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Coincidence theory and KKM type maps

Donal O'Regan

Abstract. In this paper we present a variety of coincidence results for classes of maps defined on Hausdorff topological vector spaces. Our theory is based on fixed point theory in the literature.

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1. Introduction

In this paper we present a coincidence theory for classes of maps defined on Hausdorff topological spaces. These classes include some of the most general type of maps in the literature, namely KKM type maps, PK type maps, DKT type maps and $HLPY$ type maps. We establish coincidence results in both situations, namely when the classes are the same and when the classes are different. Our theory is based on fixed point theory in the literature (some due to the author [1], [12], [13]) and on selection theorems in the literature. Our results generalize and extend many results in the literature; see [1], [3], [5], [4], [6], [7], [11], [14], [15] and the references therein.

Now we describe the maps considered in this paper. Let H be the Čech homology functor with compact carriers and coefficients in the field of rational numbers K from the category of Hausdorff topological spaces and continuous maps to the category of graded vector spaces and linear maps of degree zero. Thus $H(X) = \{H_q(X)\}$ (here X is a Hausdorff topological space) is a graded vector space, $H_q(X)$ being the q–dimensional Čech homology group with compact carriers of X. For a continuous map $f : X \to X$, $H(f)$ is the induced linear map $f_\star = \{f_{\star\, q}\}$ where $f_{\star\, q} : H_q(X) \to$

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$H_q(X)$. A space X is acyclic if X is nonempty, $H_q(X) = 0$ for every $q \geq 1$, and $H_0(X) \approx K$.

Let X, Y and Γ be Hausdorff topological spaces. A continuous single valued map $p : \Gamma \to X$ is called a Vietoris map (written $p : \Gamma \Rightarrow X$) if the following two conditions are satisfied:
(i). for each $x \in X$, the set $p^{-1}(x)$ is acyclic
(ii). p is a perfect map i.e. p is closed and for every $x \in X$ the set $p^{-1}(x)$ is nonempty and compact.

Let $\phi : X \to Y$ be a multivalued map (note for each $x \in X$ we assume $\phi(x)$ is a nonempty subset of Y). A pair (p, q) of single valued continuous maps of the form $X \stackrel{p}{\leftarrow} \Gamma \stackrel{q}{\rightarrow} Y$ is called a selected pair of ϕ (written $(p, q) \subset \phi$) if the following two conditions hold:
(i). p is a Vietoris map
and
(ii). $q\left(p^{-1}(x)\right) \subset \phi(x)$ for any $x \in X$.

Now we define the admissible maps of Gorniewicz [10]. A upper semicontinuous map $\phi : X \to Y$ with compact values is said to be admissible (and we write $\phi \in Ad(X, Y)$) provided there exists a selected pair (p, q) of ϕ. An example of an admissible map is a Kakutani map. A upper semicontinuous map $\phi : X \to CK(Y)$ is said to be Kakutani (and we write $\phi \in Kak(X, Y)$); here Y is a Hausdorff topological vector space and $CK(Y)$ denotes the family of nonempty, convex, compact subsets of Y.

We also discuss the following classes of maps in this paper. Let Z be a subset of a Hausdorff topological space Y_1 and W a subset of a Hausdorff topological vector space Y_2 and G a multifunction. We say $F \in HLPY(Z, W)$ [11] if W is convex and there exists a map $S : Z \to W$ with $co\,(S(x)) \subseteq F(x)$ for $x \in Z$, $S(x) \neq \emptyset$ for each $x \in Z$ and $Z = \bigcup \{\, int\, S^{-1}(w) : \ w \in W\}$; here $S^{-1}(w) = \{z \in Z : w \in S(z)\}$ and note $S(x) \neq \emptyset$ for each $x \in Z$ is redundant since if $z \in Z$ then there exists a $w \in W$ with $z \in int\, S^{-1}(w) \subseteq S^{-1}(w)$ so $w \in S(z)$ i.e. $S(z) \neq \emptyset$. These maps are related to the DKT maps in the literature and $F \in DKT(Z, W)$ [7] if W is convex and there exists a map $S : Z \to W$ with $co\,(S(x)) \subseteq F(x)$ for $x \in Z$, $S(x) \neq \emptyset$ for each $x \in Z$ and the fibre $S^{-1}(w)$ is open (in Z) for each $w \in W$. Note these maps were motivated from the $\Phi^\star$ maps. We say $G \in \Phi^\star(Z, W)$ [3] if W is convex and there exists a map $S : Z \to W$ with $S(x) \subseteq G(x)$ for $x \in Z$, $S(x) \neq \emptyset$ and has convex values for each $x \in Z$ and the fibre $S^{-1}(w)$ is open (in Z) for each $w \in W$.

Now we consider a general class of maps, namely the PK maps of Park. Let X and Y be Hausdorff topological spaces. Given a class $\mathbf{X}$ of maps, $\mathbf{X}(X, Y)$ denotes the set of maps $F : X \to 2^Y$ (nonempty subsets of Y) belonging to $\mathbf{X}$, and $\mathbf{X}_c$ the set of finite compositions of maps in $\mathbf{X}$. We let

$$\mathbf{F}(\mathbf{X}) = \{Z : \ Fix\, F \neq \emptyset \ \text{ for all } \ F \in \mathbf{X}(Z, Z)\}$$

where $Fix\, F$ denotes the set of fixed points of F.

The class $\mathbf{U}$ of maps is defined by the following properties:

(i). $\mathbf{U}$ contains the class C of single valued continuous functions;

(ii). each $F \in \mathbf{U}_c$ is upper semicontinuous and compact valued; and

(iii). $B^n \in \mathbf{F}(\mathbf{U}_c)$ for all $n \in \{1, 2, ...\}$; here $B^n = \{x \in \mathbf{R}^n : \ \|x\| \leq 1\}$.

We say $F \in PK(X, Y)$ if for any compact subset K of X there is a $G \in \mathbf{U}_c(K, Y)$ with $G(x) \subseteq F(x)$ for each $x \in K$. Recall PK is closed under compositions [12].

For a subset K of a topological space X, we denote by $Cov_X\,(K)$ the directed set of all coverings of K by open sets of X (usually we write $Cov\,(K) = Cov_X\,(K)$). Given two maps F, $G : X \to 2^Y$ and $\alpha \in Cov\,(Y)$, F and G are said to be α–close if for any $x \in X$ there exists $U_x \in \alpha$, $y \in F(x) \cap U_x$ and $w \in G(x) \cap U_x$.

