Stud. Univ. Babeş-Bolyai Math. 67(2022), No. 4, 739–747
DOI: 10.24193/subbmath.2022.4.06

Coefficient estimates for a subclass of analytic functions by Srivastava-Attiya operator

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Abstract. In this paper, we investigate bounds of the coefficients for subclass of analytic and bi-univalent functions. The results presented in this paper would generalize and improve some recent works and other authors.

Mathematics Subject Classification (2010): 30C45, 30C50.

Keywords: Analytic functions, bi-univalent functions, coefficient estimates, Srivastava-Attiya operator, subordination.

1. Introduction

Let $\mathcal{A}$ be a class of functions of the form

$$f(z) = z + \sum_{n=2}^{\infty} a_n z^n, \tag{1.1}$$

which are analytic in the open unit disk $\mathbb{U} = \{z \in \mathbb{C} : |z| < 1\}$. Further, let $\mathcal{S}$ denote the class of functions $f \in \mathcal{A}$ which are univalent in $\mathbb{U}$.

For $f(z)$ defined by (1.1) and $h(z)$ defined by

$$h(z) = z + \sum_{n=2}^{\infty} b_n z^n,$$

the Hadamard product $(f * h)(z)$ of the functions $f(z)$ and $h(z)$ defined by

$$(f * h)(z) = z + \sum_{n=2}^{\infty} a_n b_n z^n,$$

In 2007, Srivastava and Attiya [21] (see also Răducanu and Srivastava [18] and Prajapat and Goyal [17]) for the class $\mathcal{A}$ introduced and investigated linear operator

Received 19 December 2019; Accepted 08 January 2020.

$\mathcal{J}_\mu^b : \mathcal{A} \to \mathcal{A}$ that defined in terms of the Hadamard product by

$$\mathcal{J}_\mu^b f(z) = z + \sum_{k=2}^{\infty} \Theta_k a_k z^k,$$

where

$$\Theta_k = \left| \left(\frac{1+b}{k+b} \right)^{\mu} \right|,$$

and (throughout this paper unless otherwise mentioned) the parameters μ, b are considered as $\mu \in \mathbb{C}$ and $b \in \mathbb{C} \setminus \{0, -1, -2, \cdots\}$, (see for more details [20]).

Remark 1.1. (1) For $\mu = 1$ and $b = \upsilon$ $(\upsilon > -1)$, we get generalized Libera-Bernardi integral operator [19];
(2) For $\mu = \sigma$ $(\sigma > 0)$ and $b = 1$, we get Jung-Kim-Srivastava integral operator [12].

For each $f \in \mathcal{S}$, the Koebe one-quarter theorem [9] ensures that the image of $\mathbb{U}$ under f contains a disk of radius $\frac{1}{4}$. Hence every function $f \in \mathcal{S}$ has an inverse f^{-1}, which is defined by

$$f^{-1}(f(z)) = z \qquad (z \in \mathbb{U}),$$

and

$$f(f^{-1}(w)) = w \qquad \left(|w| < r_0(f); r_0(f) \geq \frac{1}{4} \right),$$

where

$$g(w) = f^{-1}(w) = w - a_2 w^2 + (2a_2^2 - a_3) w^3 + \cdots . \tag{1.2}$$

A function $f \in \mathcal{A}$ is said to be bi-univalent in $\mathbb{U}$ if both f and f^{-1} are univalent in $\mathbb{U}$. Let Σ denote the class of bi-univalent functions in $\mathbb{U}$ given by (1.1).

Recently many researchers have introduced and investigated several interesting subclasses of the bi-univalent function class Σ and they have found non-sharp estimates on the first two Taylor-Maclaurin coefficients $|a_2|$ and $|a_3|$ and other problems, see for example, [3, 2, 4, 5, 6, 7, 8, 10, 11, 13, 14, 15, 22, 23, 24].

For two functions f and g that are analytic in $\mathbb{U}$, we say that the function f is *subordinate* to g and write $f(z) \prec g(z)$, if there exists a Schwarz function ω, that is analytic in $\mathbb{U}$ with $\omega(0) = 0$ and $|\omega(z)| < 1$ such that $f(z) = g(\omega(z))$ for all $z \in \mathbb{U}$.

