Wl + \Pi - D - \Theta + \hat{Q} - \Psi)\right],$$

$$\mathcal{L}_1^u = \left(\int_0^1 c_1^u(z)^{\frac{\sigma-1}{\sigma}} dz\right)^{\frac{\sigma}{\sigma-1}} - \lambda_1^u \left[\int_0^1 p_1(z) c_1^u(z) dz - \alpha(\Pi + \hat{Q})\right],$$

and

$$\mathcal{L}_2^u = \left(\int_0^1 c_2^u(z)^{\frac{\sigma-1}{\sigma}} dz\right)^{\frac{\sigma}{\sigma-1}} - \lambda_2^u \left[\int_0^1 p_2(z) c_2^u(z) dz - \alpha(\Pi + \hat{Q})\right].$$

λ_1^e, λ_2^e, λ_1^u and λ_2^u are Lagrange multipliers. Solving these maximization problems, the following demand functions of employed and unemployed consumers are derived.

$$c_1^e(z) = \left(\frac{p_1(z)}{P_1}\right)^{-\sigma} \frac{\alpha(Wl + \Pi - D - \Theta + \hat{Q} - \Psi)}{P_1},$$

$$c_2^e(z) = \left(\frac{p_2(z)}{P_2}\right)^{-\sigma} \frac{(1-\alpha)(Wl + \Pi - D - \Theta + \hat{Q} - \Psi)}{P_2},$$

$$c_1^u(z) = \left(\frac{p_1(z)}{P_1}\right)^{-\sigma} \frac{\alpha(\Pi + \hat{Q})}{P_1},$$

and

$$c_2^u(z) = \left(\frac{p_2(z)}{P_2}\right)^{-\sigma} \frac{(1-\alpha)(\Pi + \hat{Q})}{P_2}.$$

About some calculations of these maximization problems please see Appendix. From these analyses we obtain the indirect utility functions of employed and unemployed consumers as follows:

$$V^e=u\left(\alpha\frac{Wl+\Pi-D-\Theta+\hat{Q}-\Psi}{P_1},(1-\alpha)\frac{Wl+\Pi-D-\Theta+\hat{Q}-\Psi}{P_2},D\right)-\Gamma(l),$$

and

$$V^u=u\left(\alpha\frac{\Pi+\hat{Q}}{P_1},(1-\alpha)\frac{\Pi+\hat{Q}}{P_2},D\right).$$

Let

$$\omega=\frac{W}{P_1},\qquad \rho=\frac{P_2}{P_1}.$$

Then, since D is constant, we can write

$$V^e=\varphi\left(\omega l+\frac{\Pi-D-\Theta+\hat{Q}-\Psi}{P_1},\rho\right)-\Gamma(l),$$

$$V^u=\varphi\left(\frac{\Pi+\hat{Q}}{P_1},\rho\right),$$

ω is the real wage rate. Denote

$$I=\omega l+\frac{\Pi-D-\Theta+\hat{Q}-\Psi}{P_1}. \tag{3}$$

The condition for maximization of V^e with respect to l given ρ is

$$\frac{\partial\varphi}{\partial I}\omega-\Gamma'(l)=0, \tag{4}$$

where

$$\frac{\partial\varphi}{\partial I}=\alpha\frac{\partial u}{\partial C_1^e}+(1-\alpha)\frac{\partial u}{\partial C_2^e}.$$

Given P_1 and ρ the labor supply is a function of ω. From (4) we get

$$\frac{dl}{d\omega}=\frac{\frac{\partial\varphi}{\partial I}+\frac{\partial^2\varphi}{\partial I^2}\omega l}{\Gamma''(l)-\frac{\partial^2\varphi}{\partial I^2}\omega^2}. \tag{5}$$

If $\frac{dl}{d\omega}>0$, the labor supply is increasing with respect to the real wage rate ω.

