

GÁBOR KASSAY – IN MEMORIAM

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Abstract. The scientific activity of Professor Gábor Kassay (1956–2021), one of the most prolific mathematician in Cluj-Napoca (Romania) is presented, through the memories of some co-authors, with whom he collaborated throughout the four decades of his scientific activity.

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1. Life and scientific activity of GÁBOR KASSAY (1956–2021)

Gábor Kassay was born on December 24, 1956 in Odorheiu Secuiesc (Székelyudvarhely). He studied elementary and high school in his hometown and mathematics at Babeş-Bolyai University in Cluj-Napoca (1976–1980). In 1994 he obtained his scientific degree in mathematics at the same university, with a thesis summarizing his researches on minimax problems, under the supervision of Professor József Kolumbán.

He started his teaching career in secondary schools in Cluj-Napoca (1980–1987) and continued at Babeş-Bolyai University as teaching assistant (1987–1990), assistant professor (1990–1995), associate professor (1995–2002, 2004–2005), professor (2005–2021). In the period 2002–2004 he was a visiting professor at Eastern Mediterranean University in Famagusta, Northern Cyprus.

His university lectures covered the following topics: mathematical analysis, optimization theory, functional analysis, operations research, convex analysis, game theory.

The list of publications of Gábor Kassay totals 87 scientific articles published in prestigious international journals such as: *Mathematical Methods of Operations Research*, *SIAM Journal on Optimization*, *Journal of Optimization Theory and Applications*, *Nonlinear Analysis*, *Journal of Global Optimization*; four books, five book



chapters and a conference proceedings volume edited by him. His recognition is also indicated by the fact that he worked together with more than thirty-five coauthors from different countries. His works total over 2000 citations, including several articles with over 100 independent citations. The complete list of his publications can be found at [36].

He was leader of successful group research programs, co-organizer of scientific conferences, leader of scientific seminars on analysis and optimization. He presented his results at several international conferences around the world.

In this article we try to present this special scientific personality through the testimonies of some of his collaborators.

2. Memories from coauthors

Gábor Kassay was a great master of scientific collaboration. He successfully established and maintained contacts with specialists involved in his fields of interest, publishing joint results with over thirty-five co-authors. The confessions presented below give us a real picture about his ability to establish scientific relationships, about his work style, as well as about the special man and friend who Gabi Kassay was for many. For more details see [36].

József Kolumbán, Babeş-Bolyai University, Cluj-Napoca, Romania:

"I noticed Gábor Kassay from the first year of his studies as one of the most diligent and passionate about mathematics. In particular, his work capacity, intuitive mindset and task-solving skills were extraordinary. Already at that time excelled in finding examples and counterexamples. Although Gábor Kassay graduated from the university with excellent results and could be very useful and necessary in our faculty, he could not be appointed to the university in the circumstances of that time.

His interest in mathematical research kept him in Cluj even after graduating. He chose a high school in this university center, in order to be able to continue actively participating in the activities within the Tiberiu Popoviciu Scientific Seminar. This seminar was very helpful in Gabi's scientific activity throughout his career, he even published some of his first papers in the volumes of this seminar [1, 2]. It was at this time that we wrote our first collaborations [3].

Gábor Kassay's 1994 doctoral (PhD) dissertation was titled "New results in minimax theory applied to variational inequalities and optimization tasks". Throughout his career, the theory of equilibrium, which includes these types of tasks, has been the focus of his attention. It includes, among other topics, optimization, minimax problems, Nash-equilibrium, complementarity, fixed point tasks, variational inequalities, and many other problems in applied mathematics. Gábor Kassay has been publishing articles on this topic since the early 1990s [4, 5, 6], when the synthesizing name "equilibrium theory" had not yet been born. Since then, this theory has evolved enormously. His practitioners have appeared all over the world, who publish hundreds of papers on this topic every year. Gabi has exploited this professional environment very cleverly. He had a working relationship with the best of the profession, from whom he learned a lot, and returned home and shared his experiences with his colleagues.

His curiosity, polite action, reliability and dear manners helped him greatly in this regard.