In particular, if the function g is univalent in $\mathbb{U}$, then $f(z) \prec g(z)$ if and only if $f(0) = g(0)$ and $f(\mathbb{U}) \subseteq g(\mathbb{U})$.

In this work, we obtain estimates of coefficients for a subclass of bi-univalent functions considered by Selvaraj et al. [20]. The results presented in this paper would generalize and improve some recent works and other authors.

2. The subclass $\mathcal{S}_{\Sigma,t}^{\mu,b}(\gamma,\lambda,\phi)$

Throughout this paper, we assume that ϕ is an analytic function with positive real part in the unit disk $\mathbb{U}$, satisfying $\phi(0)=1$, $\phi'(0)>0$ and symmetric with respect to the real axis. Such a function has series expansion of the form

$$\phi(z)=1+B_1z+B_2z^2+B_3z^3+\cdots \quad (B_1>0), \tag{2.1}$$

Let that $u(z)$ and $v(z)$ are Schwarz function in $\mathbb{U}$ with

$$u(0)=v(0)=0,\ |u(z)|<1,\ |v(z)|<1$$

and suppose that

$$u(z)=\sum_{n=1}^{\infty}p_nz^n \quad \text{and} \quad v(z)=\sum_{n=1}^{\infty}q_nz^n \quad (z\in\mathbb{U}). \tag{2.2}$$

Then [16, p. 172]

$$|p_1|\le 1, \quad |p_2|\le 1-|p_1|^2, \quad |q_1|\le 1, \quad |q_2|\le 1-|q_1|^2. \tag{2.3}$$

By (2.1), we get

$$\phi(u(z))=1+B_1p_1z+(B_1p_2+B_2p_1^2)z^2+\cdots \quad (z\in\mathbb{U}) \tag{2.4}$$

and

$$\phi(v(w))=1+B_1q_1w+(B_1q_2+B_2q_1^2)w^2+\cdots \quad (w\in\mathbb{U}). \tag{2.5}$$

In 2014, Selvaraj et al. [20] introduced subclass of Σ and obtained estimates on the coefficients $|a_2|$ and $|a_3|$ for functions in this subclass as follows:

Definition 2.1. [20] A function $f\in\Sigma$ given by (1.1) is said to be in the class $\mathcal{S}_{\Sigma,t}^{\mu,b}(\gamma,\lambda,\phi)$ if the following conditions are satisfied:

$$1+\frac{1}{\gamma}\left(\frac{[(1-t)z]^{1-\lambda}(\mathcal{J}_\mu^bf(z))'}{[\mathcal{J}_\mu^bf(z)-\mathcal{J}_\mu^bf(tz)]^{1-\lambda}}-1\right)\prec\phi(z),$$

and

$$1+\frac{1}{\gamma}\left(\frac{[(1-t)w]^{1-\lambda}(\mathcal{J}_\mu^bg(w))'}{[\mathcal{J}_\mu^bg(w)-\mathcal{J}_\mu^bg(tw)]^{1-\lambda}}-1\right)\prec\phi(w),$$

where $|t|\le 1$ $(t\neq 1)$; $\gamma\in\mathbb{C}\backslash\{0\}$; $\lambda\ge 0$; $z,w\in\mathbb{U}$ and g is given by (1.2).

Theorem 2.2. [20] *Let the function $f(z)$ given by (1.1) be in the class $\mathcal{S}_{\Sigma,t}^{\mu,b}(\gamma,\lambda,\phi)$. Then*

$$|a_2|\le\frac{|\gamma|B_1\sqrt{2B_1}}{\sqrt{|\gamma B_1^2\Lambda(\lambda,t)\Xi(\lambda,t)-2(B_2-B_1)[\Lambda(\lambda,t)+2]^2\Theta_2^2+2\gamma B_1^2\Upsilon(\lambda,t)\Theta_3|}} \tag{2.6}$$

and

$$|a_3|\le\frac{B_1|\gamma|}{\Upsilon(\lambda,t)\Theta_3}+\left(\frac{B_1|\tau|}{[\Lambda(\lambda,t)+2]\Theta_2}\right)^2, \tag{2.7}$$

where

$$\Lambda(\lambda,t)=(\lambda-1)(1+t),\ \Upsilon(\lambda,t)=[(\lambda-1)(1+t+t^2)+3]$$

and

$$\Xi(\lambda,t)=[(\lambda-2)(1+t)+4].$$

3. Coefficient estimates

In the section, we get that the following theorem which is an refinement of inequalities (2.6) and (2.7).

