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Book reviews

Lance Fortnow, The Golden Ticket: P, NP, and the Search for the Impossible, Princeton University Press, 2013, ISBN 978-0-691-15649-1.

The class P is the class of problems which can be solved in polynomial time on deterministic machines. In Complexity Theory tractable is synonym to having solution algorithm with polynomial runtime. The class NP is the class of problems which can be solved in polynomial time on nondeterministic machines, or equivalently having solutions which can be checked in polynomial time.

The P=NP problem is the most important open problem in computer science, if not all of mathematics. In colloquial language, it asks whether every problem whose solution can be quickly checked by computer can also be quickly solved by computer. The Clay Mathematics Institute offers a million-dollar prize for the solution of this problem.

The title *The Golden Ticket* is inspired from Roald Dahl's book, *Charlie and the Chocolate Factory.*

The first chapter introduces in a nontechnical way the concepts of P and NP and gives some examples: the traveling salesman problem and the partition problem as NP-hard problems; shortest path as an exemplar for P.

In Chapter 2, entitled "The Beautiful World", Fortnow does a fanciful spiritual exercise, analyzing the hypothetical consequences of positive answer to P=NP. The unreal "Urbana algorithm" would leads to world changing: progresses in cancer cure, weather prediction, and so on. As a negative consequence the author mentioned the fall of present-day cryptography methods.

Chapter 3 is dedicated to the introduction of standard problems such as cliques, Hamiltonian path, map coloring, and max-cut. The author starts from "freenemy graph", a graph of friendships (and enemies) in a world where every pair of people is either a friend or an enemy. The author clearly emphasizes the idea that solutions to these problems are easy to check, but difficult to find. He also talks of polynomial algorithms: shortest paths, matching, Eulerian paths, and minimum cut.

The history of "P=NP" problem is presented in Chapters 4 and 5. S. Cook showed in 1971 that the satisfiability problem (SAT) is, in a well-defined sense, as difficult as any problem in NP and if somebody could solve satisfiability in polynomial time, then every other problem in NP could also be solved in polynomial time. Richard Karp then showed that not only satisfiability, but another twenty problems taken from real world had the property that if you solved one of them quickly, all the other could be solved quickly. Earlier (in 1960), Jack Edmonds discussed the polynomial/exponential divide in algorithms and pointed out the need for a formalism of this issue. Besides the western contributions, Fortnow discusses the results of Soviet mathematicians Levin, Yablonsky and Kolmogorov.

Some approaches concerning how to deal with NP-complete problems are covered in Chapter 6. Heuristics and approximations, with examples from problems in Chapter 3 are treated with some attention.

Chapter 7 treats topics related to the proof of "P=NP". Fortnow discusses the "undecidability" and the relationship between circuit complexity and the P versus NP problem; he outlines the difficulties in this direction. The chapter ends in a pessimistic note; Fortnow notes the lack of a path toward resolving the problem and that it is not clear what directions to choose.

The Chapters 8 and 9 are about cryptography and quantum computing, respectively. The quantum chapter in particular is interesting for the effect of quantum computing on complexity classes. Having a workable quantum computer would not, as known so far, allow the solution of NP-complete problems in polynomial time: the best known speedup only changes the complexity of an algorithm by taking the square root of its running time. It would, however, affect problems like factoring, which closely relates the final two chapters.

In the conclusions (Chapter 10), "The Future", Fortnow makes predictions, highlighting parallel computing and big data.

Last, but not least, the book has a nice bibliography of sources which deserve to be consulted.

Intended audience: undergraduates, the general popular science audience (nonspecialists which wish an introduction to subject), lecturers who want to liven their courses.

Radu Trîmbiţaş

Leiba Rodman, Topics in Quaternion Linear Algebra, Princeton Series in Applied Mathematics, Princeton University Press, Princeton, 2014, xii+363 pp, ISBN 978-0-691-16185-3 (hardback).

