STUDIA UBB MUSICA, LVI, 1, 2011 (p. 99-108)

SELF-SIMILARITY IN PITCH ORGANIZATION

ADRIAN BORZA[1]

SUMMARY. In this study we intend to discuss the self-similarity concept, which can be thought of as an organizing principle of the pitch of the sound. We will put emphasis on the self-similarity aspect of pitch organization in different series created by Anton Webern, and in several modes conceived by Olivier Massiaen, and Wilhelm Georg Berger. Also, we will investigate the author's self-similar, non-octave-based, and full-chromatic mode of twenty-three notes, which led to the author's compositions, such as *Chaconne* for guitar solo (1999), *Fractus III* for percussion and computer (2001), and *Point. Line. Spot* for string orchestra (2003).

Keywords: self-similarity, tone-row, series, mod

1. Pitch organization in serial music

Many analysts and music theoreticians have mentioned in the past that the geometric transformations, claimed by the Renaissance polyphony, are echoed into the serial composition practice. As an example, Iannis Xenakis assessed that "the serial music proposed a system whose substance was built through his geometric and quantitative properties. For instance, four forms of the series for geometric properties, the interval's number of semitones for quantitative properties. The pure mathematicians' thought was therefore deliberately reintroduced in music composition."[2]

Among the well-known exponents of Serialism, Anton Webern has been apparently more preoccupied with imposing a distinct order in the micro-universe of his series, revealing thus the geometric properties of the series' segments itself.

1.1. Webern's self-similar series

Indeed, the interval structure of several Webern's series is not arbitrary. In one of his lectures given before a group of music lovers in Vienna, in early '30, the composer assumed that in organizing the series "we will tend to rely on some relations – symmetry, analogy, groups of three or four notes".[3]

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[1] Currently he is a professor at the Gheorghe Dima Academy of Music. Address: 25, I.C. Bratianu, Cluj-Napoca. E-mail: aborza@gmail.com

[2] Xenakis, Iannis, *Muzica. Arhitectura (Music Arhitecture)*, Editura Muzicală, Bucharest, 1997, p. 7.

3 Webern, Anton, *Calea spre muzica nouă (The Path to the New Music)*, Editura Muzicală, Bucharest, 1988, p. 67.

For instance, the series of his *Concert for Nine Instruments Op. 24* for flute, oboe, clarinet, horn, trumpet, trombone, violin, viola, and piano has at its foundation an isomorphic figure[4] – two intervals, remainders[5] of the division modulo 12: minor second (1) and major third (4).

The group of intervals (1, 4) can be seen as the initiator of the series, and the generators will produce in fact the segments of the series or the sets of notes {h, b, d}, {e-flat, g, f-sharp}, {g-sharp, e, f}, {c, c-sharp, a}. These sets are joined disjunctive into a self-similar series:

Table 1

Webern – *Concert for Nine Instruments Op 24*: Interval Structure

Initiator	**(1, 4)**
Generators	(–1 +4), (+4 –1), (–4 +1), and (+1 –4)
Structure	(–1 +4) +1 (+4 –1) +2 (–4 +1) –5 (+1 –4)

We note that the segments are distinct and understandable details of the series, linked together in such a way that will emerge the total of twelve chromatic pitch-classes. The series' segments are in prime form or the original state, retrograde form, retrograde inversion form, and inversion form. The initial intervals (1, 4) are "metamorphosed in all four possible forms and extended over the whole series." [6]

The inner structuring process of series on the self-similarity concept continues in other Webern's compositions. The series of *String Quartet Op 28* has isomorphic and symmetric interval structures, seen as the generators of the series, and it is the result of relating two transformations of the initial group of intervals (1, 3, 1). The generators project, however, three sets of notes, {b, a, c, h}, {d-sharp, e, c-sharp, d}, {f-sharp, f, a-flat, g}, concatenated in a disjunctive way. The geometric transformations of the series' segments are prime, and inverse. The self-similar series is represented as remainders of the division modulo 12, and the interval structures are:

[4] The array of the primary intervals (minor second, minor second, major third, and perfect fourth) "gives a single isomorphic figure, founded on the minor second and major third sequence" (Niculescu, Ştefan, *Reflecții despre muzică (Reflections on Music)*, Editura Muzicală, Bucharest, 1980, p. 217-218.)

