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Book reviews

Alexander B. Kharazishvili, Set Theoretical Aspects of Real Analysis, CRC Press, Taylor & Francis Group, Boca Raton 2015, xxii + 433 pp, ISBN: 13: 978-1-4822-4201-0.

As it is well known the Axiom of Choice (**AC**) plays a fundamental role in mathematics, many fundamental results depending on it, or even being equivalent to it. At the same time the acceptance of **AC**, i.e. working within **ZFC** (Zermelo-Frenkel set theory + **AC**), leads to counterintuitive and paradoxical results, the most intriguing being the Banach-Tarski paradox. On the other side, the restriction to **ZF** axioms also leads to paradoxical situations as, for instance, the possibility to represent the set **R** of real numbers as a countable union of countable sets (see page 11 of the book). For these reasons the mathematicians tried to understand to what extent some results depend essentially on **AC**, or on some weaker variants: Countable Choice (**CC**), Dependent Choice (**DC**), Product Countable Choice (**PCC**), Continuum Hypothesis (**CH**), Martin's Axiom (**MA**). Some of the results equivalent to **AC** are: (1) Tychonov's theorem on the compactness of the product of compact spaces; (2) Kuratowski-Zorn lemma on the existence of maximal chains in partially ordered sets; (3) the total ordering of cardinal numbers; (3) the existence of a Hamel basis in any vector space. Other results, as (i) the existence of nonmeasurable subsets of **R**; (ii) the existence of subsets of $\mathbb{R}$ not having the Baire property; (iii) the existence of a Hamel basis in **R** (considered as a vector space over **Q**); the Hahn-Banach theorem, cannot be proved within **ZF** and need some uncountable forms of the **AC**. As it is known, Solovay constructed a mathematical theory based on **ZF** plus the existence of a nonmeasurable Lebesgue set, taken as an axiom.

The book contains a detailed presentation of various aspects relating the foundation of mathematics with some fundamental results in real analysis, measure theory, set theory and topology. Among the topics included in the book we mention: measurability properties of sets and functions, the existence of nonmeasurable sets (Vitali sets, Berstein sets) and nonmeasurable functions and functions having some pathological properties (e.g. the Sierpinski-Zygmund function), measurability and continuity (Luzin-type results), the existence of nonmeasurable additive functions, measurability properties of well-orderings, etc.

The exposition is completed with five appendices containing brief but thorough presentations of various topics, that are essential for the understanding of the main text: A1. *The axioms of set theory*; A2. *The Axiom of Choice and the Continuum*

Hypothesis; A3. *Martin's Axiom and its consequences in real analysis*; A4. ω_1-*dense subsets of the real line*; A5. *The beginning of the descriptive set theory.* These appendices make the book fairly self-contained, preventing the reader to browse through specialized volumes.

The main text is completed with Exercises containing further results, ranging from routine to very difficult. For these ones, marked by stars, some hints are supplied. The bibliography at the end of the book counts 279 items.

The author is a well known expert in the area with numerous journal contributions and 6 books (3 in Russian published at Tbilisi, his home university, and other 3 in English published with various international editors), treating various aspects of the interconnections between set theory, measure theory and real analysis.

The book, containing a lot of results of interest to a broad spectrum of readers, in fact to every mathematician involved in research or teaching, is written in a didactic manner with clear proofs and many examples and comments. It can be used as a reference text or as a complementary text for courses on real analysis and measure theory.

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