Let Q be a class of topological spaces. A space Y is an extension space for Q (written $Y \in ES(Q)$) if for any pair (X, K) in Q with $K \subseteq X$ closed, any continuous function $f_0 : K \to Y$ extends to a continuous function $f : X \to Y$. A space Y is an approximate extension space for Q (written $Y \in AES(Q)$) if for any $\alpha \in Cov\,(Y)$ and any pair (X, K) in Q with $K \subseteq X$ closed, and any continuous function $f_0 : K \to Y$ there exists a continuous function $f : X \to Y$ such that $f|_K$ is α–close to f_0.

Let V be a subset of a Hausdorff topological vector space E. Then we say V is Schauder admissible if for every compact subset K of V and every covering $\alpha \in Cov_V\,(K)$ there exists a continuous function $\pi_\alpha : K \to V$ such that

(i). π_α and $i : K \to V$ are α–close;

(ii). $\pi_\alpha(K)$ is contained in a subset $C \subseteq V$ with $C \in AES(\text{compact})$.

The next results can be found in [1] and [12] respectively.

Theorem 1.1. *Let $X \in ES(\text{compact})$ and $\Psi \in PK(X, X)$ a compact map. Then there exists a $x \in X$ with $x \in \Psi(x)$.*

Theorem 1.2. *Let X be a Schauder admissible subset of a Hausdorff topological vector space and $\Psi \in PK(X, X)$ a compact upper semicontinuous map with closed values. Then there exists a $x \in X$ with $x \in \Psi(x)$.*

Remark 1.3. (i). Other variations of Theorem 1.2 can be found in [13].

(ii). It is of interest to note that Theorem 1.1 is based on the following fixed point result. If T is the Tychonoff cube and $\Phi \in PK(T, T)$ then Φ has a fixed point and to see the proof of this note since T is compact then there exists a $G \in \mathbf{U}_c(K, Y)$ with $G(x) \subseteq \Phi(x)$ for $x \in T$, so a standard result (see [4, Theorem 3.1]) guarantees a $x \in T$ with $x \in G(x)$, so $x \in \Phi(x)$.

Next we describe a class of maps more general than the PK maps in our setting. Let X be a convex subset of a Hausdorff topological vector space and Y a Hausdorff topological space. If S, $T : X \to 2^Y$ are two set valued maps such that $T(co\,(A)) \subseteq S(A)$ for each finite subset A of X then we call S a generalized KKM mappping w.r.t. T. Now the set valued map $T : X \to 2^Y$ is said to have the KKM property if for any generalized KKM map $S : X \to 2^Y$ w.r.t. T the family $\{\,\overline{S(x)} : \ x \in X\}$ has the finite intersection property (the intersection of each finite subfamily is nonempty).

We let
$$KKM(X,Y) = \{T : X \to 2^Y \mid T \text{ has the } KKM \text{ property}\}.$$
Note $PK(X,Y) \subset KKM(X,Y)$ (see [6]). Next we recall the following result [6].

Theorem 1.4. *Let X be a convex subset of a Hausdorff topological vector space and Y, Z be Hausdorff topological spaces.*
(i). $T \in KKM(X,Y)$ iff $T|_\triangle \in KKM(\triangle, Y)$ for each polytope $\triangle$ in X;
(ii). if $T \in KKM(X,Y)$ and $f \in C(Y,Z)$ then $f\,T \in KKM(X,Z)$;
(iii). if Y is a normal space, $\triangle$ a polytope of X and if $T : \triangle \to 2^Y$ is a set valued map such that for each $f \in C(Y,\triangle)$ we have that $f\,T$ has a fixed point in $\triangle$, then $T \in KKM(\triangle, Y)$.

Next we recall the following fixed point result for KKM maps. Recall a nonempty subset W of a Hausdorff topological vector space E is said to be admissible if for any nonempty compact subset K of W and every neighborhood V of 0 in E there exists a continuous map $h : K \to W$ with $x - h(x) \in V$ for all $x \in K$ and $h(K)$ is contained in a finite dimensional subspace of E (for example every nonempty convex subset of a locally convex space is admissible).

The next result can be found in [4].

Theorem 1.5. *Let X be an admissible convex set in a Hausdorff topological vector space E and $T \in KKM(X,X)$ be a closed compact map. Then T has a fixed point in X.*

Remark 1.6. One could also consider $s - KKM$ maps [5], [4], [15] in this paper and we could obtain similar results to those in section 2.

Next we will present an analogue of Theorem 1.4 (ii) for $T\,f$ and this composition will be needed in a few results in section 2.

Theorem 1.7. *Let X be an admissible convex set in a Hausdorff topological vector space and Z a subset of a Hausdorff topological space. If $T \in KKM(X,Z)$ is a upper semicontinuous compact map with closed (in fact compact) values and $f \in C(Z,X)$ then $T\,f$ has a fixed point in Z.*

Proof. Now $T \in KKM(X,Z)$, $f \in C(Z,X)$ and Theorem 1.4 (ii) implies $f\,T \in KKM(X,X)$. Also $f\,T$ is a compact upper semicontinuous map with compact values (so $f\,F$ is a closed map [2]). Now Theorem 1.5 guarantees that $f\,T$ has a fixed point in X and consequently $T\,f$ has a fixed point in Z. □

Theorem 1.8. *Let X be an admissible convex set in a Hausdorff topological vector space, Y a convex set in a Hausdorff topological vector space and Y a normal space. If $T \in KKM(X,Y)$ is a upper semicontinuous map with compact values and $f \in C(Y,X)$ then $T\,f \in KKM(Y,Y)$.*

Proof. Note $T\,f : Y \to 2^Y$. From Theorem 1.4 (i), (iii) we need to show that for each polytope $\triangle$ in Y that $g(T\,f)$ has a fixed point in $\triangle$ for any $g \in C(Y,\triangle)$. Note from Theorem 1.4 (ii) since $T \in KKM(X,Y)$ and $g \in C(Y,\triangle)$ that $g\,T \in KKM(X,\triangle)$.

Now from Theorem 1.7 (note $Z = \triangle$ is compact and $gT : X \to 2^{\triangle}$ is a upper semicontinuous compact map with compact values) guarantees that $(gT)\,f$ has a fixed point in $\triangle$. $\square$

In section 2 we will make use of the following two properties. Let C and X be convex subsets of a Hausdorff topological vector space E with $C \subseteq X$ and Y a Hausdorff topological space.