[5] „The octave (12) is reduced to unison (0). The minor ninth (13) is reduced to semitone (1); alike, the major ninth (14) is reduced to whole tone (2), and so on. An arbitrary interval *n* is reduced by dividing it by 12; the remainder will designate the interval. The notes contained into an octave can be associated to integers as well. Any integer can be thus associated to a note, according his class of remainders (modulo 12) who belongs to." (Vieru, Anatol, *Cartea modurilor (The Book of Modes)*, vol. I, Editura Muzicală, Bucharest, 1980, p. 10)

[6] Niculescu, Ştefan, *Reflecții despre muzică (Reflections on Music)*, Editura Muzicală, Bucharest, 1980, p. 218.

Table 2

Webern – *String Quartet Op 28*: Interval Structure

Initiator	**(1, 3, 1)**
Generators	(–1 +3 –1) and (+1 –3 +1)
Structure	(–1 +3 –1) +4 (+1 –3 +1) +4 (–1 +3 –1)

It is remarkable the elegance of the Webern's series and its segments. The composer chosen the palindrome as an initiator, and again the palindrome for the whole series, not to mention the occurrence of the name Bach, translated into the first segment of the series.

Another example of self-similarity in organizing the pitch of the sound is confirmed in *Variations for Orchestra Op 30*. The *Variations*' series is made by chaining two transformations of the initial intervals (1, 3, 1, 1, 3). Thus, the generators produce the series' segments in prime, and retrograde form, {a, b, d-flat, c, h, d}, and {e-flat, g-flat, f, e, g, a-flat}, jointed disjunctive. The self-similar series is represented as remainders of the division modulo 12:

Table 3

Webern – *Variations for Orchestra Op 30*: Interval Structure

Initiator	**(1, 3, 1, 1, 3)**
Generators	(+1 +3 –1 –1 +3), and (+3 –1 –1 +3 +1)
Structure	(+1 +3 –1 –1 +3) +1 (+3 –1 –1 +3 +1)

2. Pitch organization in modes

2.1. Messiaen and the modes of limited transposition

Particular examples of self-similarity are found into the modes of limited transposition. Speaking about his musical language, especially about the mechanism which led to the construction of his modes, Olivier Messiaen assumed that the modes "consist of several symmetric groups; the last note of each group is always identical with the first of the next group".[7] All seven modes of limited transposition are self-similar:

Table 4

Messiaen – Modes of Limited Transposition: Interval Structures

Initiator	**(2)**	**Generator** +2
Structure	+2 +2 +2 +2 +2 +2	
Initiator	**(1, 2)**	**Generator** (+1 +2)
Structure	(+1 +2) (+1 +2) (+1 +2) (+1 +2)	
Initiator	**(2, 1, 1)**	**Generator** (+2 +1 +1)
Structure	(+2 +1 +1) (+2 +1 +1) (+2 +1 +1)	
Initiator	**(1, 1, 3, 1)**	**Generator** (+1 +1 +3 +1)
Structure	(+1 +1 +3 +1) (+1 +1 +3 +1)	

[7] Messiaen, Olivier, *Technique de mon langage musical*, Ed. Leduc, Paris, 1942, p. 85.

Initiator	(**1, 4, 1)**	**Generator**	(+1 +4 +1)
Structure	(+1 +4 +1) (+1 +4 +1)		
Initiator	**(2, 2, 1, 1)**	**Generator**	(+2 +2 +1 +1)
Structure	(+2 +2 +1 +1) (+2 +2 +1 +1)		
Initiator	**(1, 1, 1, 2, 1)**	**Generator**	(+1 +1 +1 +2 +1)
Structure	(+1 +1 +1 +2 +1) (+1 +1 +1 +2 +1)		

It is noted that there is an initiator of each mode, materialized into a group of intervals. Applying the generator, it is acquired the prime form of the mode's segments. The segments are correlated conjunctive, by two, three, four, and six, in order to emerge an octave. The segments of the same mode are isomorphic and symmetric, which means that the mode is self-similar.

The number of transpositions of the mode is limited to the number of the semitones of each initiator, in other words, to the sum of the semitones of each set of notes.

