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Book reviews

René L. Schilling, Mass und Integral - Eine Einführung für Bachelor-Studenten, x+172 pp, Walter de Gruyter, Berlin/Bsoton, 2015, ISBN: 978-3-11-034814-9/pbk; eISBN(PDF): 978-3-11-035064-7; eISBN(EPUB): 978-3-11-038332-4.

This is a text of a course on measure theory and integration for students in mathematics and physics. The main goal of the book is to present the basic properties of the Lebesgue measure and integral needed in higher analysis, probability theory and mathematical physics.

The presentation is based on an abstract approach – σ-algebras, Dynkin systems, monotone classes. The measure is introduced using the Carathéodori extension theorem from semi-rings of sets, allowing a quick definition of Lebesgue measure on $\mathbb{R}$. The Lebesgue measure and integral on $\mathbb{R}^d$ are introduced as a product measure and integral, via the Fubini-Tonelli theorem.

The integrals of positive measurable functions f are defined as suprema of the integrals of measurable positive step functions majorized by f, and for real and extended real-valued measurable functions in the usual way, writing them as differences of positive measurable functions. The Lebesgue criterium of Riemann integrability is proved as well.

The convergence theorems (monotone convergence theorem, Lebesgue dominated convergence theorem, Egorov's theorem) are applied to the study of integrals with parameters – continuity and differentiability. The basic properties of L^p-spaces – Riesz-Fischer completeness theorem, Riesz theorem on the convergence of sequences of functions in L^p $(\|f_n - f\|_p \to 0 \iff \|f_n\|_p \to \|f\|_p$, provided $f_n(x) \to f(x)$ a.e.), Jensen inequality – are presented in detail. Applications are given to the convolution of functions and measures and to Fourier transform (Riemann-Lebesgue lemma, Wiener algebra, Plancherel's theorem).

The Lebesgue-Nikodým theorem is applied to the change of coordinate formula for integrals. The book ends with the study of functional analytic properties of the spaces L^p and $C(T)$ (for T a local compact metric space). One proves the density of some classes of functions in L^p, Riesz representation theorems for the duals of L^p and $C(T)$, and one studies the weak convergence of measures (an important topic in stochastic analysis).

Some additional questions are discussed in an Appendix: the existence of non-measurable sets, the integration of complex-valued functions, separability of $C(T)$, regularity of measures. Also, the exercises included at the end of each chapter complete the main text with further results and examples.

The book is very well organized, with clearly written conditions in all theorems, succeeding to present by a cleaver choice of the included topics, in a relatively small number of pages, some basic results of measure theory and integration, with emphasis on Lebesgue measure and integral in $\mathbb{R}^d$ and applications. For further results and applications, author's book, R. L. Schilling, *Measures, Integrals and Martingales*, Cambridge University Press, Cambridge 2001 (3rd printing), is highly recommended.

Hannelore Lisei

Daniel Alpay, An advanced complex analysis problem book. Topological vector spaces, functional analysis, and Hilbert spaces of analytic functions, Birkhäuser/Springer, New York, NY 2015, ix+525 p., ISBN 978-3-319-16058-0/pbk.

Usually in a first course on Complex Analysis analytic functions are considered as individuals, not as elements of some Hilbert, Banach or Fréchet spaces. Also some topological notions are introduced intuitively, without any rigorous topological foundation. Here one can mention the definition of the Riemann sphere $\widehat{\mathbb{C}}$ simply as $\widehat{\mathbb{C}} = \mathbb{C} \cup \{\infty\}$, the introduction of the uniform convergence on compact sets via Morera's theorem, proofs of the Riemann mapping theorem without appealing to compactness arguments.

The aim of this book is to fill in this gap, i.e. to get students familiar with some notions of functional analysis in the context of spaces of analytic functions, based on the unifying idea of reproducing kernel Hilbert space. By an adequate choice of the reproducing kernel one obtains the basic spaces of analytic functions: the Bargmann-Segal-Fock space, the Bergman space and the Hardy space. Besides the analytic description a geometric geometric one is considered as well.