(i). If $T \in KKM(X, Y)$ then $G \equiv T|_C \in KKM(C, Y)$.

This can be seen from Theorem 1.4 (i). Note $T \in KKM(X, Y)$ so $T|_\triangle \in KKM(\triangle, Y)$ for each polytope $\triangle$ in X from Theorem 1.4 (i). Thus in particular for any polytope $\triangle$ in C we have $T|_\triangle \in KKM(\triangle, Y)$ so from Theorem 1.4 (i) we have $T|_C \in KKM(C, Y)$.

Alternatively we can prove it directly as follows. Let $S : C \to 2^Y$ be a generalized KKM map w.r.t. G i.e. $G(co\,(A)) \subseteq S(A)$ for each finite subset A of C. We must show $\{\,\overline{S(x)} :\ x \in C\}$ has the finite intersection property. To see this let $S^\star : X \to 2^Y$ be given by

$$S^\star(x) = \begin{cases} S(x), & x \in C \\ Y, & x \in X \setminus C. \end{cases}$$

We claim $T(co\,(D)) \subseteq S^\star(D)$ for each finite subset D of X. Now either (a). $x \in C$ for all $x \in D$ or (b). there exists a $y \in D$ with $y \notin C$. Suppose first case (b) occurs. Then since $S^\star(y) = Y$ we have

$$T(co\,(D)) \subseteq Y = S^\star(y) = S^\star(D).$$

It remains to consider case (a). Then since C is convex we have $co\,(D) \subseteq C$ and since $S^\star(z) = S(z)$ for $z \in C$ we have

$$T(co\,(D)) = G(co\,(D)) \subseteq S(D) = S^\star(D).$$

Thus $S^\star : X \to 2^Y$ is a generalized KKM map w.r.t. T. Since $T \in KKM(X, Y)$ then $\{\,\overline{S^\star(x)} :\ x \in X\}$ has the finite intersection property. Now for any finite subset Ω of C (note $S^\star(z) = S(z)$ for $z \in C$) we have

$$\cap_{x \in \Omega}\ \overline{S(x)} = \cap_{x \in \Omega}\ \overline{S^\star(x)} \neq \emptyset,$$

so $G = T|_C \in KKM(C, Y)$.

(ii). If $T \in KKM(X, Y)$, $T(X) \subseteq Z \subseteq Y$ and Z is closed in Y then $T \in KKM(X, Z)$.

Let $S : X \to 2^Z$ be a generalized KKM map w.r.t. T i.e. $T(co\,(A)) \subseteq S(A)$ for each finite subset A of X. We must show $\{\,\overline{S(x)}^Z :\ x \in X\}$ has the finite intersection property. Note since $S : X \to 2^Y$ is a generalized KKM map w.r.t. T then since $T \in KKM(X, Y)$ we have that $\{\,\overline{S(x)}\,(= \overline{S(x)}^Y) :\ x \in X\}$ has the finite intersection property. However note for $x \in X$ that

$$\overline{S(x)}^Z = \overline{S(x)}^Y \cap Z = \overline{S(x)}^Y\ (= \overline{S(x)})$$

since Z is closed in Y (note $S(X) \subseteq Z$ so $\overline{S(x)}^Y \subseteq Z$). Thus $\{\,\overline{S(x)}^Z :\ x \in X\} = \{\,\overline{S(x)}\,(= \overline{S(x)}^Y) :\ x \in X\}$ has the finite intersection property.

2. Coincidence results

In this section we present coincidence results between two different classes of set–valued maps. Results on coincidence points for similar classes of maps is also discussed at the end of section 2.

Theorem 2.1. *Let X be a convex subset of a Hausdorff topological vector space E and Y a subset of a Hausdorff topological space. Suppose $F \in KKM(X,Y)$ is a upper semicontinuous compact map with compact values and $G \in DKT(Y,X)$. Then there exists a $x \in X$ with $G^{-1}(x) \cap F(x) \neq \emptyset$.*

Proof. By assumption there exists a compact set K of Y with $F(X) \subseteq K$. Also since $G \in DKT(Y,X)$ we have $G \in DKT(K,X)$. To see this note there exists a map $S : Y \to X$ with $co\,(S(y)) \subseteq G(y)$ for $y \in Y$, $S(y) \neq \emptyset$ for each $y \in Y$ and $S^{-1}(x)$ is open (in Y) for each $x \in X$. Let $S^\star$ denote the restriction of S to K and note $co\,(S^\star(y)) = co\,(S(y)) \subseteq G(y)$ for $y \in K$ and for $x \in X$ then (note $K \subseteq Y$)

$$\begin{aligned}(S^\star)^{-1}(x) &= \{z \in K : x \in S^\star(z)\} = \{z \in K : x \in S(z)\}\\ &= K \cap \{z \in Y : x \in S(z)\} = K \cap S^{-1}(x)\end{aligned}$$

which is open in $K \cap Y = K$. Thus $G \in DKT(K,X)$ with K compact so from [7] there exists a continuous single valued selection $g : K \to X$ (i.e. $g \in C(K,X)$) of G and there exists a finite subset A of X with $g(K) \subseteq co\,(A)$, so $g \in C(K, co\,(A)$. Note from Section 1 (see (i) and (ii)) that $F \in KKM(co\,(A), K)$ so $g\,F \in KKM(co\,(A), co\,(A))$ (see Theorem 1.4 (ii)) is a upper semicontinuous map with compact values, so a closed map [2]. Also note $co\,(A)$ is a compact convex subset in a finite dimensional subspace of E. Then Theorem 1.5 guarantees a $x \in co\,(A) \subseteq X$ with $x \in g\,(F(x))$. As a result $G^{-1}(x) \cap F(x) \neq \emptyset$. □

Remark 2.2. Since $PK(X,Y) \subseteq KKM(X,Y)$ then one could replace KKM with PK in Theorem 2.1.

Our next result replaces $G \in DKT(Y,X)$ with $G \in HLPY(Y,X)$ in Theorem 2.1.

