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Geometric characteristics and properties of a two-parametric family of Lie groups with almost contact B-metric structure of the smallest dimension

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Abstract. Almost contact B-metric manifolds of the lowest dimension 3 are constructed by a two-parametric family of Lie groups. Our purpose is to determine the class of considered manifolds in a classification of almost contact B-metric manifolds and their most important geometric characteristics and properties.

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1. Introduction

The study of the differential geometry of the almost contact B-metric manifolds has initiated in [5]. The geometry of these manifolds is a natural extension of the geometry of the almost complex manifolds with Norden metric [3, 6] in the case of odd dimension. Almost contact B-metric manifolds are investigated and studied for example in [5, 11, 12, 14, 15, 17, 18, 20].

Here, an object of special interest are the Lie groups considered as three-dimensional almost contact B-metric manifolds. For example of such investigation see [19].

The aim of the present paper is to make a study of the most important geometric characteristics and properties of a family of Lie groups with almost contact B-metric structure of the lowest dimension 3, belonging to the main vertical classes. These classes are $\mathcal{F}_4$ and $\mathcal{F}_5$, where the fundamental tensor F is expressed explicitly by the metric g, the structure (φ, ξ, η) and the vertical components of the Lee forms θ and θ^*, i.e. in this case the Lee forms are proportional to η at any point. These classes contain some significant examples as the time-like sphere of g and the light cone of the associated metric of g in the complex Riemannian space, considered in [5], as well as the Sasakian-like manifolds studied in [7].

The paper is organized as follows. In Sec. 2, we give some necessary facts about almost contact B-metric manifolds. In Sec. 3, we construct and study a family of Lie groups as three-dimensional manifolds of the considered type.

2. Almost contact manifolds with B-metric

Let (M,φ,ξ,η,g) be a $(2n+1)$-dimensional *almost contact B-metric manifold*, i.e. (φ,ξ,η) is a triplet of a tensor (1,1)-field φ, a vector field ξ and its dual 1-form η called an almost contact structure and the following identities holds:

$$\varphi\xi = 0, \quad \varphi^2 = -\mathrm{Id} + \eta\otimes\xi, \quad \eta\circ\varphi = 0, \quad \eta(\xi) = 1,$$

where Id is the identity. The B-metric g is pseudo-Riemannian and satisfies

$$g(\varphi x, \varphi y) = -g(x,y) + \eta(x)\eta(y)$$

for arbitrary tangent vectors $x, y \in T_pM$ at an arbitrary point $p \in M$ [5].

Further, x, y, z, w will stand for arbitrary vector fields on M or vectors in the tangent space at an arbitrary point in M.

Let us note that the restriction of a B-metric on the contact distribution $H = \ker(\eta)$ coincides with the corresponding Norden metric with respect to the almost complex structure and the restriction of φ on H acts as an anti-isometry on the metric on H which is the restriction of g on H.

The associated metric $\tilde{g}$ of g on M is given by $\tilde{g}(x,y) = g(x,\varphi y) + \eta(x)\eta(y)$. It is a B-metric, too. Hence, $(M,\varphi,\xi,\eta,\tilde{g})$ is also an almost contact B-metric manifold. Both metrics g and $\tilde{g}$ are indefinite of signature $(n+1,n)$.

The structure group of (M,φ,ξ,η,g) is $\mathcal{G}\times\mathcal{I}$, where $\mathcal{I}$ is the identity on span(ξ) and $\mathcal{G} = \mathcal{GL}(n;\mathbb{C})\cap\mathcal{O}(n,n)$.

The (0,3)-tensor F on M is defined by $F(x,y,z) = g\big((\nabla_x\varphi)\,y, z\big)$, where ∇ is the Levi-Civita connection of g. The tensor F has the following properties:

$$F(x,y,z) = F(x,z,y) = F(x,\varphi y,\varphi z) + \eta(y)F(x,\xi,z) + \eta(z)F(x,y,\xi).$$

A classification of the almost contact B-metric manifolds is introduced in [5], where eleven basic classes $\mathcal{F}_i$ $(i = 1,2,\ldots,11)$ are characterized with respect to the properties of F. The special class $\mathcal{F}_0$ is defined by the condition $F(x,y,z) = 0$ and is contained in each of the other classes. Hence, $\mathcal{F}_0$ is the class of almost contact B-metric manifolds with ∇-parallel structures, i.e. $\nabla\varphi = \nabla\xi = \nabla\eta = \nabla g = \nabla\tilde{g} = 0$.