Theorem 3.1. *Let the function* $f(z)$ *given by* (1.1) *be in the class* $\mathcal{S}^{\mu,b}_{\Sigma,t}(\gamma,\lambda,\phi)$, $|t|\le 1$ $(t\neq 1)$, $\gamma\in\mathbb{C}\backslash\{0\}$ *and* $\lambda\ge 0$. *Then*

$$|a_2|\le\frac{|\gamma|B_1\sqrt{2B_1}}{\sqrt{2B_1[\Lambda(\lambda,t)+2]^2\Theta_2^2+|\gamma B_1^2\Lambda(\lambda,t)\Xi(\lambda,t)-2B_2[\Lambda(\lambda,t)+2]^2\Theta_2^2+2\gamma B_1^2\Upsilon(\lambda,t)\Theta_3|}}$$

and

$$|a_3|\le\begin{cases}\dfrac{|\tau|B_1}{\Upsilon(\lambda,t)\Theta_3} & B_1\le\dfrac{[(\lambda-1)(1+t)+2]^2\Theta_2^2}{|\gamma|\Theta_3[(\lambda-1)(1+t+t^2)+3]}\\[2ex] \dfrac{\Phi(\Theta_1,\Theta_2,\lambda,t)}{\Psi(\Theta_1,\Theta_2,\lambda,t)\Upsilon(\lambda,t)\Theta_3} & B_1>\dfrac{[(\lambda-1)(1+t)+2]^2\Theta_2^2}{|\gamma|\Theta_3[(\lambda-1)(1+t+t^2)+3]}\end{cases}.$$

where

$$\begin{aligned}\Phi(\Theta_1,\Theta_2,\lambda,t)=&|\tau|B_1\left|\gamma B_1^2\Lambda(\lambda,t)\Xi(\lambda,t)-2B_2[\Lambda(\lambda,t)+2]^2\Theta_2^2+2\gamma B_1^2\Upsilon(\lambda,t)\Theta_3\right|\\&+2|\gamma|^2\Theta_3\Upsilon(\lambda,t)B_1^3,\end{aligned}$$

and

$$\begin{aligned}\Psi(\Theta_1,\Theta_2,\lambda,t)=&2B_1[\Lambda(\lambda,t)+2]^2\Theta_2^2\\&+|\gamma B_1^2\Lambda(\lambda,t)\Xi(\lambda,t)-2B_2[\Lambda(\lambda,t)+2]^2\Theta_2^2+2\gamma B_1^2\Upsilon(\lambda,t)\Theta_3|.\end{aligned}$$

Proof. Let $f\in\mathcal{S}^{\mu,b}_{\Sigma,t}(\gamma,\lambda,\phi)$ and $g=f^{-1}$. Then there are analytic functions $u,v:\mathbb{U}\to\mathbb{U}$, with $u(0)=v(0)=0$, given by (2.2) such that

$$1+\frac{1}{\gamma}\left(\frac{[(1-t)z]^{1-\lambda}(\mathcal{J}^b_\mu f(z))'}{[\mathcal{J}^b_\mu f(z)-\mathcal{J}^b_\mu f(tz)]^{1-\lambda}}-1\right)=\phi(u(z)),\tag{3.1}$$

and

$$1+\frac{1}{\gamma}\left(\frac{[(1-t)w]^{1-\lambda}(\mathcal{J}^b_\mu g(w))'}{[\mathcal{J}^b_\mu g(w)-\mathcal{J}^b_\mu g(tw)]^{1-\lambda}}-1\right)=\phi(v(w)).\tag{3.2}$$