Theorem 2.3. *Let X be a convex subset of a Hausdorff topological vector space E and Y a subset of a Hausdorff topological space. Suppose $F \in KKM(X,Y)$ is a upper semicontinuous compact map with compact values and $G \in HLPY(Y,X)$. Then there exists a $x \in X$ with $G^{-1}(x) \cap F(x) \neq \emptyset$.*

Proof. There exists a compact set K of Y with $F(X) \subseteq K$. We claim $G \in HLPY(K,X)$. To see this note there exists a map $S : Y \to X$ with $co\,(S(y)) \subseteq G(y)$ for $y \in Y$ and $Y = \bigcup\{\,int\,S^{-1}(w) : \ w \in X\}$. Let S also denote the restriction of S to K. We now show $K = \bigcup\{\,int_K\,S^{-1}(w) : \ w \in X\}$. To see this first notice that

$$\begin{aligned}K = K \cap Y &= K \cap \left(\bigcup\{\,int\,S^{-1}(w) : \ w \in X\}\right)\\ &= \bigcup\{K \cap \,int\,S^{-1}(w) : \ w \in X\},\end{aligned}$$

so $K \subseteq \bigcup \{ int_K\, S^{-1}(w) :\ w \in X\}$ since for each $w \in X$ we have that $K \cap int\, S^{-1}(w)$ is open in K. On the other hand clearly $\bigcup \{ int_K\, S^{-1}(w) :\ w \in X\} \subseteq K$ so as a result

$$K = \bigcup \{ int_K\, S^{-1}(w) :\ w \in X\}.$$

Thus $G \in HLPY(K, X)$ so from [11] there exists a selection $g \in C(K, X)$ of G and a finite subset A of X with $g(K) \subseteq co\,(A)$. As in Theorem 2.1 we have that $g\,F \in KKM(co\,(A), co\,(A))$ is a upper semicontinuous map with compact values. Now apply Theorem 1.5. □

Remark 2.4. Note $F \in KKM(X, Y)$ can be replaced by $F \in PK(X, Y)$ in Theorem 2.3.

Next we will consider the case where F is a compact map is replaced by G is a compact map in Theorem 2.1.

Theorem 2.5. *Let X be an admissible convex set in a Hausdorff topological vector space and Y a paracompact subset of a Hausdorff topological space. Suppose $F \in KKM(X, Y)$ is a upper semicontinuous map with compact values and $G \in DKT(Y, X)$ is a compact map. Then there exists a $x \in X$ with $G^{-1}(x) \cap F(x) \neq \emptyset$.*

Proof. Since Y is paracompact then from [7] there exists a selection $g \in C(Y, X)$ of G. Now from Theorem 1.4 (ii) we have that $g\,F \in KKM(X, X)$ is a compact (since G is compact) upper semicontinuous map with compact values, so a closed map [2]. Theorem 1.5 guarantees a $x \in X$ with $x \in g\,(F(x))$. □

Remark 2.6. (i). Note we could replace $G \in DKY(Y, X)$ with $G \in HLPY(Y, X)$ in Theorem 2.5 since [11] guarantees a selection $g \in C(Y, X)$ of G.

(ii). In fact one could replace X an admissible set with $D = co\,(\overline{G(Y)})$ an admissible set in Theorem 2.5. To see this note $G \in DKT(Y, D)$ [there exists a map $S : Y \to X$ with $co\,(S(y)) \subseteq G(y)$ for $y \in Y$, $S(y) \neq \emptyset$ for each $y \in Y$ and $S^{-1}(x)$ is open (in Y) for each $x \in X$ so since $G(Y) \subseteq D$ then $S : Y \to D$ since $S(y) \subseteq co\,(S(y)) \subseteq G(y) \subseteq D$ for $y \in Y$ and trivially $S^{-1}(x)$ is open (in Y) for each $x \in D$ since $D \subseteq X$]. Now from [7] there exists a selection $g \in C(Y, D)$ of G. Also note since $D \subseteq X$ and $F \in KKM(X, Y)$ then $F \in KKM(D, Y)$ (see (i) in Section 1). Then from Theorem 1.4 (ii) we have that $g\,F \in KKM(D, D)$ and now apply Theorem 1.5. A similar comment applies if $G \in DKY(Y, X)$ is replaced by $G \in HLPY(Y, X)$.

Remark 2.7. Note $F \in KKM(X, Y)$ can be replaced by $F \in PK(X, Y)$ in Theorem 2.5. However it is possible to obtain other results for PK maps as the next theorem shows.

Theorem 2.8. *Let X be a convex set in a Hausdorff topological vector space and Y a paracompact subset of a Hausdorff topological space. Suppose $F \in PK(X, Y)$ and $G \in DKT(Y, X)$ is a compact map. Also assume one of the following hold:*

(a). $X \in ES(compact)$,

(b). X is Schauder admissible and F is upper semicontinuous with closed values. Then there exists a $x \in X$ with $G^{-1}(x) \cap F(x) \neq \emptyset$.

Proof. Let $g \in C(Y,X)$ be a selection of G (guaranteed since Y is paracompact) and note $g\,F \in PK(X,X)$ since the composition of PK maps is PK. Finally note $g\,F$ is a compact map since G is a compact map. If (a) holds then apply Theoren 1.1 to guarantee a $x \in X$ with $x \in g\,(F(x))$. If (b) holds note $g\,F$ is a upper semicontinuous map with closed (in fact compact) values. Now apply Theorem 1.2. □

Remark 2.9. (i). Note we could replace $G \in DKY(Y,X)$ with $G \in HLPY(Y,X)$ in Theorem 2.8.

(ii). In (a) in Theorem 2.8 we could replace $X \in ES$(compact) with $D = co\,(\overline{G(Y)}) \in ES$(compact) (respectively, in (b) in Theorem 2.8 we could replace X is Schauder admissible with D is Schauder admissible). To see this note $G \in DKT(Y,D)$ so there exists a selection $g \in C(Y,D)$ of G. Also note since $D \subseteq X$ and $F \in PK(X,Y)$ then $F \in PK(D,Y)$. Thus $g\,F \in PK(D,D)$.

In Theorem 2.5 and Theorem 2.8 we considered $g\,F$. It is also possible to consider $F\,g$ to obtain other results. We first consider the analogue of Theorem 2.8.

Theorem 2.10. *Let X be a convex set in a Hausdorff topological vector space and Y a paracompact subset of a Hausdorff topological space E. Suppose $F \in PK(X,Y)$ is a upper semicontinuous map with compact values and $G \in DKT(Y,X)$ is a compact map. Also assume one of the following hold:*

(a). $Y \in ES(compact)$,

(b). E is a uniform space and Y is a Schauder admissible subset of E.