The algebra $\mathbf{H}$ of quaternions, meaning the space $\mathbf{R^4}$ endowed with a nocommutative multiplication rule, was discovered by William Rowan Hamilton in 1844 (the notation $\mathbf{H}$ comes from his name). As it is known there only four possibilities to endow $\mathbf{R^k}$ with a multiplicative structure: $k = 1$ the real numbers, $k = 2$ the complex numbers, $k = 4$ the quaternions and $k = 8$ Cayley's algebra (or the octonions). The multiplication in $\mathbf{H}$ is noncommutative and in Cayley's algebra is noncommutative and nonassociative.

In spite of their numerous applications in quantum physics, engineering (control systems), computer graphics, chemistry (molecular symmetry), the quaternions were considered only in chapters of some algebra books or in survey articles. The aim of the present book is to fill in this gap, being the first one dedicated entirely to a thorough and detailed presentations of linear algebra over quaternions. Besides classical results it contains new, previously unpublished, results with full proofs as well as results appearing for the first time in book form.

The book can be divided into two parts. The first one (Chapters 2-7), written at upper undergraduate or graduate level, contains the basic properties of quaternions, vector spaces and matrices, matrix decompositions, invariant subspaces and Jordan form, Kroneccker form, Smith form, determinants, numerical ranges. The second one, Chapters 8-14, is concerned with pencils (meaning matrices of the form $A + tB$, for A, B $m \times n$ matrices over $\mathbf{H}$, i.e. first degree matrix polynomials) of Hermitian and skewhermitian matrices and their canonical forms, indefinite inner products and conjugation, involutions for matrix pencils and for inner products. Applications are given to systems of linear differential equations with symmetries and to matrix equations. This part is written at the level of a research monograph.

The book contains also over than 200 exercises and problems of various levels of difficulties, ranging from routine to open research problems. They give opportunity to do original research – concrete, specific problems for undergraduate research and theses, and the research problems for professional mathematicians and PhD theses. The prerequisites are modest - familiarity with linear algebra, complex analysis and some calculus will suffice.

Written in a clear style, with full proofs to almost all included results, the first part of the book can be used for courses in advanced linear algebra, complemented with chapters from the second part. For working mathematicians, interested vector calculus, linear and partial differential equations, as well as practitioners (scientists and engineers) using quaternions in their research, the book is a fairly complete and accessible reference tool.

Cosmin Pelea

Boris Makarov and Anatolii Podkorytov, Real Analysis : Measures, Integrals and Applications, Universitext, Springer, London - Heidelberg - New York - Dordrecht, 2013, ISBN 978-1-4471-5121-0; ISBN 978-1-4471-5121-7 (eBook); DOI 10.1007/978-1-4471-5122-7, xix + 772 pp.

The specific feature of the present book consists in the presentation of abstract measure and integration theory alongside with the Lebesgue and Lebesgue-Stieltjes measure and integral and a lot of applications in analysis and geometry. This approach facilitates reader's access to these applications on a complete rigorous basis, usually lacking in the books treating applications, while most of the books devoted to an abstract development of the theory neglect consistent applications.

The basics of measure theory are developed in the first chapter, starting with measures defined on semi-rings of subsets of a given set and using Carathéodori extension and definition of measure to extend them to σ-algebras. One proves the fundamental properties of the Lebesgue measure – regularity, invariance with respect to rigid motions, behavior under linear maps (used later in the proof of the change of variables). Hausdorff measures and Vitali coverings are considered as well and, as application, a proof of the Brun-Minkowski inequalities and a study of some isoperimetric problems are included.

The definition and basic properties of measurable functions are treated in the third chapter, including various kinds of convergence for sequences of measurable functions and the fundamental theorems relating them, as well as Luzin's theorem on the approximation of Lebesgue measurable functions by continuous functions.