From (2.4), (2.5), (3.1) and (3.2), we obtain

$$[(\lambda-1)(1+t)+2]\Theta_2a_2=\gamma B_1p_1,\tag{3.3}$$

$$\begin{aligned}&[(\lambda-1)(1+t+t^2)+3]\Theta_3a_3+\frac{1}{2}(\lambda-1)(1+t)[(\lambda-2)(1+t)+4]\Theta_2^2a_2^2\\=&\gamma[B_1p_2+B_2p_1^2],\end{aligned}\tag{3.4}$$

$$-[(\lambda-1)(1+t)+2]\Theta_2a_2=\gamma B_1q_1,\tag{3.5}$$

and

$$[(\lambda-1)(1+t+t^2)+3]\Theta_3(2a_2^2-a_3) + \frac{1}{2}(\lambda-1)(1+t)[(\lambda-2)(1+t)+4]\Theta_2^2a_2^2 = \gamma[B_1q_2+B_2q_1^2]. \quad (3.6)$$

From (3.3) and (3.5), we get

$$p_1 = -q_1. \quad (3.7)$$

Adding (3.4) and (3.6), and using (3.7), we have

$$\big((\lambda-1)(1+t)[(\lambda-2)(1+t)+4]\Theta_2^2 + 2\Theta_3[(\lambda-1)(1+t+t^2)+3]\big)\, a_2^2 - 2\gamma B_2p_1^2 = \gamma B_1(p_2+q_2). \quad (3.8)$$

From (3.3), we have

$$\big(\gamma B_1^2\{(\lambda-1)(1+t)[(\lambda-2)(1+t)+4]\Theta_2^2 + 2\Theta_3[(\lambda-1)(1+t+t^2)+3]\} -2B_2[(\lambda-1)(1+t)+2]^2\Theta_2^2)a_2^2 = \gamma^2B_1^3(p_2+q_2).$$

By (2.3) and (3.3), we get

$$\begin{aligned}&|\big(\gamma B_1^2\{(\lambda-1)(1+t)[(\lambda-2)(1+t)+4]\Theta_2^2 + 2\Theta_3[(\lambda-1)(1+t+t^2)+3]\} \\ &- 2B_2[(\lambda-1)(1+t)+2]^2\Theta_2^2)a_2^2| \le |\tau|^2B_1^3(|p_2|+|q_2|) \\ &\le 2|\gamma|^2B_1^3(1-|p_1|^2) \\ &= 2|\gamma|^2B_1^3 - 2B_1[(\lambda-1)(1+t)+2]^2\Theta_2^2|a_2|^2.\end{aligned}$$

Therefore,

$$|a_2| \le \quad (3.9)$$

$$\le \frac{|\gamma|B_1\sqrt{2B_1}}{\sqrt{2B_1[\Lambda(\lambda,t)+2]^2\Theta_2^2 + |\gamma B_1^2\Lambda(\lambda,t)\Xi(\lambda,t) - 2B_2[\Lambda(\lambda,t)+2]^2\Theta_2^2 + 2\gamma B_1^2\Upsilon(\lambda,t)\Theta_3|}},$$

where

$$\Lambda(\lambda,t) = (\lambda-1)(1+t),\ \ \Upsilon(\lambda,t) = [(\lambda-1)(1+t+t^2)+3]$$

and

$$\Xi(\lambda,t) = [(\lambda-2)(1+t)+4].$$

Next, in order to find the bound on the coefficient $|a_3|$, by subtracting (3.6) from (3.4), and using (3.7), we get

$$2[(\lambda-1)(1+t+t^2)+3]\Theta_3a_3 = 2\Theta_3[(\lambda-1)(1+t+t^2)+3]a_2^2 + \tau B_1(p_2-q_2). \quad (3.10)$$

Using (2.3) and (3.7), we have

$$\begin{aligned}&2[(\lambda-1)(1+t+t^2)+3]\Theta_3|a_3| \\ &\le |\gamma|B_1(|p_2|+|q_2|) + 2\Theta_3[(\lambda-1)(1+t+t^2)+3]|a_2|^2 \\ &\le 2|\gamma|B_1(1-|p_1|^2) + 2\Theta_3[(\lambda-1)(1+t+t^2)+3]|a_2|^2.\end{aligned}$$