Then there exists a $x \in X$ with $G^{-1}(x) \cap F(x) \neq \emptyset$.

Proof. Let $g \in C(Y,X)$ be a selection of G and note $F\,g \in PK(Y,Y)$. Also note $F\,g$ is a upper semicontinuous, compact map (since G is a compact map and F is upper semicontinuous map with compact values) with closed (in fact compact) values. If (a) holds apply Theorem 1.1 whereas if (b) holds apply Theorem 1.2. □

Remark 2.11. Note in Theorem 2.10 (a), F is upper semicontinuous map with compact values is only needed to guarantee that $F\,g$ is a compact map (this is all that is needed to apply Theorem 1.1). As a result in Theorem 2.10 (a), F is upper semicontinuous map with compact values and G is a compact map can be replaced by the condition that $F\,G$ is a compact map.

Remark 2.12. Note we could replace $G \in DKY(Y,X)$ with $G \in HLPY(Y,X)$ in Theorem 2.10.

We now consider the analogue of Theorem 2.5 when we use $F\,g$ instead of $g\,F$.

Theorem 2.13. *Let X be a convex set in a Hausdorff topological vector space E and Y an admissible convex paracompact subset in a Hausdorff topological vector space. Suppose $F \in KKM(X,Y)$ is a upper semicontinuous map with compact values and $G \in DKT(Y,X)$ is a compact map. Let $D = co\,(\overline{G(Y)})$ and assume D is an admissible subset of E. Then there exists a $x \in X$ with $G^{-1}(x) \cap F(x) \neq \emptyset$.*

Proof. We claim $G \in DKT(Y, D)$. To see this note there exists a map $S : Y \to X$ with $co\,(S(y)) \subseteq G(y)$ for $y \in Y$, $S(y) \neq \emptyset$ for each $y \in Y$ and $S^{-1}(x)$ is open (in Y) for each $x \in X$. Since $G(Y) \subseteq D$ then $S : Y \to D$ since $S(y) \subseteq co\,(S(y)) \subseteq G(y) \subseteq D$ for $y \in Y$. Trivially $S^{-1}(x)$ is open (in Y) for each $x \in D$ since $D \subseteq X$. Thus $G \in DKT(Y, D)$ so from [7] there exists a selection $g \in C(Y, D)$ of G (note Y is paracompact). Also note since $D \subseteq X$ and $F \in KKM(X, Y)$ then $F \in KKM(D, Y)$ (see (i) in Section 1). Now $g \in C(Y, D)$, $F \in KKM(D, Y)$ and Theorem 1.8 guarantee that $F\,g \in KKM(Y, Y)$ (note D is an admissible convex set in E and Y is normal since Hausdorff paracompact spaces are normal [9]). Also note $F\,g$ is a upper semicontinuous compact map with compact values, so it is a closed map [2]. Now apply Theorem 1.5 to guarantee a $y \in Y$ with $y \in F\,(g(y))$. □

Remark 2.14. Note we could replace $G \in DKY(Y, X)$ with $G \in HLPY(Y, X)$ in Theorem 2.13. To see this we only need to show that $G \in HLPY(Y, D)$. To see this note there exists a map $S : Y \to X$ with $co\,(S(y)) \subseteq G(y)$ for $y \in Y$ and $Y = \bigcup\{\, int\, S^{-1}(w) : \; w \in X\}$. Now note for any $y \in Y$ there exists a $w \in X$ with $y \in int\, S^{-1}(w)$ so $w \in S(y) \subseteq co\,(S(y)) \subseteq G(y) \subseteq D$. Thus $Y = \bigcup\{\, int\, S^{-1}(w) : w \in D\}$, so $G \in HLPY(Y, D)$.

Remark 2.15. In Theorem 2.13 we used Theorem 1.8 to get $F\,g \in KKM(Y, Y)$ and then we applied Theorem 1.5. If D is an admissible subset of E was replaced by X is an admissible subset of E then since one could obtain a selection $g \in C(Y, X)$ of G and since $F \in KKM(X, Y)$, then from Theorem 1.8 we have that $F\,g \in KKM(Y, Y)$.

In our next set of results we will remove the condition that Y is paracompact in Theorem 2.5, Theorem 2.8, Theorem 2.10 and Theorem 2.13. We begin with the analogue of Theorem 2.8.

Theorem 2.16. *Let X be a convex set and Y a closed set in a Hausdorff topological vector space E. Suppose $F \in PK(X, Y)$ and $G \in DKT(Y, X)$ is a compact map. Let K be a compact subset of X with $G(Y) \subseteq K$, let $L(K)$ be the linear span of K (i.e. the smallest linear subspace of E that contains K) and assume $F(X) \subseteq L(K) \cap Y$. Also suppose one of the following hold:*

(a). $X \in ES(compact)$,

(b). X is Schauder admissible and F is upper semicontinuous with closed values.

Then there exists a $x \in X$ with $G^{-1}(x) \cap F(x) \neq \emptyset$.

Proof. Let $L(K)$ be as described above. We claim $G \in DKT(Y \cap L(K), X)$. To see this note there exists a map $S : Y \to X$ with $co\,(S(y)) \subseteq G(y)$ for $y \in Y$, $S(y) \neq \emptyset$ for each $y \in Y$ and $S^{-1}(x)$ is open (in Y) for each $x \in X$. Let $S^\star$ denote the restriction of S to $Y \cap L(K)$ and note if $x \in X$ then

$$\begin{aligned} (S^\star)^{-1}(x) &= \{z \in Y \cap L(K) : \, x \in S^\star(z)\} = \{z \in Y \cap L(K) : \, x \in S(z)\} \\ &= L(K) \cap \{z \in Y : \, x \in S(z)\} = L(K) \cap S^{-1}(x) \end{aligned}$$

which is open in $L(K) \cap Y$. Thus $G \in DKT(Y \cap L(K), X)$. Now recall $L(K)$ is Lindelöf so paracompact [9] and since $Y \cap L(K)$ is closed in $L(K)$ then $Y \cap L(K)$ is paracompact. Now from [7] there exists a selection $g \in C(Y \cap L(K), X)$ of G. Also since $F(X) \subseteq L(K) \cap Y$ then $F \in PK(X, Y \cap L(K))$. Thus $g\,F \in PK(X, X)$ is a compact maps (since G is a compact map). If (a) holds then apply Theoren 1.1 to guarantee a $x \in X$ with $x \in g\,(F(x))$. If (b) holds note $g\,F$ is a upper semicontinuous map with closed (in fact compact) values. Now apply Theorem 1.2. □

Remark 2.17. In Theorem 2.16 we could replace $F(X) \subseteq L(K) \cap Y$ with $F(L(K)) \subseteq L(K) \cap Y$ provided we assume $F \in PK(E, Y)$ and $G \in DKT(Y, E)$ is a compact map (here K is a compact subset of E with $G(Y) \subseteq K$). Also we need to replace (a) and (b) in Theorem 2.16 with one of the following:

(a). $L(K) \in ES(\text{compact})$,

(b). $L(K)$ is Schauder admissible and F is upper semicontinuous with closed values.