2.2 Firms

Let $d_1(z)$ be the total demand for good z by younger generation consumers in Period 1. Then,

$$d_1(z) = \left(\frac{p_1(z)}{P_1}\right)^{-\sigma} \frac{\alpha(WLl + L_f\Pi - LD - L\Theta + L_f\hat{Q} - L\Psi)}{P_1}$$
$$= \left(\frac{p_1(z)}{P_1}\right)^{-\sigma} \frac{\alpha\left(WLl + L_f\Pi - L_fD + L_f\hat{Q} - L_fQ\right)}{P_1}.$$

This is the sum of the demand of employed and unemployed consumers. Note that $\hat{Q}$ is the pay-as-you-go pension for younger generation consumers in their Period 2. Similarly, their total demand for good z in Period 2 is written as

$$d_2(z) = \left(\frac{p_2(z)}{P_2}\right)^{-\sigma} \frac{(1-\alpha)\left(WLl + L_f\Pi - L_fD + L_f\hat{Q} - L_fQ\right)}{P_2}.$$

Let $\overline{d_2(z)}$ be the demand for good z by the older generation. Then,

$$\overline{d_2(z)} = \left(\frac{p_1(z)}{P_1}\right)^{-\sigma} \frac{(1-\bar{\alpha})\left(\bar{W}\bar{L}\bar{l} + L_f\bar{\Pi} - L_f\bar{D} + L_fQ - L_f\bar{Q}\right)}{P_1},$$

where $\bar{W}$, $\bar{\Pi}$, $\bar{L}$, $\bar{l}$, $\bar{D}$ and $\bar{Q}$ are the nominal wage rate, the profits of firms, the employment, the individual labor supply, the debt of an individual, and the pay-as-you-go pension, respectively, during the previous period. $\bar{\alpha}$ is the value of α for the older generation. Q is the pay-as-you-go pension for consumers of the older generation themselves. Let

$$M = (1-\bar{\alpha})\left(\bar{W}\bar{L}\bar{l} + L_f\bar{\Pi} - L_f\bar{D} + L_fQ - L_f\bar{Q}\right).$$

This is the total savings or the total consumption of the older generation consumers including the pay-as-you-go pensions that they receive in their Period 2. It is the planned consumption that is determined in Period 1 of the older generation consumers. Net savings is the difference between M and the pay-as-you-go pensions in their Period 2, as follows:

$$M - L_fQ.$$

Their demand for good z is written as $\left(\frac{p_1(z)}{P_1}\right)^{-\sigma} \frac{M}{P_1}$. Government expenditure constitutes the national income as well as the consumptions of the younger and older generations. Then, the total demand for good z is written as

$$d(z) = \left(\frac{p_1(z)}{P_1}\right)^{-\sigma} \frac{Y}{P_1}, \tag{6}$$

where Y is the effective demand defined by

$$Y = \alpha\left(WLl + L_f\Pi - L_fD + L_f\hat{Q} - L_fQ\right) + G + L_f\hat{D} + M.$$

Note that $\hat{D}$ is consumption in the childhood period of a next generation consumer. G is the government expenditure, except for the pay-as-you-go pensions, scholarships and unemployment benefits (see Otaki (2007), (2015) about this demand function). Now, we assume that G is financed by seigniorage similarly to Otaki (2007), (2009). In a later section, we will consider the government's budget constraint with respect to taxes.

Let L and Ll be employment and the "employment $\times$ labor supply" of firm z. The total employment and the total "employment $\times$ labor supply" are also

$$\int_0^1 Ldz = L, \qquad \int_0^1 Lldz = Ll.$$

The output of firm z is $Lly(Ll)$. At the equilibrium $Lly(Ll) = d(z)$. Then, we have

$$\frac{\partial d(z)}{\partial(Ll)} = y(Ll) + Lly'.$$

From (6)

$$\frac{\partial p_1(z)}{\partial d(z)} = -\frac{p_1(z)}{\sigma d(z)}.$$

Thus

$$\frac{\partial p_1(z)}{\partial(Ll)} = -\frac{p_1(z)(y(Ll) + Lly')}{\sigma d(z)} = -\frac{p_1(z)(y(Ll) + Lly')}{\sigma Lly(Ll)}.$$