The problems in the book are labeled *Exercise*, for which solutions are given, or *Question* or *Problem*, left without solutions or with solutions given in a previous book of the author:

[CAPB] D. Alpay, *A Complex Analysis Problem Book*, Birkhäuser/Springer Basel AG, Basel, 2011.

The first chapter of the book, 1. *Algebraic prerequisites*, contains some results on sets, functions, groups, matrices. The second one, 2. *Analytic functions*, contains some elements of complex analysis, a more detailed presentation being given in [CAPB].

The presentation of topological and functional analytic aspects is done in the second part of the book, II. *Topology and Functional Analysis*, having the chapters: 3. *Topological spaces*, 4. *Normed spaces* (Banach and Hilbert spaces, operators - bounded and unbounded), 5. *Locally convex topological vector spaces* (countably normed and Fréchet spaces, topologies on spaces of analytic functions and their duals, normal families), 6. *Some functional analysis* (Fourier transform, Stieltjes integral, density results in L_2-spaces). The third part, III. *Hilbert Spaces of Analytic Functions*, contains the chapters 7. *Reproducing kernel Hilbert spaces*, 8. *Hardy spaces*, 9. *de Branges-Rovnyak Spaces*, 10. *Bergman spaces* , and 11. *Fock spaces.*

Written by an expert in the area, the book is dedicated to beginning graduate students aiming a specialization in complex analysis. Teachers of complex analysis will find some supplementary material here and those of functional analysis a source of concrete examples. The presentation is restricted to one variable but, as the author promises in the Prologue to the book, a volume dedicated to several variables is in preparation.

S. Cobzaş

Marek Jarnicki and Petter Pflug, Continuous nowhere differentiable functions. The monsters of analysis, Springer Monographs in Mathematics, Springer - Cham, Heidelberg, New York, Dordrecht, London, 2015, xii+299 p., ISBN 978-3-319-12669-2/hbk; 978-3-319-12670-8/ebook.

After Newton put the basis of the differential calculus and applied it to the study the physical world, there was a general belief between mathematicians that a continuous function must be differentiable excepting a finite number of points. The famous mathematician and physicist A.-M. Ampère even published a proof of this result (based on some intuitively justified geometric reasonings on the behavior of curves) which was generally accepted by the mathematical community and included in almost every calculus book of that time. So the presentation by Karl Weierstrass in 1872 in front of the Königliche Akademie der Wissenschaften, Berlin, of his famous nowhere differentiable continuous function $\sum_{k=0}^{\infty} a^k \cos(b^k \pi x)$, $x \in \mathbb{R}$, came as a great surprise, not very pleasant for some of them. Emile Picard said that if Newton had known about such functions he would never create calculus, and Ch. Hermite wrote to Stieltjes: "Je me détourne avec effroi et horreur de cette plaie lamentable des fonctions continues qui nont point de dérivées.". H. Poincaré – who was the first to call such functions monsters – claimed that the functions were an arrogant distraction, and of little use to the subject: "They are invented on purpose to show that our ancestors reasoning was at fault, and we shall never get anything more out of them."

But, finally, the mathematicians had to accept the existence of these functions and to reconsider some results based on the idea of differentiability of continuous functions as well as the idea of proof – replace the geometric intuitive reasonings by rigorous analytic ones, as this was done by Weierstrass in his proof of nondifferentiability. These comments and others are nicely presented in the introductory chapter of the book, 1. *Introduction: A Historical Journey.*

The present book present in a rigorous and systematic way results related to this kind of functions, starting with classical and ending with some very recent and some open problems.

The first part I. *Classical results,* contains results obtained from the middle of the nineteenth century up to about 1950. Although the proofs are based on complicated arguments, they are accessible to undergraduate students.