Let g_{ij}, $i,j \in \{1,2,\ldots,2n+1\}$, be the components of the matrix of g with respect to a basis $\{e_i\}_{i=1}^{2n+1} = \{e_1,e_2,\ldots,e_{2n+1}\}$ of T_pM at an arbitrary point $p\in M$, and g^{ij} – the components of the inverse matrix of (g_{ij}). The Lee forms associated with F are defined as follows:

$$\theta(z) = g^{ij}F(e_i,e_j,z), \quad \theta^*(z) = g^{ij}F(e_i,\varphi e_j,z), \quad \omega(z) = F(\xi,\xi,z).$$

In [12], the *square norm of* $\nabla\varphi$ is introduced by:

$$\|\nabla\varphi\|^2 = g^{ij}g^{ks}g\big((\nabla_{e_i}\varphi)\,e_k, \big(\nabla_{e_j}\varphi\big)\,e_s\big). \tag{2.1}$$

If (M,φ,ξ,η,g) is an $\mathcal{F}_0$-manifold then the square norm of $\nabla\varphi$ is zero, but the inverse implication is not always true. An almost contact B-metric manifold satisfying the condition $\|\nabla\varphi\|^2=0$ is called an *isotropic-$\mathcal{F}_0$-manifold.* The square norms of $\nabla\eta$ and $\nabla\xi$ are defined in [13] by:

$$\|\nabla\eta\|^2=g^{ij}g^{ks}\left(\nabla_{e_i}\eta\right)e_k\left(\nabla_{e_j}\eta\right)e_s,\quad \|\nabla\xi\|^2=g^{ij}g\left(\nabla_{e_i}\xi,\nabla_{e_j}\xi\right). \tag{2.2}$$

Let R be the curvature tensor of type (1,3) of Levi-Civita connection ∇, i.e. $R(x,y)z=\nabla_x\nabla_y z-\nabla_y\nabla_x z-\nabla_{[x,y]}z$. The corresponding tensor of R of type (0,4) is defined by $R(x,y,z,w)=g(R(x,y)z,w)$.

The Ricci tensor ρ and the scalar curvature τ for R as well as their associated quantities are defined by the following traces $\rho(x,y)=g^{ij}R(e_i,x,y,e_j)$, $\tau=g^{ij}\rho(e_i,e_j)$, $\rho^*(x,y)=g^{ij}R(e_i,x,y,\varphi e_j)$ and $\tau^*=g^{ij}\rho^*(e_i,e_j)$, respectively.

An almost contact B-metric manifold is called *Einstein* if the Ricci tensor is proportional to the metric tensor, i.e. $\rho=\lambda g$, $\lambda\in\mathbb{R}$.

Let α be a non-degenerate 2-plane (section) in T_pM. It is known from [20] that the special 2-planes with respect to the almost contact B-metric structure are: a *totally real section* if α is orthogonal to its φ-image $\varphi\alpha$ and ξ, a *φ-holomorphic section* if α coincides with $\varphi\alpha$ and a *ξ-section* if ξ lies on α.

The sectional curvature $k(\alpha;p)(R)$ of α with an arbitrary basis $\{x,y\}$ at p regarding R is defined by

$$k(\alpha;p)(R)=\frac{R(x,y,y,x)}{g(x,x)g(y,y)-g(x,y)^2}. \tag{2.3}$$

It is known from [12] that a linear connection D is called a *natural connection* on an arbitrary manifold (M,φ,ξ,η,g) if the almost contact structure (φ,ξ,η) and the B-metric g (consequently also $\tilde{g}$) are parallel with respect to D, i.e. $D\varphi=D\xi=D\eta=Dg=D\tilde{g}=0$. In [18], it is proved that a linear connection D is natural on (M,φ,ξ,η,g) if and only if $D\varphi=Dg=0$. A natural connection exists on any almost contact B-metric manifold and coincides with the Levi-Civita connection if and only if the manifold belongs to $\mathcal{F}_0$.

Let T be the torsion tensor of D, i.e. $T(x,y)=D_xy-D_yx-[x,y]$. The corresponding tensor of T of type (0,3) is denoted by the same letter and is defined by the condition $T(x,y,z)=g(T(x,y),z)$.