The profit of firm z is

$$\pi(z) = p_1(z)Lly(Ll) - LlW.$$

The condition for profit maximization is

$$\frac{\partial \pi(z)}{\partial(Ll)} = p_1(z)(y(Ll) + Lly') - Lly(Ll)\frac{p_1(z)(y(Ll) + Lly')}{\sigma Lly(Ll)} - W$$
$$= p_1(z)(y(Ll) + Lly') - \frac{p_1(z)(y(Ll) + Lly')}{\sigma} - W = 0.$$

Therefore, we obtain

$$p_1(z) = \frac{1}{(1 - \frac{1}{\sigma})(1 + \zeta)y(Ll)}W.$$

Let $\mu = \frac{1}{\sigma}$. Then,

$$p_1(z) = \frac{1}{(1 - \mu)(1 + \zeta)y(Ll)}W.$$

This means that the real wage rate is

$$\omega = (1 - \mu)(1 + \zeta)y(Ll). \qquad (7)$$

With increasing (decreasing or constant) returns to scale, ω is increasing (decreasing or constant) with respect to "employment $\times$ labor supply" Ll.

From (3), (4) and (7), we have

$$\frac{\partial \varphi}{\partial I}(1 - \mu)(1 + \zeta)y(Ll) - \Gamma'(l) = 0,$$

with

$$I = (1-\mu)(1+\zeta)y(Ll)l + \frac{\Pi - D - \Theta + \hat{Q} - \Psi}{P_1}.$$

Then, from (5)

$$\frac{dl}{d(Ll)} = \frac{dl}{d\omega}\frac{d\omega}{d(Ll)} = \frac{\left[\frac{\partial\varphi}{\partial I} + \frac{\partial^2\varphi}{\partial I^2}(1-\mu)(1+\zeta)y(Ll)l\right](1-\mu)(1+\zeta)y'}{\Gamma''(l) - \frac{\partial^2\varphi}{\partial I^2}[(1-\mu)(1+\zeta)y']^2}.$$

Assuming $\frac{dl}{d\omega} > 0$, with increasing (decreasing) returns to scale $y' > 0$ $(y < 0)$, this is positive (negative). Since

$$\frac{d(Ll)}{dL} = l + L\frac{dl}{dL}, \tag{8}$$

we have

$$\frac{dl}{dL} = \frac{dl}{d(Ll)}\frac{d(Ll)}{dL} = \left(l + L\frac{dl}{dL}\right)\frac{dl}{d(Ll)}.$$

Thus,

$$\frac{dl}{dL} = \frac{l}{1 - L\frac{dl}{d(Ll)}}\frac{dl}{d(Ll)}.$$

Usually $\frac{dl}{dL}$ and $\frac{dl}{d(Ll)}$ have the same sign, and we assume $\frac{d(Ll)}{dL} > 0$ in (8). Also, since $-1 < \zeta < 1$, we have

$$\frac{d(Lly(Ll))}{Ll} = y(Ll) + Lly' = y(Ll)(1+\zeta) > 0.$$

Then, the output $Lly(Ll)$ increases by an increase in L.
Since all firms are symmetric,

$$P_1 = p_1(z) = \frac{1}{(1-\mu)(1+\zeta)y(Ll)}W. \tag{9}$$

2.3 Involuntary unemployment

The (nominal) aggregate supply of the goods is equal to

$$WL + L_f\Pi = P_1Lly(Ll).$$

The (nominal) aggregate demand is

$$\alpha\left(WL + L_f\Pi - L_fD + L_f\hat{Q} - L_fQ\right) + G + L_f\hat{D} + M$$
$$= \alpha\left[P_1Lly(Ll) - L_fD + L_f\hat{Q} - L_fQ\right] + G + L_f\hat{D} + M.$$

Since they are equal,

$$P_1Lly(Ll) = \alpha\left[P_1Lly(Ll) - L_fD + L_f\hat{Q} - L_fQ\right] + G + L_f\hat{D} + M,$$

or

$$P_1 Lly(Ll) = \frac{\alpha\left(-L_f D + L_f \hat{Q} - L_f Q\right) + G + L_f \widehat{D} + M}{1-\alpha}.$$