Part II. *Topological methods*, shows that these strange functions not even that do exist, but they are in big quantities, in the sense of Baire category. For instance, the set of nowhere differentiable continuous functions are of second Baire category (and so contain a dense G_δ-set) in the space $C[a, b]$ (Banach-Mazurkiewicz-Jarnik and Saks theorems). The same is true about the set of functions $f \in A(\mathbb{D})$ (holomorphic in the open unit disk $\mathbb{D}$ and continuous on $\bar{\mathbb{D}}$ – the disk algebra) such that $f|_{\partial\mathbb{D}}$ is nowhere differentiable.

Part III. *Modern approach*, requires some more advanced tools from analysis, as measure theory and Fourier transform. The last chapter of this part (Chapter 12) is concerned with the existence of (closed) linear spaces of such functions (properties called lineability and spaceability - see the next review).

B. Riemann claimed that the function $\mathbf{R}(x) = \sum_{n=1}^{\infty} n^{-2}\sin(\pi n^2 x)$, $x \in \mathbb{R}$, is also nowhere differentiable, a result that turned out to be false. The difficult problem of the points of differentiability or nondifferentiability of this function, that preoccupied many famous mathematicians as, e.g., G. H. Hardy, is treated in detail in the fourth part of the book, having only one chapter, 13. *Riemann function.*

For reader's convenience 9 appendices, dealing with topics as Fourier transform, harmonic and holomorphic functions, Poisson summation formula, etc, are included.

By bringing together results scattered in various publications, some of them hardly to find or/and hardly to read (I mean old papers), presenting them in a unitary and rigorous way (using a modern language and style) with pertinent historical comments, the authors have done a great service to the mathematical community. The book presents interest for all mathematicians, but also for people (engineers, physicists, etc) having a basic background in calculus, interested in the evolution of some fascinating problems in this area, simply to formulate, but hardly to solve. Undergraduate, graduate students and teachers will find an accessible source of interesting examples, and possible be attracted by some hard problems remained unsolved till now.

S. Cobzaş

Richard M. Aron, Luis Bernal-González, Daniel M. Pellegrino and Juan B. Seoane Sepúlveda; Lineability. The search for linearity in mathematics, Monographs and Research Notes in Mathematics, CRC Press, Boca Raton, FL, 2015, xix+308 p, ISBN: 978-1-4822-9909-0/hbk;
978-1-4822-9910-6/ebook.

For a long time many mathematicians (including great names like A. M. Ampère) believed that any continuous function must be differentiable on a large subset of its domain of definition. So was a great shock when K. Weierstrass presented in 1872 his famous continuous and nowhere differentiable function: $\sum_{k=0}^{\infty} a^k \cos(b^k \pi x)$, where $0 < a < 1$, b is an odd integer and $ab > 1+2\pi/2$. Although such functions were devised before by B. Bolzano (1830), B. Riemann (1861), H. Hankel (1870), Weierstrass was the first who published such a result. In spite of the natural assumption that the existence of these "pathological" (or "strange") functions is an exception, it turned out that they form large sets in the sense of Baire category. S. Banach (1931) and S. Mazurkiewicz (1936) proved that the set $\mathcal{ND}[a, b]$ of nowhere differentiable functions is of second Baire category (and so dense) in the space $C[a, b]$. The situations is the same with the set $(S)(I)$ of C^∞-functions with are nowhere real-analytic on the interval $I \subset \mathbb{R}$ (called singular functions). A classical example is that of the function

$f(x) = \exp(-1/x^2)$ which is of class C^∞ but no analytic at 0. Du Bois-Raymond (1876) constructed a function in $(S)(I)$ and H. Salzmann and K. Zeller (1955) proved that the set of singular functions is of second Baire category in $C^\infty(I)$.

The purpose of the present monograph is to analyze the situations when a class of functions in a given space contains a linear subspace. More exactly, a subset M of a topological vector space X is called:

- μ-lineable if $M \cup \{0\}$ contains a vector subspace of dimension μ;
- μ-spaceable if $M \cup \{0\}$ contains a closed vector subspace of dimension μ;
- μ-dense 0-lineable if $M \cup \{0\}$ contains a dense vector subspace of dimension μ.