2.2. Berger's full-chromatic, non-octave-based modes

The organization of modes on geometric basis has continued to be in attention of composers; the research of Wilhelm Georg Berger leading, in this respect, to highlight a category of "modes obtained through synthesis". "The modes of this category become visible in the music of this century. As preliminary phenomena, I remind the hexatonic scale, then later, the serial organization of the chromatic scale. Olivier Messiaen's modes, for example, are configured by using specific interval sequences, consistently distributed into the modes." [8]

Of the many full-chromatic modes, twenty of them are conceived by assembling two, three, four, and six interval structures, known as modal structures[9]. In our opinion these non-octave-based modes are self-similar.

In what follows, we are proposing a different representation of the Berger's modes, by restricting the scope of the modes – sometimes extended to seven octaves – to an acceptable range, but avoiding altering their non-octave-based property. Also, we consider that this is a simpler way to find the symmetric form of a mode, relation that has been noticed by Berger: "the inversion of each interval leads to a new species, related through the nature of the proportions." [10]

[8] Berger, Wilhelm Georg, *Dimensiuni modale (Modal Dimensions)* , Editura Muzicală, Bucharest, 1979, p. 10

[9] "It is called a modal structure the sequence of intervals associated to a mode". This definition highlights the relational aspect of a mode, in opposition to its quantitative aspect: "The transition from modes to intervallic thought is the leap made by mathematical understanding of music, from a purely quantitative view to a relational, structural view. A modal structure is a function, unlike a mode, which is a set." (Marcus, Solomon, *Artă şi Ştiinţă (Arts and Science)*, Editura Eminescu, Bucharest, 1986, p. 159)

[10] Berger, Wilhelm Georg, *Dimensiuni modale (Modal Dimensions)*, Editura Muzicală, Bucharest, 1979, p. 180

The self-similar modes are presented below:

Table 5

Berger – Self-similar Modes: Interval Structures

Initiator	**(5)**	**Generator**	(+5 –7)
Structure	(+5 –7) (+5 –7) (+5 –7) (+5 –7) (+5 –7) (+5 –7)		
Initiator	**(7)**	**Generator**	(+7 –5)
Structure	(+7 –5) (+7 –5) (+7 –5) (+7 –5) (+7 –5) (+7 –5)		
Initiator	**(1, 9)**	**Generator**	(+1 –3)
Structure	(+1 –3) (+1 –3) (+1 –3) (+1 –3) (+1 –3) (+1 –3)		
Initiator	**(11, 3)**	**Generator**	(–1 +3)
Structure	(–1 +3) (–1 +3) (–1 +3) (–1 +3) (–1 +3) (–1 +3)		
Initiator	**(3, 7)**	**Generator**	(+3 –5)
Structure	(+3 –5) (+3 –5) (+3 –5) (+3 –5) (+3 –5) (+3 –5)		
Initiator	**(9, 5)**	**Generator**	(–3 +5)
Structure	(–3 +5) (–3 +5) (–3 +5) (–3 +5) (–3 +5) (–3 +5)		
Initiator	**(3, 11)**	**Generator**	(+3 –1)
Structure	(+3 –1) (+3 –1) (+3 –1) (+3 –1) (+3 –1) (+3 –1)		
Initiator	**(9, 1)**	**Generator**	(–3 +1)
Structure	(–3 +1) (–3 +1) (–3 +1) (–3 +1) (–3 +1) (–3 +1)		
Initiator	**(5, 9)**	**Generator**	(+5 –3)
Structure	(+5 –3) (+5 –3) (+5 –3) (+5 –3) (+5 –3) (+5 –3)		
Initiator	**(7, 3)**	**Generator**	(–5 +3)
Structure	(–5 +3) (–5 +3) (–5 +3) (–5 +3) (–5 +3) (–5 +3)		
Initiator	**(1, 10, 4)**	**Generator**	(+1 –2 +4)
Structure	(+1 –2 +4) (+1 –2 +4) (+1 –2 +4) (+1 –2 +4)		
Initiator	**(11, 2, 8)**	**Generator**	(–1 +2 –4)
Structure	(–1 +2 –4) (–1 +2 –4) (–1 +2 –4) (–1 +2 –4)		
Initiator	**(2, 5, 2)**	**Generator**	(+2 –7 +2)
Structure	(+2 –7 +2) (+2 –7 +2) (+2 –7 +2) (+2 –7 +2)		
Initiator	**(10, 7, 10)**	**Generator**	(–2 +7 –2)
Structure	(–2 +7 –2) (–2 +7 –2) (–2 +7 –2) (–2 +7 –2)		
Initiator	**(5, 2, 2)**	**Generator**	(–7 +2 +2)
Structure	(–7 +2 +2) (–7 +2 +2) (–7 +2 +2) (–7 +2 +2)		
Initiator	**(7, 10, 10)**	**Generator**	(+7 –2 –2)
Structure	(+7 –2 –2) (+7 –2 –2) (+7 –2 –2) (+7 –2 –2)		
Initiator	**(6, 5, 6, 3)**	**Generator**	(–6 +5 –6 +3)
Structure	(–6 +5 –6 +3) (–6 +5 –6 +3) (–6 +5 –6 +3)		
Initiator	**(6, 7, 6, 9)**	**Generator**	(+6 –5 +6 –3)
Structure	(+6 –5 +6 –3) (+6 –5 +6 –3) (+6 –5 +6 –3)		
Initiator	**(3, 10, 3, 10, 3, 1)**	**Generator**	(+3 –2 +3 –2 +3 +1)
Structure	(+3 –2 +3 –2 +3 +1) (+3 –2 +3 –2 +3 +1)		
Initiator	**(9, 2, 9, 2, 9, 11)**	**Generator**	(–3 +2 –3 +2 –3 –1)
Structure	(–3 +2 –3 +2 –3 –1) (–3 +2 –3 +2 –3 –1)		