In [15], it is introduced a natural connection $\dot{D}$ on (M,φ,ξ,η,g) in all basic classes by

$$\dot{D}_xy=\nabla_xy+\tfrac{1}{2}\left\{(\nabla_x\varphi)\,\varphi y+(\nabla_x\eta)\,y\cdot\xi\right\}-\eta(y)\nabla_x\xi. \tag{2.4}$$

This connection is called a *φB-connection* in [16]. It is studied for the main classes $\mathcal{F}_1,\mathcal{F}_4,\mathcal{F}_5,\mathcal{F}_{11}$ in [15, 10, 11]. Let us note that the φB-connection is the odd-dimensional analogue of the B-connection on the almost complex manifold with Norden metric, studied for the class $\mathcal{W}_1$ in [4].

In [17], a natural connection $\ddot{D}$ is called a *φ-canonical connection* on $(M, \varphi, \xi, \eta, g)$ if its torsion tensor $\ddot{T}$ satisfies the following identity:

$$\begin{aligned}
&\ddot{T}(x,y,z) - \ddot{T}(x,z,y) - \ddot{T}(x,\varphi y,\varphi z) + \ddot{T}(x,\varphi z,\varphi y) \\
&= \eta(x)\left\{\ddot{T}(\xi,y,z) - \ddot{T}(\xi,z,y) - \ddot{T}(\xi,\varphi y,\varphi z) + \ddot{T}(\xi,\varphi z,\varphi y)\right\} \\
&+ \eta(y)\left\{\ddot{T}(x,\xi,z) - \ddot{T}(x,z,\xi) - \eta(x)\ddot{T}(z,\xi,\xi)\right\} \\
&- \eta(z)\left\{\ddot{T}(x,\xi,y) - \ddot{T}(x,y,\xi) - \eta(x)\ddot{T}(y,\xi,\xi)\right\}.
\end{aligned}$$

It is established that the φB-connection and the φ-canonical connection coincide if and only if $(M, \varphi, \xi, \eta, g)$ is in the class $\mathcal{F}_1 \oplus \mathcal{F}_2 \oplus \mathcal{F}_4 \oplus \mathcal{F}_5 \oplus \mathcal{F}_6 \oplus \mathcal{F}_8 \oplus \mathcal{F}_9 \oplus \mathcal{F}_{10} \oplus \mathcal{F}_{11}$.

In [8] it is determined the class of all three-dimensional almost contact B-metric manifolds. It is $\mathcal{F}_1 \oplus \mathcal{F}_4 \oplus \mathcal{F}_5 \oplus \mathcal{F}_8 \oplus \mathcal{F}_9 \oplus \mathcal{F}_{10} \oplus \mathcal{F}_{11}$.

3. A family of Lie groups as three-dimensional $(\mathcal{F}_4 \oplus \mathcal{F}_5)$-manifolds

In this section we study three-dimensional real connected Lie groups with almost contact B-metric structure. On a three-dimensional connected Lie group G we take a global basis of left-invariant vector fields $\{e_0, e_1, e_2\}$ on G.

We define an almost contact structure on G by

$$\begin{aligned}
&\varphi e_0 = o, \quad \varphi e_1 = e_2, \quad \varphi e_2 = -e_1, \quad \xi = e_0; \\
&\eta(e_0) = 1, \quad \eta(e_1) = \eta(e_2) = 0,
\end{aligned} \tag{3.1}$$

where o is the zero vector field and define a B-metric on G by

$$\begin{aligned}
&g(e_0, e_0) = g(e_1, e_1) = -g(e_2, e_2) = 1, \\
&g(e_0, e_1) = g(e_0, e_2) = g(e_1, e_2) = 0.
\end{aligned} \tag{3.2}$$

We consider the Lie algebra $\mathfrak{g}$ on G, determined by the following non-zero commutators:

$$[e_0, e_1] = -be_1 - ae_2, \quad [e_0, e_2] = ae_1 - be_2, \quad [e_1, e_2] = 0, \tag{3.3}$$

where $a, b \in \mathbb{R}$. We verify immediately that the Jacobi identity for $\mathfrak{g}$ is satisfied. Hence, G is a 2-parametric family of Lie groups with corresponding Lie algebra $\mathfrak{g}$.