From (3.3), we get

$$\begin{aligned}&|\gamma|B_1[(\lambda-1)(1+t+t^2)+3]\Theta_3|a_3|\\ \leq&\left[|\gamma|\Theta_3[(\lambda-1)(1+t+t^2)+3]B_1-[(\lambda-1)(1+t)+2]^2\Theta_2^2\right]|a_2|^2+|\gamma|^2B_1^2.\end{aligned}$$

From (3.9), for $\left[|\gamma|\Theta_3[(\lambda-1)(1+t+t^2)+3]B_1-[(\lambda-1)(1+t)+2]^2\Theta_2^2\right]>0$ we have

$$\begin{aligned}&|\gamma|B_1[(\lambda-1)(1+t+t^2)+3]\Theta_3|a_3|\\ &\leq\left[|\gamma|\Theta_3[(\lambda-1)(1+t+t^2)+3]B_1-[(\lambda-1)(1+t)+2]^2\Theta_2^2\right]\\ &\times\frac{2|\gamma|^2B_1^3}{2B_1[\Lambda(\lambda,t)+2]^2\Theta_2^2+|\gamma B_1^2\Lambda(\lambda,t)\Xi(\lambda,t)-2B_2[\Lambda(\lambda,t)+2]^2\Theta_2^2+2\gamma B_1^2\Upsilon(\lambda,t)\Theta_3|}\\ &+|\gamma|^2B_1^2.\end{aligned}$$

Therefore,

$$\begin{aligned}|a_3|\leq&\left[|\gamma|\Theta_3[(\lambda-1)(1+t+t^2)+3]B_1-[(\lambda-1)(1+t)+2]^2\Theta_2^2\right]\\ &\times\frac{2|\gamma|B_1^2}{\Psi(\Theta_1,\Theta_2,\lambda,t)[(\lambda-1)(1+t+t^2)+3]\Theta_3}+\frac{|\gamma|B_1}{[(\lambda-1)(1+t+t^2)+3]\Theta_3},\end{aligned}$$

where

$$\begin{aligned}\Psi(\Theta_1,\Theta_2,\lambda,t)=&2B_1[\Lambda(\lambda,t)+2]^2\Theta_2^2\\ &+|\gamma B_1^2\Lambda(\lambda,t)\Xi(\lambda,t)-2B_2[\Lambda(\lambda,t)+2]^2\Theta_2^2+2\gamma B_1^2\Upsilon(\lambda,t)\Theta_3|.\end{aligned}$$

Consequently,

$$|a_3|\leq\begin{cases}\dfrac{|\gamma|B_1}{[(\lambda-1)(1+t+t^2)+3]\Theta_3} & B_1\leq\dfrac{[(\lambda-1)(1+t)+2]^2\Theta_2^2}{|\gamma|\Theta_3[(\lambda-1)(1+t+t^2)+3]}\\[2ex] \dfrac{\Phi(\Theta_1,\Theta_2,\lambda,t)}{\Psi(\Theta_1,\Theta_2,\lambda,t)[(\lambda-1)(1+t+t^2)+3]\Theta_3} & B_1>\dfrac{[(\lambda-1)(1+t)+2]^2\Theta_2^2}{|\gamma|\Theta_3[(\lambda-1)(1+t+t^2)+3]},\end{cases}$$

where

$$\begin{aligned}\Phi(\Theta_1,\Theta_2,\lambda,t)=&|\tau|B_1\left|\gamma B_1^2\Lambda(\lambda,t)\Xi(\lambda,t)-2B_2[\Lambda(\lambda,t)+2]^2\Theta_2^2+2\gamma B_1^2\Upsilon(\lambda,t)\Theta_3\right|\\ &+2|\gamma|^2\Theta_3[(\lambda-1)(1+t+t^2)+3]B_1^3.\end{aligned}$$