3. Self-similar mode of twenty-three notes

For the purpose of this study we define the self-similarity as being a principle of generating a mode, by interrelating two, three or more interval structures or sequences of intervals of the same kind. All the structures of a mode have two essential properties, symmetry and isomorphism. Also, the structures carry the attribute modal; consequently, they are named modal structures. The structures are the result of applying transformations, such as rotation, inversion, translation, algebraic sum etc to an initial group of intervals, arbitrary chosen. The initial group is called initiator, and the transformations are the generators of the sets of notes of the self-similar mode.

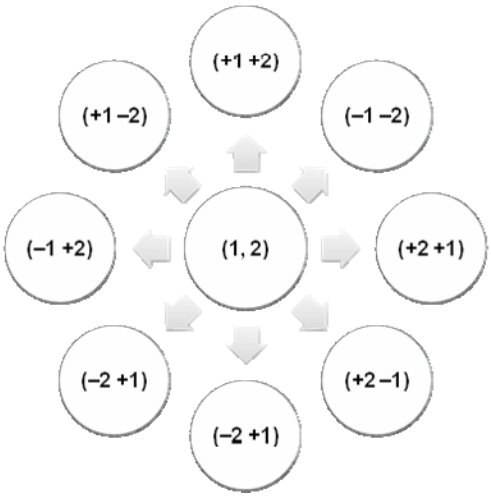
Let's consider that the intervals of an octave are equally-tempered, and the semitone is the measurement unit of the division of an octave. The value 1 signify a semitone, the + and – symbols indicate the sense of the interval, that is ascending or descending.

The initiator of the mode of twenty-three notes is the initial group of intervals (1, 2). As simple in configuration, this group is widespread in music of any king, representing the structure of motifs. For instance, the set of notes {d, e-flat, f} has a constitutive semitone and conjunctive whole tone, thus the initiator is (1, 2).

The generators of the mode are identified as eight transformations, such as (+1 +2), (–1 –2), (+2 +1), (–2 –1), (+2 –1), (–2 +1), (–1 +2), (+1 –2). In other words, any of the eight modal structures can be directly generated from the initial group of intervals by applying the algebraic sum. The resulting sets of notes are {d, e-flat, f}, {d, c-sharp, h}, {d, e, f}, {d, c, h}, {d, e, e-flat}, {d, c, c-sharp}, {c, c-sharp, e-flat}, and {d, e-flat, c-sharp}.