The integration is treated in the fourth chapter, first for positive simple functions and then for positive measurable functions f as the supremum of the integrals of positive simple functions majorized by f. Various theorems on the passage to limit under the integral sign are proved and a detailed study of Lebesgue integral, of functions with bounded variation and of Lebesgue-Stieltjes integral is included. As special topics, we mention the maximal function of Littlewood-Hardy and Lebesgue's theorem on the differentiation of integrals with respect to sets.

Chapter 5, *Product measures*, is concerned mainly with finite products of measure, infinite products being discussed briefly at the end of the chapter. As applications one proves the Cavalieri principle and Gagliardo-Nirenberg-Sobolev inequality relating the integrals of a smooth function and of its gradient.

The delicate problem of the change of variables in multiple Lebesgue integrals is treated in the fifth chapter. This chapter contains also a proof of Poincaré's recurrence theorem for measure preserving transformations and a study of distribution functions and zero-one laws in probability theory. Milnor's proof of Brouwer's fixed point theorem based on the change of variables is also included.

Chapter 7 contains a detailed study of integrals (both proper and improper) depending on parameters with application to the study of Gamma function.

In the seventh chapter, *The surface area*, after a quick introduction to smooth manifolds, one proves the key properties of the k-dimensional surface area in $\mathbb{R}^m$ and Gausss-Ostrogradski formula. The area on Lipschitz manifolds is considered in the last section of the chapter. The theoretical results are applied to harmonic functions.

The last theoretical chapter is Chapter 11. *Charges. The Radon-Nikodym theorem,* devoted to this important result in measure theory. Applications are given to the differentiation of measures and to the differentiability of Lipschitz functions (Rademacher's theorem).

The last of the chapters, 9. *Approximation and convolution in the spaces* $\mathcal{L}^p$, 10. *Fourier series and the Fourier transform,* and 12. *Integral representation of linear functionals,* are devoted to applications.

A consistent chapter (64 pages), *Appendices,* surveys some notions and results (most with proofs) used in the main body of the book – regular measures, extensions of continuous functions, integration of vector functions, smooth mappings and Sard's theorem, convexity.

The book is based on the courses taught by the authors at the Department of Mathematics and Mechanics of St. Petersburg State University, at various levels. Since the volume of the included material exceeds the limits of a course in measure theory, they suggest in the Preface how different chapters (or sections) can be used for introductory courses on measure theory, or for more advanced ones, at Master level: maximal functions and the differentiation of measures, Fourier series and Fourier transform, approximate identities and their applications. Some sections, containing more specialized topics, marked with $\star$, can be skipped at the first reading. A diagram on the dependence of the chapters is also included. Each section ends with a set of exercises of various difficulties.

Written in a didactic style, with clear proofs and intuitive motivations for the abstract notions, the book is a valuable addition to the literature on measure theory and integration and their applications to various areas of analysis and geometry. The numerous nontrivial examples and applications are of great importance for those interested in various domains of modern analysis and geometry, or in teaching.

S. Cobzaş

William Kirk and Naseer Shahzad, Fixed Point Theory in Distance Spaces, Springer, Heidelberg New-York Dordrecht London, 2014, xi + 173 pp, ISBN 978-3-319-10926-8; ISBN 978-3-319-10927-5 (eBook); DOI 10.1007/978-3-319-10927-5.

The book is devoted to various aspects of fixed point theory in metric spaces and their generalizations. Fixed points for contractions, a topic treated in many places, is omitted from this presentation. The book is divided into three parts: I. *Metric spaces*, II. *Length spaces and geodesic spaces*, and III. *Beyond metric spaces.*

The main topic in the first part is Caristi's fixed point theorem and its generalizations. A special attention is paid to the question whether a proof depends on the axiom of choice or on some its weaker forms (Dependent Choice (DC), Countable Choice (CC)), a theme which appears recurrently throughout the book. The fixed point for nonexpansive mappings is proved within the context of metric spaces endowed with a compact and normal convexity structure. The proof is based on Zermelo's fixed point theorem in ordered sets, requiring only ZF+DC (Zermelo-Fraenkel set theory plus the axiom of Dependent Choice). The first part closes with a presentation of fixed points for nonexpansive mappings on hyperconvex metric spaces, with emphasis on hyperconvex ultrametric spaces (metric spaces (X,d) with $d(x,y)\le\max\{d(x,z),d(z,y)\}$ for all $x,y,z\in X$).