Here μ is a cardinal number. If $\mu \geq \aleph_0$, then the set M is called simply lineable, spaceable, or dense-lineable. The first who proved the existence of such a subspace was V. I. Gurariy (1966): the set $\mathcal{ND}[0,1]$ contains an infinite dimensional vector space. Although some scattered results in this area were obtained in the last third of the preceding century, a systematic study of these problems started at the beginning of the current millennium. The survey paper by the last three authors, Bull. Amer. Math. Soc. 51 (2014), 71–130, can be considered as a forerunner of the present volume.

The authors examine the existence of linear subspaces in various classes of functions, as reflected by the headings of the chapters: 1. *Real analysis*, 2. *Complex analysis*, 3. *Sequence spaces, measure theory and integration*, 4. *Universality, hypercyclicity and chaos*, 5. *Zeros of polynomials in Banach spaces.* Other situations (divergent Fourier series, norm attaining functionals, etc) are discussed in Chapter 6. *Miscellaneous.*

The book is fairly self-contained - each chapter starts with a section, *What one needs to know*, and the first chapter (unnumbered), *Preliminary notions and tools*, contains also some additional notions and results used throughout the book.

Written by four experts in the area, whose substantial contributions are included in the book, most of them obtained in this millennium, the present monograph is addressed to postgraduate, but also to young or senior researchers wanting to enter the subject. Mathematicians interested in analysis, understood in a broad sense, will find a lot of interesting results collected in it.

Valeriu Anisiu

Valeriu Soltan; Lectures on Convex Sets, World Scientific Publishing Co. Pte. Ltd., Singapore 2015, x+405 pp, ISBN 978-9814656689; ISBN 978-9814656696.

The present book is devoted to a systematic study of algebraic and topological properties of convex subsets of the Euclidean space $\mathbb{R}^n$. As it is known these objects form the background of various mathematical disciplines, as convex geometry and operation research.

After two preliminary chapters 0. *Preliminaries*, and 1. *The affine structure of* $\mathbb{R}^n$, the study of convex sets starts in the second chapter with some algebraic and topological properties (relative interior, closure and relative boundary). Convex hulls, convex cones and conic hulls are treated in Chapters 3 and 4. Chapter 5 is concerned with some important topics in optimization and operation research – recession cones, normal cones and barrier cones. The separation and support properties of convex sets are discussed in Chapter 6. *Extreme points*, extreme faces and representations of convex sets in terms of extreme points are discussed in Chapter 7, while Chapter 8 is concerned with the exposed structure of convex sets and representation theorems (Straszewicz, Klee, Soltan). In the last chapter of the book, Chapter 9. *Polyhedra*, the results obtained in the previous chapters are applied to the study of this important class of convex sets.

The book is very well written. Each chapter ends with a section of notes and comments, and a set of exercises, with solutions given at the end of the book, completing the main text. Carefully done drawings illustrates the main notions introduced throughout the book.

The book is written at undergraduate level or entry-level graduate courses on geometry and convexity, the prerequisites being undergraduate courses on linear algebra, analysis and elementary topology. In spite of its relatively elementary level, the book contains many important results in finite dimensional convexity, necessary in many other mathematical areas. By the detailed and rigorous presentation of the material it can be recommended for self-study as well.

Nicolae Popovici

Arthur Benjamin, Gary Chartrand and Ping Zhang; The Fascinating World of Graph Theory, Princeton University Press, Princeton NJ, 2015, xii+322 p., ISBN 978-0-691-16381-9/hbk; 978-1-4008-5200-0/ebook).

The book is designed to introduce the field of Graph Theory to a broad audience and to also serve as an introductory textbook. Although the content is traditional for an undergraduate course, the way of presentation is not: the authors manage to motivate all topics with interesting applications, historical problems and discussion of concepts from an intuitive point of view.

After a funny prologue, chapter one deals exclusively with games, puzzles and problems that may be modeled using graphs. The basic notions of graphs and multigraphs are introduced and the way they capture the situations from reality are explained.

