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Book reviews

Ravi P. Agarwal, Erdal Karapinar, Donal O'Regan and Antonio Francisco Roldán-López-de-Hierro; Fixed Point Theory in Metric Type Spaces, Cham: Springer, 2015, xvii+385 p. ISBN 978-3-319-24080-0/hbk; 978-3-319-24082-4/ebook).

The book is devoted to fixed points in generalized metric (G-metric) spaces. A G-metric on a set X is a function $G : X^3 \to [0,\infty)$ satisfying the following conditions:

(G1) $G(x,y,z) = 0 \iff x = y = z$;

(G2) $G(x,x,y) > 0)$ for all $x,y \in X$ with $x \neq y$;

(G3) $G(x,x,y) \leq G(x,y,z)$ for all $x,y,z \in X$ with $x \neq y$;

(G4) $G(x,y,z) = G(\pi(x,y,z))$ for every pemutation π of x,y,z;

(G5) $G(x,y,z) \leq G(x,w,w) + G(w,y,z)$ for all $x,y,z,w \in X$.

A typical example of a G-metric is the perimeter of a triangle in $\mathbb{R}^n$.

Generalized metric spaces were introduced by Dhage in the nineteens of the last century, but his papers contained some flaws (mainly concerning the topological properties of these spaces), which were fixed by Z. Mustafa and B. Sims in J. Nonlinear Convex Anal. (2006).

As the authors consider fixed point results for mappings T on G-metric spaces satisfying a condition of the type

$$G(Tx,Ty,Tz) \leq G(x,y,y) - \phi(G(,x,y,y)),$$

where $\phi : [0,\infty) \to [0,\infty)$ is called a control function, the second chapter, *Preliminaries*, contains a detailed study of these control functions.

One considers 18 properties that could be satisfied by a control function (like being monotone, subadditive, semi-continuous, or satisfying for all $t > 0$, $\phi(t) < t$, $\lim_n \phi^n(t) = 0$, or the convergence of the series $\sum_n \phi^n(t)$, etc). Combining some of these properties (usually three of them) one obtains various control functions considered in the fixed point theory of mapping on metric spaces as Matkowski comparison function, Geraghty, Boyd-Wong, Ćirić, Krasnoselski functions, etc. This chapter contains a detailed study of the relations between these properties and between the classes of functions they define.

The basic properties of G-metric spaces are studied in Ch. 3, *G-Metric Spaces*, while the rest of the book is devoted to various fixed point results in this class of spaces: Ch. 4, *Basic Fixed Point Results in the Setting of G-Metric Spaces*, Ch. 5 *Fixed Point Theorems in Partially Ordered G-Metric Spaces*, Ch. 6, *Further Fixed Point Results on*

G-Metric Spaces, Ch. 7, *Fixed Point Theorems via Admissible Mappings*, Ch. 8, *New Approaches to Fixed Point Results on G-Metric Spaces,* Ch. 9, *Expansive Mappings*, Ch. 10, *Reconstruction of G-Metrics: G^*-Metrics*, Ch. 11, *Multidimensional Fixed Point Theorems on G-Metric Spaces*, Ch. 12, *Recent Motivating Fixed Point Theory.*

Some supplementary material is collected in an Appendix, *Some Basic Definitions and Results in Metric Spaces.*

The book, including many contributions of its authors, provides an accessible and up-to-date source of information for researchers in fixed point theory in metric spaces and in various of their generalizations, for mappings satisfying some very general conditions.

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Afif Ben Amar and Donal O'Regan; Topological Fixed Point Theory for Singlevalued and Multivalued Mappings and Applications, Cham: Springer, 2016, , x+194 p.
ISBN: 978-3-319-31947-6/hbk; 978-3-319-31948-3/ebook.

The present book on fixed points focusses on applications to integral equations and to nonlinear eigenvalue problems. Since the main context is functional analytic, the authors devoted the first chapter of the book, *Basic concepts*, to the presentation of some basic notions and results in functional analysis – normed spaces, ordered vector spaces and ordered normed spaces, locally convex spaces, weak topologies (weak compactness, Dunford-Pettis property), compact and weakly compact operators. The chapter ends with some fixed point theorems – Krasnoselskii's, Leray-Scahuder theory, and fixed points for multivalued maps. Although some proofs are included, the majority of the results are presented without proofs.

The second chapter is dedicated to nonlinear eigenvalue problems in Banach spaces satisfying the Dunford-Pettis property. The third chapter is concerned with Leray-Schauder type theorems for mappings which are condensing with respect to De Blasi measure of weak noncompactness. This study is continued in the fourth chapter for mappings with sequentially closed graph. Applications are given to Volterra-type integral equations under Henstock-Kurzweil-Pettis integrability and to integral equations in Lebesgue spaces.

The fifth chapter is devoted to fixed points for applications of the form $AxBx + Cx$, $x \in X$, defined on a Banach algebras X, under appropriate conditions on the operators A, B, C and on the Banach algebra X. Applications are given to some nonlinear functional integral equations.

The sixth chapter is concerned with a class of operators $F : D \to X$, X a Banach space and $D \subset X$, introduced by Gowda and Isac in 1993, called by them (ws)-compact and meaning that F is $\|\cdot\|$-continuous and $(F(x_n))$ contains a $\|\cdot\|$-convergent sequence for every weakly convergent sequence (x_n) in D.

The last chapter of the book (Ch. 7) presents some results on approximate fixed point sequences (i.e. sequences satisfying $x_n - Fx_n \to 0$) for multivalued mappings with applications to Nash equilibrium in noncooperative games.

The book, including original contributions of the authors, is addressed to researchers interested in applications of fixed point results (in functional analytic context) to integral equations, ordinary and partial differential equations, game theory, etc. The detailed exposition of the subject and the prerequisites make it appropriate for graduate courses in linear and in nonlinear functional analysis.

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