In real terms

$$Lly(Ll) = \frac{\alpha\left(-L_f D + L_f \hat{Q} - L_f Q\right) + G + L_f \widehat{D} + M}{(1-\alpha)P_1}, \tag{10}$$

or

$$Ll = \frac{\alpha\left(-L_f D + L_f \hat{Q} - L_f Q\right) + G + L_f \widehat{D} + M}{(1-\alpha)P_1 y(Ll)}.$$

$\frac{1}{1-\alpha}$ is a multiplier. From (4) and (5) the individual labor supply l is a (usually increasing) function of ω. From (7) ω is a function of Ll. With increasing (decreasing or constant) returns to scale technology it is increasing (decreasing or constant) with respect to Ll or with respect to L given l. The individual labor supply l may be increasing or decreasing in L or Ll. However, we assume that Ll is increasing in L. This requires

$$\frac{dLl}{dL} = l + \frac{dl}{dL} > 0.$$

It means $Ll < L_f l$ for $L < L_f$. The equilibrium value of Ll cannot be larger than $L_f l$. However, it may be strictly smaller than $L_f l$. Then, we have $L < L_f$ and involuntary umemployment exists.

If the government collects a lump-sum tax T from the younger generation consumers, the aggregate supply and demand satisfy

$$P_1 Lly(Ll) = \alpha\left[P_1 Lly(Ll) - T - L_f D + L_f \hat{Q} - L_f Q\right] + G + L_f \widehat{D} + M. \tag{11}$$

2.4 Discussion summary

The real wage rate depends on the elasticity of the labor productivity with respect to “employment × labor supply” and the employment level. But the employment level does not depend on the real wage rate. The real aggregate demand and the employment level are determined by the value of

$$\frac{\alpha\left(-L_f D + L_f \hat{Q} - L_f Q\right) + G + L_f \widehat{D} + M}{P_1}. \tag{12}$$

If the employment is smaller than the labor population, then involuntary unemployment exists.

2.5 Positive and negative real balance effects

The net savings of the older generation consumers is the difference between their savings and the pay-as-you-go pensions. It is written as

$$M - L_f Q.$$

On the other hand, the debts of the younger generation consumers is L_fD. There are two cases about the relation between $M - L_fQ$ and L_fD as follows:

1. Case 1: $M - L_fQ > L_fD$, that is, $M > L_fQ + L_fD$. Then, the net savings of the older generation consumers is larger than the debts of consumers in the childhood period. In this case the real balance effect due to a decline in the price of the goods is positive.

2.Case 2: $M - L_fQ < L_fD$, that is, $M < L_fQ + L_fD$. Then, the net savings of the older generation consumers is smaller than the debts of consumers in the childhood period. In this case the real balance effect due to a decline in the price of the goods is negative.

2.6 The case of full-employment

If $Ll = L_fl$, full-employment is achieved. Then, (10) is re-written as

$$L_fly(L_fl) = \frac{\alpha(-L_fD + L_f\hat{Q} - L_fQ) + G + L_f\hat{D} + M}{(1-\alpha)P_1}. \tag{13}$$

Since L_f and L_fl are constant (if $L = L_f$, ω is constant), this is an identity not an equation. On the other hand, (10) is an equation not an identity. (13) should be written as

$$\frac{\alpha(-L_fD + L_f\hat{Q} - L_fQ) + G + L_f\hat{D} + M}{(1-\alpha)P_1} \equiv L_fly(L_fl).$$

This yields:

$$P_1 = \frac{1}{(1-\alpha)L_fly(L_fl)}[\alpha(-L_fD + L_f\hat{Q} - L_fQ) + G + L_f\hat{D} + M].$$