Table 6

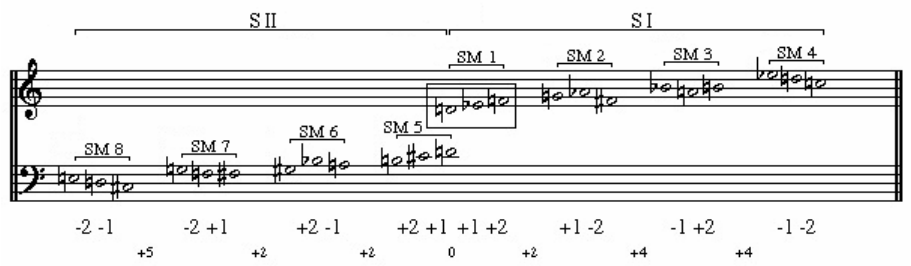
Network of Transformations



The modal structures, symmetrical and isomorphic, are interrelated in order to create the self-similar mode of twenty-three notes (Ex. 1). The highest note of a modal structure is set up at a distance of one semitone from the lowest note of the right-neighboring modal structure, less two central modal structures that have a joint note. This note {d} is the center of the mode, equidistant otherwise from {e-flat} in upper register, and {d-sharp} in lower register, at a distance of thirteen semitones. The mode includes the twelve notes of the chromatic scale, and the corresponding eight sets of notes are scattered over roughly two octaves, as truly melodic formulas.

Ex. 1

Self-similar, non-octave-based, full-chromatic mode of twenty-three notes



The self-similar mode is not a one-level construction process, a building with a ground floor. By analogy, the access to the first floor of the building – the generator – is allowed after finishing the ground floor – the initiator –, and we cannot jump to the second floor – the self-similar mode – only if the first floor is completed.

Table 7

Generating the Self-similar Mode

Initiator
(1, 2)
Generators
(+1 +2), (–1 –2), (+2 +1), (–2 –1), (+2 –1), (–2 +1), (–1 +2), (+1 –2)
Self-similar Mode: Interval Structure
(–2 –1) +5 (–2 +1) +2 (+2 –1) +2 (+2 +1) (+1 +2) +2 (+1 –2) +4 (–1 +2) +4 (–1 –2)

Section I of the mode, S I, contains modal structures, SM 1, SM 2, SM 3, and SM 4, that have the same sequence of intervals, despite their sense, meaning that the structures are isomorphic. Alike, Section II, S II, has isomorphic modal structures, SM 5, SM 6, SM 7, and SM 8.

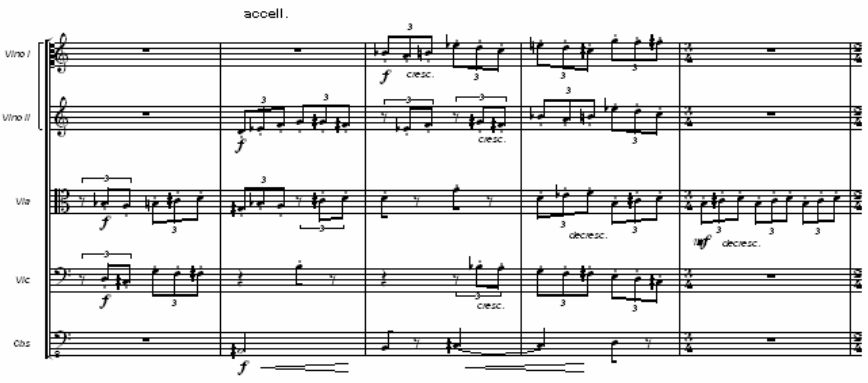
S I holds symmetrical modal structures, SM 1 and SM 4, on one hand, and SM 2 and SM 3, on the other hand. In the same way, S II holds SM 5 and SM 8, respectively SM 6 and SM 7 symmetrical structures. Four of them

are geometric transformations, SM 1, SM 4, SM 5, and SM 8 – prime, inversion, retrograde, and retrograde inversion forms – coupled with their transformation through translation.

A meaningful use of the self-similar mode of twenty-three notes can be found in the author's *Point. Line. Spot* composition, from which we present the following excerpt:

Ex. 2

Adrian Borza – *Point. Line. Spot*: Modal Structures Distribution

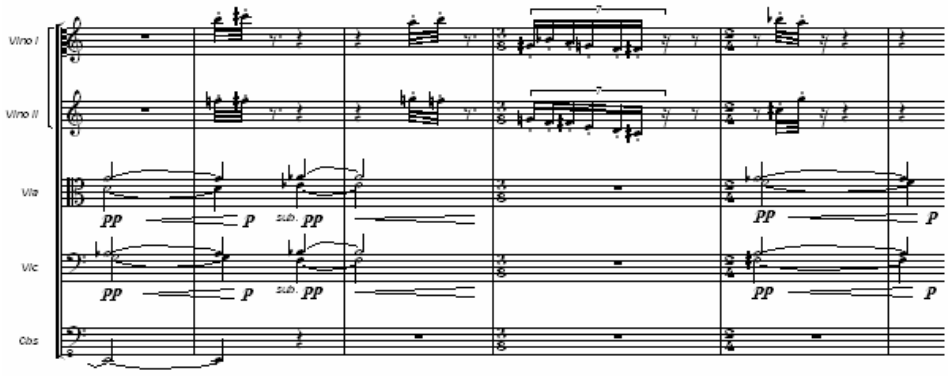


It is to be remarked that all eight modal structures become visible at the instruments of the string ensemble, through the melodic progression, which follows a linear path, from lower register to upper register, and through the polyphonic accumulation.

Other excerpts (Ex. 3, Ex. 4, and Ex. 5) from the author's works are completing our examples of distribution of the modal structures discussed above. A detailed analysis of the works is beyond the purpose of this study.

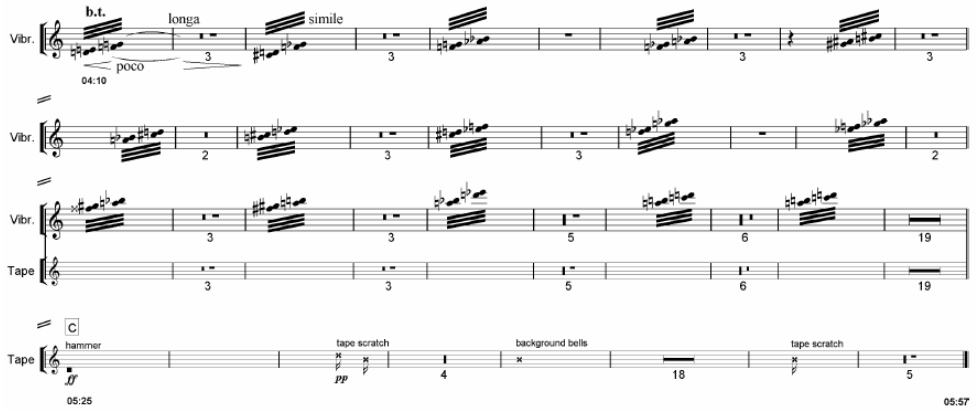
Ex. 3

Adrian Borza – *Point. Line. Spot* for string orchestra (2003)



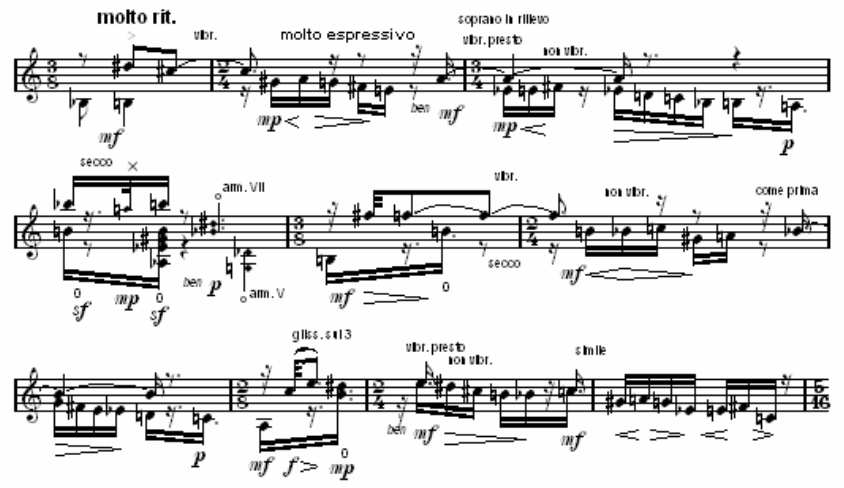
Ex. 4

Adrian Borza – *Fractus III* for percussion and computer (2001)



Ex. 5

Adrian Borza – *Chaconne* for guitar solo (1999)



4. Conclusion

This study does not claim to be exhaustive on the topic of self-similarity, but it has proven that self-similarity can be a practical approach in generating new modes useful in the composition practice.

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