The second part is concerned with spaces which, in addition to their metric structure, have also a geometric structure – length spaces, geodesic spaces, Busemann spaces, CAT(k) spaces, Ptolemaic spaces and $\mathbb{R}$-trees (or metric trees).

A semimetric is a function $d: X\times X\to\mathbb{R}_+$ (X being a nonempty set) such that (i) $d(x,y)=0 \iff x=y$ and (ii) $d(x,y)=d(y,x)$, for all $x,y\in X$. A b-space is a semimetric space (X,d) such that $d(x,y)\le s[d(x,y)+d(z,y)]$ for all $x,y\in X$. Obviously, X is a metric space for $s=1$. One considers also semimetric spaces satisfying a quadrilateral inequality (instead of the triangle inequality) and partial metric spaces. Various aspects of fixed point theory in these generalized metric spaces is examined in the third part of the book.

The book is clearly written and contains a very good selection of results in this rapidly growing area of research – fixed points in metric spaces and their generalizations. The sources of the presented results are carefully mentioned as well as references to related results and further investigation (the bibliography at the end of the book contains 223 items). The book will be an essential reference tool for researchers working in fixed point theory as well as for those interested in applications of metric spaces and their generalizations to other areas – computer science, biology, etc.

S. Cobzaş

Petr Hájek, Michal Johanis, Smooth Analysis in Banach Spaces, De Gruyter Series in Nonlinear Analysis and Applications, Vol. 19, xvi + 465 pp, Walter de Gruyter, Berlin - New York, 2014, ISBN: 978-3-11-025898-1, e-ISBN: 978-3-11-025899-8, ISSN: 0941-813X.

Smoothness is one of the most important and most studied topic in both finite and infinite dimensional analysis. It turns out that in the infinite dimensional case the existence and the properties of smooth mappings between Banach spaces are tightly interconnected withy the structural properties of the underlying spaces. In some cases the existence of a smooth norm forces the Banach space to be isomorphic to a Hilbert one. A decisive role in this study is played by the classical Banach spaces c_0 and ℓ_p (mainly ℓ_1), as well as by other properties as Radon-Nikodym, super-reflexivity, or being Asplund. In fact many new tools in geometric Banach space theory, as e.g. ultraproducts, were devised to solve, among others, problems related to smoothness.

The present book is devoted to a thorough and detailed presentation of various aspects of smoothness in Banach space setting. In this case, like in the finite dimensional one, a prominent role is played by polynomials, via Taylors formula – the most important result concerning smooth mappings. As the authors point out in the Introduction: "In the infinite dimensional setting the role of polynomials is brought even further, as polynomials also provide the vital link with the structure of underlying Banach space." For these reasons three chapters, 2. *Basic properties of polynomials on* $\mathbb{R}^n$, 3. *Weak continuity of polynomials and estimates of coefficients,* and 4. *Asymptotic properties of polynomials,* are entirely devoted to the presentation of various properties of polynomials, both in finite and infinite dimension.

The first chapter, 1. *Fundamental properties of smoothness,* contains a thorough, fairly detailed introduction to smoothness in Banach spaces, including high order smoothness, polynomials, Taylor's formula and converses, power series and analytic mappings. Here both real and complex cases are considered, some real results being transferred to the complex case via complexification techniques.