This completes the proof. □

Remark 3.2. Theorem 3.1 is an improvement of the estimates obtained by Selvaraj et al. [20] in Theorem 2.2. For the coefficient $|a_2|$, it is clear that

$$\begin{aligned}&\frac{|\gamma|B_1\sqrt{2B_1}}{\sqrt{2B_1[\Lambda(\lambda,t)+2]^2\Theta_2^2+|\gamma B_1^2\Lambda(\lambda,t)\Xi(\lambda,t)-2B_2[\Lambda(\lambda,t)+2]^2\Theta_2^2+2\gamma B_1^2\Upsilon(\lambda,t)\Theta_3|}}\\ \leq&\frac{|\gamma|B_1\sqrt{2B_1}}{\sqrt{|\gamma B_1^2\Lambda(\lambda,t)\Xi(\lambda,t)-2(B_2-B_1)[\Lambda(\lambda,t)+2]^2\Theta_2^2+2\gamma B_1^2\Upsilon(\lambda,t)\Theta_3|}}.\end{aligned}$$

On the other hand, for the coefficient $|a_3|$, we make the following cases:

(i) For $B_1 \leq \dfrac{[(\lambda-1)(1+t)+2]^2\Theta_2^2}{|\gamma|\Theta_3[(\lambda-1)(1+t+t^2)+3]}$, it is clear that

$$\frac{|\gamma|B_1}{\Upsilon(\lambda,t)\Theta_3} \leq \frac{B_1|\gamma|}{\Upsilon(\lambda,t)\Theta_3} + \left(\frac{B_1|\tau|}{[\Lambda(\lambda,t)+2]\Theta_2}\right)^2.$$

(ii) For $B_1 > \dfrac{[(\lambda-1)(1+t)+2]^2\Theta_2^2}{|\gamma|\Theta_3[(\lambda-1)(1+t+t^2)+3]}$, it is clear that

$$\frac{\Phi(\Theta_1,\Theta_2,\lambda,t)}{\Psi(\Theta_1,\Theta_2,\lambda,t)\Upsilon(\lambda,t)\Theta_3} \leq \frac{B_1|\gamma|}{\Upsilon(\lambda,t)\Theta_3} + \left(\frac{B_1|\tau|}{[\Lambda(\lambda,t)+2]\Theta_2}\right)^2.$$

Remark 3.3. If we set $\lambda = 0$ in Theorem 3.1, then we get an improvement of the estimates obtained by Selvaraj et al. [20, Corollary 2.1].

Remark 3.4. If we set $\lambda = 1$ in Theorem 3.1, then we get an improvement of the estimates obtained by Selvaraj et al. [20, Corollary 2.2].

Remark 3.5. If $\mathcal{J}_\mu^b f(z)$ be the identity map and $\lambda = 0$ in Theorem 3.1, then we get an improvement of the estimates obtained by Selvaraj et al. [20, Corollary 2.3].

Remark 3.6. If $\mathcal{J}_\mu^b f(z)$ be the identity map and $\lambda = 1$ in Theorem 3.1, then we get an improvement of the estimates obtained by Selvaraj et al. [20, Corollary 2.4].

Remark 3.7. If $\mathcal{J}_\mu^b f(z)$ be the identity map and $\gamma = 1,\ t = 0$ in Theorem 3.1, then we get an improvement of the estimates obtained by Deniz [8, Theorem 2.8].

Remark 3.8. If $\mathcal{J}_\mu^b f(z)$ be the identity map and $\gamma = 1,\ \lambda = 1$ in Theorem 3.1 is an improvement of the estimates obtained by Ali et al. in [3, Theorem 2.1].

Remark 3.9. If we take

$$\phi(z) = \frac{1+Az}{1+Bz} = 1 + (A-B)z + (B-A)Bz^2 + \cdots (-1 \leq B < A \leq 1, z \in \mathbb{U})$$

and

$$\varphi(z) = \left(\frac{1+z}{1-z}\right)^\alpha = 1 + 2\alpha z + 2\alpha^2 z^2 + \cdots \quad (0 < \alpha \leq 1,\ z \in \mathbb{U}),$$

which gives $B_1 = A - B,\ B_2 = (B-A)B$ and $B_1 = 2\alpha,\ B_2 = 2\alpha^2$, in Theorem 3.1, then we can deduce interesting results analogous, respectively. Also, for suitable choices the parameter μ and b in Theorems 3.1 and some Remarks above we have an improvement of results involving Libera-Bernardi integral operator [19] and Jung-Kim-Srivastava integral operator [12].

Acknowledgments. The authors thank from the Najafabad Branch, Islamic Azad University for their financial support.

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