Theorem 3.1. *Let $(G, \varphi, \xi, \eta, g)$ be a three-dimensional connected Lie group with almost contact B-metric structure determined by* (3.1), (3.2) *and* (3.3). *Then it belongs to the class $\mathcal{F}_4 \oplus \mathcal{F}_5$.*

Proof. The well-known Koszul equality for the Levi-Civita connection ∇ of g

$$2g\left(\nabla_{e_i} e_j, e_k\right) = g\left(\left[e_i, e_j\right], e_k\right) + g\left(\left[e_k, e_i\right], e_j\right) + g\left(\left[e_k, e_j\right], e_i\right) \tag{3.4}$$

implies the following form of the components $F_{ijk} = F(e_i, e_j, e_k)$ of F:

$$\begin{aligned}
2F_{ijk} = g\left(\left[e_i, \varphi e_j\right] - \varphi\left[e_i, e_j\right], e_k\right) &+ g\left(\varphi\left[e_k, e_i\right] - \left[\varphi e_k, e_i\right], e_j\right) \\
&+ g\left(\left[e_k, \varphi e_j\right] - \left[\varphi e_k, e_j\right], e_i\right).
\end{aligned} \tag{3.5}$$

Using (3.5) and (3.3) for the non-zero components F_{ijk}, we get:

$$\begin{aligned} F_{101} = F_{110} = -F_{202} = -F_{220} = a, \\ F_{102} = F_{120} = F_{201} = F_{210} = b. \end{aligned} \tag{3.6}$$

Immediately we establish that the components in (3.6) satisfy the condition $F = F^4 + F^5$ which means that the manifold belongs to $\mathcal{F}_4 \oplus \mathcal{F}_5$. Here, the components F^s of F in the basic classes $\mathcal{F}_s$ $(s = 4, 5)$ have the following form (see [8])

$$\begin{aligned} F_4(x, y, z) &= \tfrac{1}{2}\theta_0 \Big\{ x^1 \left(y^0 z^1 + y^1 z^0\right) - x^2 \left(y^0 z^2 + y^2 z^0\right) \Big\}, \\ &\tfrac{1}{2}\theta_0 = F_{101} = F_{110} = -F_{202} = -F_{220}; \\ F_5(x, y, z) &= \tfrac{1}{2}\theta_0^* \big\{ x^1 \left(y^0 z^2 + y^2 z^0\right) + x^2 \left(y^0 z^1 + y^1 z^0\right) \big\}, \\ &\tfrac{1}{2}\theta_0^* = F_{102} = F_{120} = F_{201} = F_{210}. \end{aligned} \tag{3.7}$$

where $\theta_0 = \theta(e_0)$ and $\theta_0^* = \theta^*(e_0)$ are determined by $\theta_0 = 2a$, $\theta_0^* = 2b$. Therefore, the induced three-dimensional manifold $(G, \varphi, \xi, \eta, g)$ belongs to the class $\mathcal{F}_4 \oplus \mathcal{F}_5$ from the mentioned classification. It is an $\mathcal{F}_0$-manifold if and only if $(a, b) = (0, 0)$ holds.

Obviously, $(G, \varphi, \xi, \eta, g)$ belongs to $\mathcal{F}_4$, $\mathcal{F}_5$ and $\mathcal{F}_0$ if and only if the parameters θ_0^* vanishes if the manifold belongs to $\mathcal{F}_4$, and θ_0 vanishes if it belong to $\mathcal{F}_5$, and $\theta_0 = \theta_0^*$ vanishes if it belong to $\mathcal{F}_0$, respectively.

According to the above, the commutators in (3.3) take the form

$$\begin{gathered} [e_0, e_1] = -\tfrac{1}{2}(\theta_0^* e_1 + \theta_0 e_2), \quad [e_0, e_2] = \tfrac{1}{2}(\theta_0 e_1 - \theta_0^* e_2), \\ [e_1, e_2] = 0, \end{gathered} \tag{3.8}$$

in terms of the basic components of the Lee forms θ and θ^*. □

According to Theorem 3.1 and the consideration in [9], we can remark that the Lie algebra determined as above belongs to the type $Bia(VII_h)$, $h > 0$ of the Bianchi classification (see [1, 2]).

