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Book reviews

Derek K. Thomas, Nikola Tuneski, and Allu Vasudevarao; Univalent Functions. A Primer, De Gruyter Studies in Mathematics 69, De Gruyter, Berlin 2018, xii + 252 p., ISBN 978-3-11-056009-1/hbk; 978-3-11-056096-1/ebook.

Although there has been a continuing interest in the theory of univalent functions since its inception in the celebrated paper of Bieberbach in 1918, only a few books have appeared during this period. The seminal books by Hayman, Pommerenke and Duren, the last published in 1983, deal with the important fundamental properties, contains much material of an advanced nature, and also gives some information concerning subclasses.

In recent years, interest in univalent functions appears to have increased, particularly in the study of subclasses, and probably as a result of the changing nature of academic publishing, many more papers have appeared. The books of Goodman are primarily concerned with subclasses, but also published in 1983, are now in many ways out of date, and an update of present knowledge is now timely.

This book is directed at those new to research in the theory of univalent functions, and thus omits some material of a deeper nature. It is also of interest to those interested in updating what is currently known about a selection of problems in the important subclasses of univalent functions. In the preface, the authors set out their aims, which are to update current information on the important subclasses of univalent functions, concentrating on what they consider to be the more important problems, whilst at the same time providing examples of the use of the central ideas and methods involved in proving theorems. The book ends with a set of 50 open problems, many of which are related to the topics considered in the book.

The book contains 17 chapters. The first three chapters contain the elementary theory of univalent functions, basic definitions, and properties of the important subclasses, and a chapter laying out some fundamental lemmas which are used later in the book.

Chapter 4 gives an in-depth survey of the important results concerning starlike and convex functions. As elsewhere in the book, complete proofs of the theorems presented are given in most instances.

The next two chapters are concerned with starlike and convex functions of order α, and strongly starlike and convex functions. Here again, a thorough account of the often complicated proofs of many of the results is given.

Chapters 7 and 8 deal with the so-called α-convex functions and gamma-starlike functions, where again an up-to-date account is given, including a detailed proof of the sharp coefficient inequality for α-convex functions.

Chapter 9 contains a very full study of most of the important problems for close-to-convex functions. Amongst items discussed is an up-to-date treatment of finding the sharp bounds for the modulus of the logarithmic coefficients, in particular the third logarithmic coefficient, which remains an outstanding and significant unsolved problem.

The next two chapters deal with Bazilević functions $\mathcal{B}(\alpha)$, Chapter 10 with the case the $\alpha \geq 0$, and Chapter 11 with the so-called $\mathcal{B}_1(\alpha)$ functions, where the associated starlike function in the definition of $\mathcal{B}(\alpha)$ is the identity function.

Chapter 12 considers the class $\mathcal{U}(\lambda)$, and contains some results concerning conditions for univalence, and recent sharp coefficient estimates.

The main aim of the next chapter is to present a proof of the Pólya-Schoenberg Conjecture concerning convolutions.

Meromorphic univalent functions are dealt with in Chapter 14, where a detailed discussion is given to the determination of the Clunie-constant. Other results for the subclasses of starlike, close-to-convex and Bazilević meromorphic functions are also included

Chapter 15 gives a brief introduction to Loewner Theory with some applications, including a proof of the Bieberbach conjecture when $n = 3$, and the solution to the Fekete-Szegő problem.

In the next chapter a selection of topics not contained in the previous chapters are briefly introduced, and some basic properties are presented.

The book ends with a list of 50 open problems, most of which are connected with the material in the book.

There is an extensive bibliography, and a detailed index.

The book is a significant addition to the study of univalent functions, and will be particularly useful to those with an interest in subclasses.

Teodor Bulboacă

Wojbor A. Woyczynski, Geometry and martingales in Banach spaces, CRC Press, Boca Raton, FL, 2019, ISBN 978-1-138-61637-0/hbk; 978-0-4298-6883-2/ebook, xiii+315 p.

The study of Banach space valued random variables is tightly connected with the geometric properties of the underlying space. In particular, martingale theory is essential in the study of Radon-Nikodým property, finite tree property and super-reflexivity, and of the local properties of Banach spaces. The UMD spaces (meaning Banach spaces X for which X-valued martingale differences are unconditionally convergent in $L^p(X)$, $1 < p < \infty$) provide the correct framework for the development of the harmonic analysis for vector-valued functions. This is masterly illustrated in two recent books: G. Pisier, *Martingales in Banach spaces*, Cambridge University Press, Cambridge, 2016, and T. Hytönen, J. van Neerven, M. Veraar, L. Weis, *Analysis in*

Banach spaces. Vol. I. *Martingales and Littlewood-Paley theory*, Springer 2016; Vol. II. *Probabilistic methods and operator theory*, Springer, 2017.

As the author points out in Introduction:

> In this volume we are providing a compact exposition of the results explaining the interrelation existing between the metric geometry of Banach spaces and probability theory of random vectors with values in those Banach spaces. In particular martingales and random series of independent random vectors are studied.

The presentation is focussed on the remarkable results obtained in the 1970s (an effervescent period in this area) by reputed mathematicians as P. Assouad, D. L. Burkholder, S. D. Chatterji, G. Pisier, J. Hoffmann-Jorgensen, B. Maurey, and the author himself. A good idea on the topics treated in the book is given by the headings of the chapters: 1. *Preliminaries: Probability and geometry in Banach spaces*; 2. *Dentability, Radon-Nikodym Theorem, and Martingale Convergence Theorem*; 3. *Uniform Convexity and Uniform Smoothness*; 4. *Spaces that do not contain* c_0; 5. *Cotypes of Banach spaces*; 6. *Spaces of Rademacher and stable type*; 7. *Spaces of type* 2; 8. *Beck convexity* (B-convex spaces, meaning spaces of type > 1); 9. *Marcinkiewicz-Zygmund Theorem in Banach spaces.*

The author provides detailed proofs of all the results concerning the interplay between the geometry and martingales. For purely geometric or probabilistic results only references are given, the prerequisites being familiarity with basic facts of functional analysis and probability theory.

The book is of interest for researchers in Banach spaces, probability theory and their applications to the analysis of vector functions.

S. Cobzaş