To see this we claim $G \in DKT(Y \cap L(K), L(K))$. Note there exists a map $S : Y \to E$ with $co\,(S(y)) \subseteq G(y)$ for $y \in Y$, $S(y) \neq \emptyset$ for each $y \in Y$ and $S^{-1}(x)$ is open (in Y) for each $x \in E$. Let $S^\star$ denote the restriction of S to $Y \cap L(K)$ and note if $x \in L(K)$ then

$$\begin{aligned} (S^\star)^{-1}(x) &= \{z \in Y \cap L(K) : x \in S^\star(z)\} = \{z \in Y \cap L(K) : x \in S(z)\} \\ &= L(K) \cap \{z \in Y : x \in S(z)\} = L(K) \cap S^{-1}(x) \end{aligned}$$

which is open in $L(K) \cap Y$. Thus $G \in DKT(Y \cap L(K), L(K))$ and so there exists a selection $g \in C(Y \cap L(K), L(K))$ of G. Also since $F(L(K)) \subseteq L(K) \cap Y$ then $F \in PK(L(K), L(K) \cap Y)$ and so $g\,F \in PK(L(K), L(K))$ is a compact map. Now apply Theorem 1.1 or Theorem 1.2.

Next we will replace DKT with $HLPY$ in Theorem 2.16.

Theorem 2.18. *Let X be a convex set and Y a closed set in a Hausdorff topological vector space E. Suppose $F \in PK(X, Y)$ and $G \in HLPY(Y, X)$ is a compact map. Let K be a compact subset of X with $G(Y) \subseteq K$, let $L(K)$ be the linear span of K and assume $F(X) \subseteq L(K) \cap Y$. Also suppose one of the following hold:*

(a). $X \in ES(\text{compact})$,

(b). X is Schauder admissible and F is upper semicontinuous with closed values.

Then there exists a $x \in X$ with $G^{-1}(x) \cap F(x) \neq \emptyset$.

Proof. Let $L(K)$ be as described above. We claim $G \in HLPY(Y \cap L(K), X)$. To see this note there exists a map $S : Y \to X$ with $co\,(S(y)) \subseteq G(y)$ for $y \in Y$ and $Y = \bigcup\{\,int\,S^{-1}(w) :\ w \in X\}$. Let S also denote the restriction of S to $L(K) \cap Y$. We now show $Y \cap L(K) = \bigcup\{\,int_{Y \cap L(K)}\,S^{-1}(w) :\ w \in X\}$. To see this first notice that

$$\begin{aligned} L(K) \cap Y &= L(K) \cap \Big(\bigcup\{\,int\,S^{-1}(w) :\ w \in X\}\Big) \\ &= \bigcup\{L(K) \cap int\,S^{-1}(w) :\ w \in X\}, \end{aligned}$$

so $L(K) \cap Y \subseteq \bigcup\{\,int_{Y \cap L(K)}\,S^{-1}(w) :\ w \in X\}$ since for each $w \in X$ we have that $Y \cap int\,S^{-1}(w) = int\,S^{-1}(w)$ so $L(K) \cap int\,S^{-1}(w) = L(K) \cap Y \cap$

$int\, S^{-1}(w) = (L(K) \cap Y) \cap int\, S^{-1}(w)$ with is open in $L(K) \cap Y$. On the other hand clearly $\bigcup \{ int_{Y \cap L(K)}\, S^{-1}(w) : \ w \in X\} \subseteq L(K) \cap Y$. Thus $L(K) \cap Y = \bigcup \{ int_{Y \cap L(K)}\, S^{-1}(w) : \ w \in X\}$ so $G \in HLPY(Y \cap L(K), X)$ and since $Y \cap L(K)$ is paracompact there exists [11] a selection $g \in C(Y \cap L(K), X)$ of G and also note $g\,F \in PK(X, X)$ is a compact map. Now apply Theorem 1.1 or Theorem 1.2. □

Remark 2.19. In Theorem 2.18 we could replace $F(X) \subseteq L(K) \cap Y$ with $F(L(K)) \subseteq L(K) \cap Y$ provided we assume $F \in PK(E, Y)$ and $G \in HLPY(Y, E)$ is a compact map (here K is a compact subset of E with $G(Y) \subseteq K$). Also we need to replace (a) and (b) in Theorem 2.18 with one of the following:

(a). $L(K) \in ES(\text{compact})$,

(b). $L(K)$ is Schauder admissible and F is upper semicontinuous with closed values.

To see this we claim $G \in HLPY(Y \cap L(K), L(K))$. First note there exists a map $S : Y \to E$ with $co\,(S(y)) \subseteq G(y)$ for $y \in Y$ and $Y = \bigcup \{ int\, S^{-1}(w) : \ w \in E\}$. Let S also denote the restriction of S to $L(K) \cap Y$. Notice

$$\begin{aligned} L(K) \cap Y &= L(K) \cap \left(\bigcup \{ int\, S^{-1}(w) : \ w \in E\} \right) \\ &= \bigcup \{ L(K) \cap int\, S^{-1}(w) : \ w \in E\}, \end{aligned}$$

so $L(K) \cap Y \subseteq \bigcup \{ int_{Y \cap L(K)}\, S^{-1}(w) : \ w \in X\}$ since for each $w \in E$ we have that $Y \cap int\, S^{-1}(w) = int\, S^{-1}(w)$ so $L(K) \cap int\, S^{-1}(w) = (L(K) \cap Y) \cap int\, S^{-1}(w)$ with is open in $L(K) \cap Y$. On the other hand clearly $\bigcup \{ int_{Y \cap L(K)}\, S^{-1}(w) : \ w \in E\} \subseteq L(K) \cap Y$. Thus