Using (3.4) and (3.3), we obtain the components of ∇:

$$\begin{aligned} \nabla_{e_1} e_0 = be_1 + ae_2, \quad \nabla_{e_1} e_1 = -be_0, \quad \nabla_{e_1} e_2 = ae_0, \\ \nabla_{e_2} e_0 = -ae_1 + be_2, \quad \nabla_{e_2} e_1 = ae_0, \quad \nabla_{e_2} e_2 = be_0. \end{aligned} \tag{3.9}$$

We denote by $R_{ijkl} = R(e_i, e_j, e_k, e_l)$ the components of the curvature tensor R, $\rho_{jk} = \rho(e_j, e_k)$ of the Ricci tensor ρ, $\rho^*_{jk} = \rho^*(e_j, e_k)$ of the associated Ricci tensor ρ^* and $k_{ij} = k(e_i, e_j)$ of the sectional curvature for ∇ of the basic 2-plane α_{ij} with a basis $\{e_i, e_j\}$, where $i, j \in \{0, 1, 2\}$. On the considered manifold $(G, \varphi, \xi, \eta, g)$ the basic 2-planes α_{ij} of special type are: a φ-holomorphic section — α_{12} and ξ-sections — α_{01}, α_{02}. Further, by (2.3), (3.2), (3.3) and (3.9), we compute

$$\begin{gathered} -R_{0101} = R_{0202} = \tfrac{1}{2}\rho_{00} = k_{01} = k_{02} = \tfrac{1}{4}(\theta_0^2 - \theta_0^{*2}), \\ R_{0102} = R_{0201} = -\rho_{12} = -\tfrac{1}{2}\rho^*_{00} = -\tfrac{1}{2}\tau^* = -\tfrac{1}{2}\theta_0\theta_0^*, \\ R_{1212} = \rho^*_{12} = k_{12} = -\tfrac{1}{4}(\theta_0^2 + \theta_0^{*2}), \quad \rho_{11} = -\rho_{22} = -\tfrac{1}{2}\theta_0^{*2}, \\ \tau = \tfrac{1}{2}(\theta_0^2 - 3\theta_0^{*2}). \end{gathered} \tag{3.10}$$

The rest of the non-zero components of R, ρ and ρ^* are determined by (3.10) and the properties $R_{ijkl} = R_{klij}$, $R_{ijkl} = -R_{jikl} = -R_{ijlk}$, $\rho_{jk} = \rho_{kj}$ and $\rho^*_{jk} = \rho^*_{kj}$.

Taking into account (2.1), (2.2), (3.1), (3.2) and (3.9), we have

$$\|\nabla\varphi\|^2 = -2\,\|\nabla\eta\|^2 = -2\,\|\nabla\xi\|^2 = \theta_0^2 - \theta_0^{*2}. \tag{3.11}$$

Proposition 3.2. *The following characteristics are valid for (G,φ,ξ,η,g):*

1. *The φB-connection $\dot{D}$ (respectively, φ-canonical connection $\ddot{D}$) is zero in the basis $\{e_0, e_1, e_2\}$.*
2. *The manifold is an isotropic-$\mathcal{F}_0$-manifold if and only if the condition $\theta_0 = \pm\theta_0^*$ is valid.*
3. *The manifold is flat if and only if it belongs to $\mathcal{F}_0$.*
4. *The manifold is Ricci-flat (respectively, $*$-Ricci-flat) if and only if it is flat.*
5. *The manifold is scalar flat if and only if the condition $\theta_0 = \pm\sqrt{3}\,\theta_0^*$ holds.*
6. *The manifold is $*$-scalar flat if and only if it belongs to either $\mathcal{F}_4$ or $\mathcal{F}_5$.*

Proof. Using (2.4), (3.1) and (3.9), we get immediately the assertion (1). Equation (3.11) implies the assertion (2). The assertions (5), (3) and (6) hold, according to (3.10). On the three-dimensional almost contact B-metric manifold with the basis $\{e_0, e_1, e_2\}$, bearing in mind the definitions of the Ricci tensor ρ and the ρ^*, we have

$$\rho_{jk} = R_{0jk0} + R_{1jk1} - R_{2jk2} \qquad \rho^*_{jk} = R_{1kj2} + R_{2jk1}.$$

By virtue of the latter equalities, we get the assertion (4). □

According to (3.6) and (3.10) we establish the truthfulness of the following

Proposition 3.3. *The following properties are equivalent for the studied manifold (G,φ,ξ,η,g):*

1. *it belongs to $\mathcal{F}_4$;*
2. *it is η-Einstein;*
3. *the Lee form θ^* vanishes.*

Using again (3.6) and (3.10)we establish the truthfulness of the following

Proposition 3.4. *The following properties are equivalent for the studied manifold (G,φ,ξ,η,g):*

1. *it belongs to $\mathcal{F}_5$;*
2. *it is Einstein;*
3. *it is a hyperbolic space form with $k = -\frac{1}{4}\theta_0^{*2}$;*
4. *the Lee form θ vanishes.*

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