The rest of the book is devoted to deeper properties of smooth mapping. In Chapter 5, *Smoothness and structure,* the structural properties of Banach spaces admitting smooth functions are studied, via the variational principles of Ekeland, Stegall, , Borwein-Preiss, Fabian-Preiss, Deville-Zizler. Chapter 6, *Structural behavior of smooth mappings,* is concerned with the relations between various classes of smooth mappings involving various notions of weak and strong uniform continuity of the derivatives. An important class of Banach spaces, denoted by $\mathcal{W}$, is introduced here, allowing the extension of some smoothness results from the Banach space $C(K)$ to spaces in the class $\mathcal{W}$.

The last chapter of the book, 7. *Smooth approximation,* is concerned with the uniform approximation of continuous functions by smooth ones, or of C^k-functions by polynomials or by real analytic functions (here only the real case is considered). This line of investigation, having its roots in the pioneering work of Jaroslav Kurzweil from 1954 and 1957, is still in the focus of intense current research, many important problems waiting for solution.

As one of the authors (PH) mentions, a source of inspiration for him (and for many people working in this domain) was a list of 50 problems compiled by V. Zizler in the early 90's, later expanded to 90 in the book by R. Deville, G. Godefroy and V. Zizler, *Smoothness and renormings in Banach spaces*, Pitman, New York 1993. In the meantime some of these problems were solved, their solutions being reflected in the present book, others, still unsolved, are also mentioned in the book along with new ones posed by the authors.

For reader's convenience, the authors have included (without proofs) auxiliary results (on tensor products, vector holomorphic functions, etc) as paragraphs and sections in the places were they are first used, preventing the reader to jump to appendices or to specialized monographs and leading so to a "smooth" reading of the text.

Written by two eminent specialists in Banach space theory, with important contributions to the field, the book will become an indispensable tool for researchers in Banach space geometry, smoothness and applications. By the detailed presentation of the subject it can be used also by graduate students or by instructors for introduction to the domain. At the same time, the nice and rewarding problems spread trough the text form a valuable source of inspiration for further investigation.

S. Cobzaş

Saleh A. R. Al-Mezel, Falleh R. M. Al-Solamy and Qamrul H. Ansari, Fixed Point Theory, Variational Analysis, and Optimization, CRC Press, Taylor & Francis Group, Boca Raton 2014, xx + 347 pp, ISBN: 13: 978-1-4822-2207-4.

The present volume grew out of an International Workshop on Nonlinear Analysis and Optimization, held at the University of Tabuk, Saudi Arabia, March 16-19, 2013, most of the contributors being participants to this event. It is divided into three parts: I. *Fixed point theory*; II. *Convex analysis and variational analysis*, and III. *Vector optimization.*

The first part contains three papers: Common fixed point in convex metric spaces (by Abdul Rahim Khan and Hafiz Fukhar-ud-din), Fixed points of nonlinear semigroups in modular function spaces (by B. A. Bin Dehaish and M. A. Khamsi), Approximation and selection methods for set-valued maps and fixed point theory (by Hichem Ben-El-Mechaiekh).

The second part consists also of three papers: Convexity, generalized convexity, and applications (by N. Hadjisavvas), New developments in quasiconvex optimization (by D. Aussel), and An introduction to variational-like inequalities (by Qamrul Hasan Ansari).

Two papers – Vector optimization : Basic concepts and solution methods (by Dinh The Luc and Augusta Raţiu) and Multi-objective combinatorial optimization (by Matthias Ehrgott and Xavier Gandibleux) - form the third and the last part of the book.

The papers included in the volume have both an introductory and an advanced character – they contain the basic concepts and results presented with full proofs, and at the same time new results situated at the frontier of current research.

Reporting on basic and new results in these tightly interrelated areas of nonlinear analysis – fixed point theory, variational analysis, and optimization – the survey papers included in this volume, written by renown experts in the domain, are of great interest to researchers in nonlinear analysis, as well as for the novices as a source of a quick and accessible introduction to some problems of great interest in contemporary research.

J. Kolumbán