$$L(K) \cap Y = \bigcup \{ int_{Y \cap L(K)}\, S^{-1}(w) : \ w \in E\}.$$

Now for any $y \in L(K) \cap Y$ there exists a $w \in E$ with $y \in int_{Y \cap L(K)}\, S^{-1}(w) \subseteq S^{-1}(w)$ so $w \in S(y) \subseteq co\,(S(y)) \subseteq G(y) \subseteq K \subseteq L(K)$. Thus

$$L(K) \cap Y = \bigcup \{ int_{Y \cap L(K)}\, S^{-1}(w) : \ w \in L(K)\},$$

so $G \in HLPY(Y \cap L(K), L(K))$ and since $Y \cap L(K)$ is paracompact there exists a selection $g \in C(Y \cap L(K), L(K))$ of G and also note $g\,F \in PK(L(K), L(K))$ is a compact map. Now apply Theorem 1.1 or Theorem 1.2.

Now we consider the analogue of Theorem 2.5.

Theorem 2.20. *Let X be an admissible convex set and Y a set in a Hausdorff topological vector space E. Suppose $F \in KKM(X, Y)$ is a upper semicontinuous map with compact values and $G \in DKT(Y, X)$ is a compact map. Let K be a compact subset of X with $G(Y) \subseteq K$, let $L(K)$ be the linear span of K and assume $F(X) \subseteq L(K) \cap Y$ and $L(K) \cap Y$ is closed in both Y and $L(K)$. Then there exists a $x \in X$ with $G^{-1}(x) \cap F(x) \neq \emptyset$.*

Proof. Let $L(K)$ be as described above and as in Theorem 2.16 we have $G \in DKT(Y \cap L(K), X)$. Note $L(K)$ is paracompact and since $Y \cap L(K)$ is closed in $L(K)$ then $Y \cap L(K)$ is paracompact. Now from [7] there exists a selection $g \in C(Y \cap L(K), X)$

of G. Now $F \in KKM(X,Y)$, $F(X) \subseteq L(K) \cap Y$ so from Section 1 (see (ii), note $Y \cap L(K)$ is closed in Y) we have $F \in KKM(X, Y \cap L(K))$. Now Theorem 1.4 (ii) guarantees that $g\,F \in KKM(X,X)$ and note $g\,F$ is a compact map. Now apply Theorem 1.5. □

Remark 2.21. Note we could replace $G \in DKY(Y,X)$ with $G \in HLPY(Y,X)$ in Theorem 2.20 (since $G \in HLPY(Y \cap L(K), X)$ as in Theorem 2.18).

Remark 2.22. In Theorem 2.20 (respectively, Remark 2.21) we could replace $F(X) \subseteq L(K) \cap Y$ with $F(L(K)) \subseteq L(K) \cap Y$ provided we assume $F \in KKM(E,Y)$ and $G \in DKT(Y,E)$ ((respectively, $G \in HLPY(Y,E)$) is a compact map (here K is a compact subset of E with $G(Y) \subseteq K$). Also we need to replace the assumption that X is an admissible subset of E with $L(K)$ is an admissible subset of E.

To see this note (see Remark 2.17, respectively, Remark 2.19) $G \in DKT(Y \cap L(K), L(K))$ (respectively, $G \in HLPY(Y \cap L(K), L(K)))$ so since $Y \cap L(K)$ is paracompact there exists a selection $g \in C(Y \cap L(K), L(K))$ of G. Also since $F(L(K)) \subseteq L(K) \cap Y$ from Section 1 (see (i) and (ii)) we have $F \in KKM(L(K), Y \cap L(K))$. Now Theorem 1.4 (ii) guarantees that $g\,F \in KKM(L(K), L(K))$ and note $g\,F$ is a compact map. Now apply Theorem 1.5.

Next we present the analogue of Theorem 2.10.

Theorem 2.23. *Let X be a convex set and Y a closed subset in a Hausdorff topological vector space E. Suppose $F \in PK(X,Y)$ is a upper semicontinuous map with compact values and $G \in DKT(Y,X)$ is a compact map. Let K be a compact subset of X with $G(Y) \subseteq K$, let $L(K)$ be the linear span of K and assume $F(X) \subseteq L(K) \cap Y$. Also suppose one of the following hold:*

(a). $Y \cap L(K) \in ES(compact)$,

(b). $Y \cap L(K)$ is a Schauder admissible subset of E.

Then there exists a $x \in X$ with $G^{-1}(x) \cap F(x) \neq \emptyset$.

Proof. As in Theorem 2.16 note $G \in DKT(Y \cap L(K), X)$ and $Y \cap L((K)$ is paracompact, so there exists a selection $g \in C(Y \cap L(K), X)$ of G. Also $F \in PK(X, Y \cap L(K))$ so $F\,g \in PK(Y \cap L(K), Y \cap L(K))$ is a upper semicontinuous compact map with compact values. Now apply Theorem 1.1 or Theorem 1.2. □

Remark 2.24. Note we could replace $G \in DKY(Y,X)$ with $G \in HLPY(Y,X)$ in Theorem 2.23 (since $G \in HLPY(Y \cap L(K), X)$ as in Theorem 2.18).

Remark 2.25. In Theorem 2.23 (respectively, Remark 2.24) we could replace $F(X) \subseteq L(K) \cap Y$ with $F(L(K)) \subseteq L(K) \cap Y$ provided we assume $F \in PK(E,Y)$ and $G \in DKT(Y,E)$ (respectively, $G \in HLPY(Y,E)$) is a compact map (here K is a compact subset of E with $G(Y) \subseteq K$). To see this notice from Remark 2.17 (respectively, Remark 2.19) that $G \in DKT(Y \cap L(K), L(K))$ (respectively, $G \in HLPY(Y \cap L(K), L(K)))$, so there exists a selection $g \in C(Y \cap L(K), L(K))$ of G. Also since $F(L(K)) \subseteq L(K) \cap Y$ then $F \in PK(L(K), L(K) \cap Y)$ and so $F\,g \in PK(Y \cap L(K), Y \cap L(K))$ is a compact map. Now apply Theorem 1.1 or Theorem 1.2.