Graph classification is the topic of the second chapter. The basic idea of isomorphism is introduced and the reconstruction problem is the first unsolved problem discussed.

Chapter three introduces basic notions revolving around connectivity and distance. Both vertex and edge cuts are discussed along with several interesting applications.

Chapter four introduces trees and their basic properties. Among important topics we mention Cayley's formula for labeled trees and minimal spanning trees.

Graph traversal is treated in chapters five and six. There is a nice discussion of both Euler tours and the Chinese Postman Problem, both of which are edge traversals, and Hamilton cycles, which is edge traversal. Chapter six concludes with a discussion of the Traveling Salesman Problem.

Chapters seven and eight deal with graph decompositions, that is, partitions of the edge set of a graph. The latter concludes with Instant Insanity puzzle.

The ninth chapter treats orientations of graphs with an emphasis on tournaments and concludes with a nice application of tournaments for voting schemes.

Drawing graphs is the subject of Chapter ten. This is a well-presented standard material on topological graph theory.

The last two chapters deal with colouring: vertex colouring (Chapter 11) and edge colouring (Chapter 12).

The good integration of biographical sketches, human aspects and mathematics serves a good pedagogy. From the total of more than 300 pages, 50 is of gradual and well-chosen exercises A remarkable feature of this book is the constant effort to reveal the beauty of Mathematics in general and Graph Theory in particular.

Drawbacks of this works are little emphasizes on algorithms, the lack of mention to NP-Completeness, and lack of proofs for some important theorems.

Intended audience: undergraduates in Mathematics and Computer Science, professors which would like to use this as a textbook, anyone interested in discrete mathematics - its results, history and evolution.

Radu Trîmbiţaş

The Best Writing on Mathematics 2015, Edited by Mircea Pitici, Princeton University Press 2016, xxvi+376 pp., ISBN: 978-0-691-16965-1.

This is the sixth volume in a series edited by M. Pitici and published with PUP (2010, 2011, 2012, 2013, 2014). As the other volumes it contains a collection of 27 essays (a greater number than in the previous volumes), first published in 2014, dealing with various topics of mathematics and its applications. In a consistent Introduction (12 pages) the author explains the reasons for writing these volumes and the need for the popularization of mathematics: "That is why each volume should be seen in conjunction with the others, part of a serialized enterprise meant to facilitate the access to and exchange of ideas concerning diverse aspects of the mathematical experience.". This introductory part contains a brief survey on the writings on mathematics - both printed and online sources.

The volume contains more contributions, in comparison with previous ones, dealing with mathematical games and puzzles – *How puzzles made us human* (P. Mutalik), *Let the game continue* (C. Mulcahyand and D. Richards), *Challenging magic squares for magicians* (A. T. Benjamin and E. J. Brown), *Candy Crush's puzzling mathematics* (T. Walsh), *A prehistory of Nim* (L. Rougetet).

Some papers discuss philosophical and foundation aspects of mathematics – *Gödel, Gentzen, Goodstein: The magic sound of a G-string* (J. von Plato), *A guide for the perplexed: What mathematicians need to know to understand philosophers of mathematics* (M. Balaguer), *Writing about Math for the perplexed and the traumatized* (S. Strogatz).

In the paper *Synthetic biology, real mathematics* (by D. Mackenzie) applications to biology are discussed.

The future of high school mathematics and an analyze of the gap between Chinese and US students is presented in two papers. The relations between mathematics and art are discussed in two papers, one on Albrecht Dürer's painting and the other one on the quaternion group as a group of symmetry, while the beauty in mathematics is discussed in a paper by C. Cellucci.

Other contributions are dealing with geometry, the pigeonhole principle, chaos and billiard, big data manipulations, the Ontario lottery retailer scandal.

Dealing with topics of general interest – as history and philosophy, teaching, the occurrence of mathematics in everyday life, etc, – presented in an attractive and accessible manner, the books appeals to a large audience, including mathematicians of all levels of instruction, but also to anyone interested in the development of science and its applications.

Horia F. Pop