Then, the nominal wage rate is determined by:

$$W = (1-\mu)(1+\zeta)y(L_fl)P_1.$$

3. Steady state with ongoing nominal wage rate decline and achievement of full-employment

3.1 Steady state with ongoing nominal wage rate decline

If there exists involuntary unemployment, the nominal wage rate may decline. By (9) a decline in the nominal wage rate induces a decline in the price of the goods. We assume that consumers correctly predict a decline in the price. Suppose that the output and the employment are constant, and the price of the goods declines at the rate $\rho - 1 < 0$ from a period to the next period. Let T be the tax revenue. We can assume that $\hat{D} = \rho D$ and $\hat{Q} = \rho Q$. Thus, (11) is written as

$$P_1Lly(Ll) = \alpha[P_1Lly(Ll) - T - L_fD + (\rho-1)L_fQ] + G + \rho L_fD + M. \tag{14}$$

In order to maintain the steady state, the total savings of the younger generation consumers including the pay-as-you-go pension that they will receive must be equal to ρM. Therefore,

$$(1-\alpha)\left[P_1 Lly(Ll) - T - L_f D + (\rho - 1)L_f Q\right] = \\ = G - T + (\rho - 1)L_f(D + Q) + M = \rho M. \quad (15)$$

This means

$$G - T = (\rho - 1)(M - L_f D - L_f Q). \quad (16)$$

We obtain the following prosposition.

Proposition 1

There are two cases.

1. If $M > L_f D + L_f Q$, that is, in the positive real balance effect case, in order to maintain the steady state where the output and the employment are constant with falling prices ($\rho < 1$), a budget surplus $G - T < 0$ is required.

2. If $M < L_f D + L_f Q$, that is, in the negative real balance effect case, in order to maintain the steady state where the output and the employment are constant with falling prices ($\rho < 1$), a budget deficit $G - T > 0$ is required.

3.2 Fiscal policy to achieve full-employment

Let G' and T' be the government expenditure and the tax to achieve full-employment. Then, (14) is written as

$$P_1 L_f ly(L_f l) = \alpha\left[P_1 L_f ly(L_f l) - T' - L_f D + (\rho - 1)L_f Q\right] + G' + \rho L_f D + M.$$

From this

$$(1-\alpha)\left[P_1 L_f ly\left(L_f l\right) - T' - L_f D + (\rho - 1)L_f Q\right] = \\ = G' - T' + (\rho - 1)L_f(D + Q) + M. \quad (17)$$

Suppose $P_1 L_f ly(L_f l) - T' > P_1 Lly(Ll) - T$ or $T = T'$, that is, the realization of full employment will increase consumers' disposable income or the tax is not changed. Then, from (15) and (17) we get

$$G' - T' > (\rho - 1)(M - L_f D - L_f Q).$$

Therefore, we have the following result.

Proposition 2

In order to achieve full-employment with ongoing nominal wage rate decline the budget surplus must be smaller, or the budget deficit must be larger than the steady state case in (16).

Let G'', T'', M' and $P_1{}'$ be the government expenditure, the tax revenue, the total savings of the younger generation consumers and the price of the consumption basket in the next period after realization of full-employment. (14) is written as

$$P_1{}'L_f ly(L_f l) = \alpha\left[P_1{}'L_f ly(L_f l) - T'' - L_f D + (\rho - 1)L_f Q\right] + G'' + \rho L_f D + M'.$$

To maintain full-employment, the total savings of the younger generation including the pay-as-you-go pension must be equal to $\rho M'$. Then, we have

$$(1-\alpha)\left[P_1{}'L_f ly\left(L_f l\right) - T'' - L_f D + (\rho - 1)L_f Q\right] =$$
$$= G'' - T'' + (\rho - 1)L_f(D + Q) + M' = \rho M',$$

and,

$$G'' - T'' = (\rho - 1)(M' - L_f D - L_f Q).$$

If the nominal wage rate and the price are constant after realization of full-employment, $\rho = 1$. Then,

$$G'' - T'' = 0.$$

Therefore, we have the following result.

Proposition 3

If the nominal wage rate is constant after full-employment has been realized, the balanced budget is required to maintain the steady state with full-employment.