We note in [14, Theorem 2.5] that inadventently we assumed $F \in Ad(X,Y)$ and $G \in DKT(Y,X)$ instead of $F \in Ad(E,Y)$ and $G \in DKT(Y,E)$.

Next we present the analogue of Theorem 2.13.

Theorem 2.26. *Let X be an admissible convex set and Y a convex set in a Hausdorff topological vector space E. Suppose $F \in KKM(X,Y)$ is a upper semicontinuous map with compact values and $G \in DKT(Y,X)$ is a compact map. Let K be a compact subset of X with $G(Y) \subseteq K$, let $L(K)$ be the linear span of K and assume $F(X) \subseteq L(K) \cap Y$. Also assume $L(K) \cap Y$ is closed in both Y and $L(K)$ and $Y \cap L(K)$ is an admissible subset of E. Then there exists a $x \in X$ with $G^{-1}(x) \cap F(x) \neq \emptyset$.*

Proof. As in Theorem 2.16 note $G \in DKT(Y \cap L(K), X)$ and $Y \cap L(K)$ is paracompact, so there exists a selection $g \in C(Y \cap L(K), X)$ of G. Also $F \in KKM(X,Y)$, $F(X) \subseteq L(K) \cap Y$ so from Section 1 (see (ii), note $Y \cap L(K)$ is closed in Y) we have $F \in KKM(X, Y \cap L(K))$. Now Theorem 1.8 (note $Y \cap L(K)$ is normal since Hausdorff paracompact spaces are normal [8] and also note X is an admissible convex subset of E) guarantees that $F\,g \in KKM(Y \cap L(K), Y \cap L(K))$ and $F\,g$ is a upper semicontinuous compact map with compact values. Now apply Theorem 1.5. □

Remark 2.27. Now $G \in DKY(Y,X)$ can be replaced by $G \in HLPY(Y,X)$ in Theorem 2.26 (since $G \in HLPY(Y \cap L(K), X)$ as in Theorem 2.18).

Remark 2.28. In Theorem 2.26 (respectively, Remark 2.27) we could replace $F(X) \subseteq L(K) \cap Y$ with $F(L(K)) \subseteq L(K) \cap Y$ provided we assume $F \in KKM(E,Y)$ and $G \in DKT(Y,E)$ (respectively, $G \in HLPY(Y,E)$) is a compact map (here K is a compact subset of E with $G(Y) \subseteq K$). Also we need to replace X is an admissible subset of E with $L(K)$ is an admissible subset of E.

To see this notice from Remark 2.17 (respectively, Remark 2.19) that $G \in DKT(Y \cap L(K), L(K))$ (respectively, $G \in HLPY(Y \cap L(K), L(K))$), so there exists a selection $g \in C(Y \cap L(K), L(K))$ of G. Also since $F(L(K)) \subseteq L(K) \cap Y$ then as in Section 1 (see (i) and (ii)) we have $F \in KKM(L(K), L(K) \cap Y)$. Now Theorem 1.8 (note $Y \cap L(K)$ is normal and $L(K)$ is an admissible convex subset of E) guarantees that $F\,g \in KKM(Y \cap L(K), Y \cap L(K))$ and $F\,g$ is a upper semicontinuous compact map with compact values. Now apply Theorem 1.5.

In all the results so far we choose to obtain coincidences between different classes of maps. When the classes of maps are the same we present the following two results.

Theorem 2.29. *Let X be a subset in a Hausdorff topological space E and Y a subset in a Hausdorff topological space. Suppose $F \in PK(X,Y)$ and $G \in PK(Y,X)$ is a compact map. Also assume one of the following hold:*

(a). $X \in ES(compact)$,

(b). E is a topological vector space, X is a Schauder admissible subset of E and $G\,F : X \to 2^X$ is upper semicontinuous with closed values.

Then there exists a $x \in X$ with $G^{-1}(x) \cap F(x) \neq \emptyset$.

Proof. Note $G\,F \in PK(X,X)$ is a compact map. Now apply Theorem 1.1 or Theorem 1.2. □

Remark 2.30. (i). If E is a Hausdorff topological vector space and X is convex then in Theorem 2.29 (a) (respectively, Theorem 2.29 (b)) we could replace $X \in ES(\text{compact})$ (respectively, X is a Schauder admissible subset of E and $G\,F : X \to 2^X$ is upper semicontinuous with closed values) with $D = co\,(\overline{G(Y)}) \in ES(\text{compact})$ (respectively, D is a Schauder admissible subset of E and $G\,F : D \to 2^D$ is upper semicontinuous with closed values). To see this we just need to note that $F \in PK(D,Y)$ and $G \in PK(Y,D)$.

(ii). Of course one could have other variations of (a) and (b) in Theorem 2.29 if one uses other results in [12], [13].

(iii). Note in Theorem 2.29 we could replace "$G \in PK(Y,X)$ is a compact map" with "$F \in PK(Y,X)$ is a compact map and $G \in PK(Y,X)$ is a upper semicontinuous map with compact values".

Theorem 2.31. *Let X be a convex set in a Hausdorff topological space E and Y a convex set in a Hausdorff topological space. Suppose $F \in HLPY(X,Y)$ is a compact map and $G \in HLPY(Y,X)$. Then there exists a $x \in X$ with $G^{-1}(x) \cap F(x) \neq \emptyset$.*

Proof. There exists a compact set K of Y with $F(X) \subseteq K$. Now since $G \in HLPY(Y,X)$ then as in Theorem 2.3 we have $G \in HLPY(K,X)$, so there exists a selection $g \in C(K,X)$ of G and a finite subset A of X with $g(K) \subseteq co\,(A)$. Note $F \in HLPY(X,Y)$ so a similar argument as in Theorem 2.3 guarantees that $F \in HLPY(co\,(A),Y)$. Now since $co\,(A)$ is compact then there exists a selection $f \in C(co\,(A),Y)$ of F. Note $f(co\,(A)) \subseteq F(co\,(A)) \subseteq F(X) \subseteq K$ so $f \in C(co\,(A),K)$ and as a result we have that $g\,f \in C(co\,(A), co\,(A))$. Now since $co\,(A)$ is a compact convex subset in a finite dimensional subspace of E then Brouwer's fixed point theorem guarantees a $x \in co\,(A)$ with $x = g\,(f(x))$. □

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Donal O'Regan
School of Mathematical and Statistical Sciences,
University of Galway,
Ireland
e-mail: donal.oregan@nuigalway.ie