4. Concluding Remarks

We have examined the existence of involuntary umemployment and the effects of fiscal policy using a three-periods OLG model under monopolistic competition with ongoing nominal wage rate decline. We considered the case of a divisible labor supply, and we assumed that the goods are produced only by labor.

In the future research, we want to analyze involuntary unemployment and fiscal policy in a situation where goods are produced by capital and labor, and there exist investments of firms.

Appendix: Some calculations

The first order condition for (2) is

$$\left(\int_0^1 c_1^e(z)^{\frac{\sigma-1}{\sigma}}dz\right)^{\frac{1}{\sigma-1}} c_1^e(z)^{-\frac{1}{\sigma}} - \lambda_1^e p_1(z) = 0. \tag{A-1}$$

From this

$$\left(\int_0^1 c_1^e(z)^{\frac{\sigma-1}{\sigma}}dz\right)^{-1} c_1^e(z)^{\frac{\sigma-1}{\sigma}} = (\lambda_1^e)^{1-\sigma} p_1(z)^{1-\sigma}.$$

Then,

$$\left(\int_0^1 c_1^e(z)^{\frac{\sigma-1}{\sigma}}dz\right)^{-1}\int_0^1 c_1^e(z)^{\frac{\sigma-1}{\sigma}}dz = (\lambda_1^e)^{1-\sigma}\int_0^1 p_1(z)^{1-\sigma}dz = 1,$$

It means

$$\lambda_1^e\left(\int_0^1 p_1(z)^{1-\sigma}dz\right)^{\frac{1}{1-\sigma}} = 1,$$

and so

$$P_1 = \frac{1}{\lambda_1^e}.$$

From (A-1)

$$\left(\int_0^1 c_1^e(z)^{\frac{\sigma-1}{\sigma}}dz\right)^{\frac{1}{\sigma-1}} c_1^e(z)^{\frac{\sigma-1}{\sigma}} = \lambda_1^e p_1(z)c_1^e(z).$$

Then,

$$\left(\int_0^1 c_1^e(z)^{\frac{\sigma-1}{\sigma}}dz\right)^{\frac{1}{\sigma-1}}\int_0^1 c_1^e(z)^{\frac{\sigma-1}{\sigma}}dz = \left(\int_0^1 c_1^e(z)^{\frac{\sigma-1}{\sigma}}dz\right)^{\frac{\sigma}{\sigma-1}}$$
$$= C_1^e = \lambda_1^e\int_0^1 p_1(z)c_1^e(z)dz = \frac{1}{P_1}\int_0^1 p_1(z)c_1^e(z)dz.$$

Therefore,

$$\int_0^1 p_1(z)c_1^e(z)dz = P_1C_1^e.$$

Similarly,

$$\int_0^1 p_2(z)c_2^e(z)dz = P_2C_2^e.$$

Thus,

$$\int_0^1 p_1(z)c_1^e(z)dz + \int_0^1 p_2(z)c_2^e(z)dz = P_1C_1^e + P_2C_2^e =$$
$$= Wl + \Pi - D - \Theta + \hat{Q} - \Psi.$$

From (A-1)

$$P_1C_1^e = \alpha(Wl + \Pi - D - \Theta + \hat{Q} - \Psi).$$

Also by (A-1)

$$\left(\int_0^1 c_1^e(z)^{\frac{\sigma-1}{\sigma}}dz\right)^{\frac{\sigma}{\sigma-1}} c_1^e(z)^{-1} = C_1^e c_1^e(z)^{-1} = (\lambda_1^e)^{\sigma}p_1(z)^{\sigma} = \left(\frac{p_1(z)}{P_1}\right)^{\sigma}.$$

From this we get

$$c_1^e(z) = \left(\frac{p_1(z)}{P_1}\right)^{-\sigma}\frac{\alpha(Wl + \Pi - D - \Theta + \hat{Q} - \Psi)}{P_1}.$$

$c_2^e(z)$, $c_1^u(z)$ and $c_2^u(z)$ are similarly obtained.

Acknowledgment

This work was supported by the Japan Society for the Promotion of Science KAKENHI (Grant Number 18K01594).

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