By leaving, Gabi left a great void in my soul. He gave me one of the most beautiful gifts of my life by being a close colleague and friend for over 40 years. Two years before his death, he presented me with a copy of the monograph on the latest results of the theory of equilibrium, including some of his own, written with Vicențiu Rădulescu [31], with the following dedication: "To my mentor, József Kolumbán, without whom this book (among many others) would not have been written. With friendly love, Gabi, Cluj, 2019 March 7." In the Acknowledgements section of the book, the following sentence is included: "Gábor Kassay is indebted to Joseph Kolumbán, his former teacher and supervisor: their joint papers and interesting discussions on equilibrium problems opened the author's interest toward this topic." These words are also evidence of Gabi's spiritual richness."

Zsolt Páles, University of Debrecen, Hungary:

"After the political changes in Hungary, in 1989, my first visit to Cluj-Napoca became possible in 1992 with a small group of mathematicians from Debrecen. In Cluj, we received a very warm welcome and immediately made friendship with many Hungarian and Romanian mathematicians. Being one of our hosts, Gábor Kassay spent a lot of time with us and we both realized that we had many fields of common interest. In particular, the theory of convexity, nonsmooth analysis and variational inequalities were in the focus of research for both of us. After this visit to Cluj, starting from the year 1995, I became a regular participant of the conferences organized by the Babeş-Bolyai University, I visited Cluj almost every year and Gábor also visited Debrecen several times to deliver seminar and conference lectures. I still have a vivid memory of our participation at the first Joint Conference of Mathematics and Computer Science in Illyefalva in 1995, where also József Kolumbán joined our discussions and the snooker games in the local pub of the village. Due to this active cooperation, we published our first paper with Gábor in 1999 [9], and then two further papers jointly also with József Kolumbán [8, 11]. These works still receive many citations, they are the most important papers for all of us.

The events that we shared keeps Gábor's memory in us. We still cannot understand and accept how and why all this happened to him. Nothing can compensate his loss."

Monica Bianchi, Catholic University of the Sacred Heart and

Rita Pini, University of Milano-Bicocca, Milan:

"Gábor has been not only a great coworker, but especially a very dear friend during the last eighteen years. We met him the first time in 2003, at the 18th International Symposium on Mathematical Programming in Copenhagen. After attending our lecture, he came to us and gave us a card with his e-mail address, since he was interested in the topic and, why not?, to begin a collaboration. We wrote the first paper [12] about the existence of equilibria via Ekeland's principle working at distance, via e-mail essentially. But since then almost every year we succeeded in getting together for one week or more, in Milan, in general, and also by attending the same conferences. We also visited a few times Cluj, where he was always a thoughtful host, pleased to

show us what he liked most in the nearby. Our studies about well-posedness [18, 20], stability of equilibria and generalized equations [15, 23, 24, 25, 26, 27, 28], regularization of variational inequalities and equilibrium problems [33], that have been finalized in twelve publications, usually took the start when we could discuss face-to-face, and went on by exchanging several e-mails. Only during the pandemia we got used to meet via web, and our last work was done completely in this way [35]. Many years passed by, but we keep vivid memories of several moments with him. We will never forget his rigor, his eye for details, his intellectual honesty, but also his consideration for others, his good manners and his extreme courtesy. We will miss him a lot.”

Hans Frenk, Sabanci University, Istanbul, Turkey:

”My scientific collaboration with Gábor Kassay lasted from 1998 until 2008. During that period I visited Gábor almost every year in Koložsvár and later for one time in Cyprus while Gábor visited me several times in Rotterdam at the Erasmus University. Our collaboration started due to our mutual acquaintance Tibor Illés from Eötvös University in Budapest. We shared a common interest in generalisations of convexity and related minmax theorems [7]. Gábor had a lot of experience in this field due to his work on generalisations of so-called K -convex functions and I was interested in extending the classical theory of minmax theorems and convexity. Also around that time I completed my work with my former Ph.D students J.Gromicho and A.I de Barros on the ellipsoid method and fractional programming involving quasiconvex functions. Since immediately we liked each other personally and felt together that our knowledge was complementary we started our collaboration. This collaboration would last for almost 10 years starting with our first paper appearing in Journal of Optimization Theory and Applications in 1999 [7] and ending with the last paper in the same journal in 2007 [16]. In total we wrote 7 joint published papers (also sometimes with other coauthors) and two book chapters of which the last one appeared in 2008 [14, 17]. During that time we also visited several conferences on generalisations of convexity presenting our work. After the publication of the last chapter in 2008 our scientific cooperation ended since we both felt that our work was finished and we continued separately with other research topics. Gábor with his work on variational inequalities and me on applications of stochastic processes and optimization in Operations Research. This was also partly caused by my transition to Sabanci University in Istanbul. Although we irregularly stayed in contact and even planned a kind of reunion to visit each other in either Istanbul or Koložsvár, this never happened due to our busy schedules. I regret now we never did this. I will remember Gábor not only as a dedicated and talented researcher but also on a personal basis as somebody who was very enthusiastic and curious about everything in life and his love for mountain climbing. A nice, friendly and curious person and a scientific friend.”

Qamrul Hasan Ansari, Aligarh Muslim University, India:

”Gábor Kassay visited Aligarh Muslim University, India in November 2017 and was a guest of honour in an open ceremony of an international conference on analysis and its applications. He also visited several times the Department of Mathematics & Statistics, King Fahd University of Petroleum and Minerals, Saudi Arabia. Prof. Kassay worked as a consultant in KFUPM funded research project at King Fahd

University of Petroleum and Minerals, Saudi Arabia with Prof. S. Al-Homidan. It is our honour to work with Prof. Kassay and we published jointly several research papers, namely [29, 30, 32, 34].”

Radu Ioan Bot, University of Vienna, Austria:

”Gábor Kassay was a good friend and a great companion from the very early days of my academic career [13]. I have great memories with him from his visits in Chemnitz, and also from the various optimization conferences we jointly attended.”

Cornel Pinteá, Babeş-Bolyai University, Cluj-Napoca, Romania:

”I first met Gabi Kassay in 1985 as a freshman student at Faculty of Mathematics, Babeş-Bolyai University, as he taught me and my group of colleagues a tutorial of Mathematical Analysis. Gabi Kassay was a teacher and researcher of high order. I certainly appreciated, during my first academic year, the rigorous and meticulous way in which he prepared and delivered his topics such as the Cantor sets, the Cantor intersection theorem, the structure of the open subsets of the real line, a Whitney type decomposition theorem, integrals and so on. At that time I also noticed his ability to enter the world of the students he taught as he considered himself and used to be considered by most of his students as part of their own world. His teaching activity has obviously reached higher and higher levels, due to its own dynamic along the last decades, and its outcome consists in several realized and well established former students. Such an accommodation with the students he taught was only possible through extraordinary communication skills. Therefore, I am also sure that he was widely appreciated by his students along the almost four decades of his teaching activity and most of them still remember his lectures.

The research component of his professional activity is also very reach and highly appreciated within the Mathematical Analysis community, with emphasis on Optimization, Variational Analysis and Equilibrium Problems, as his published scientific papers have great impact in this community. Indeed, Gabi has extensively published in national and especially in international journals with good standards and was the author of several books and book chapters among which we just mention here the monographs *The Equilibrium Problem and Related Topics* [10] and *Equilibrium Problems and Applications* [31]. The outcome of his research activity was significantly influenced, in my opinion, by his communication skills as he used to have direct contacts with his collaborators on a regular basis. In this respect he traveled a lot and used these opportunities, not only for mathematical production, but also to understand the local culture and the history of the communities he visited. I had several opportunities to observe this face of his cultural interests when we both traveled for common scientific events such as those in Isfahan (Iran) for a conference on Nonlinear Analysis and Optimization in 2009, in Pisa (Italy) for a workshop on Variational Analysis, Equilibria and Optimization organized, in May 2017, in the honor of his 60th birthday or in Granada (Spain) for a conference on Minimax Inequalities and Equilibrium Problems in May 2019. In fact Gabi was one of the greatest fruitful travelers, in professional purposes, in our department. Indeed the outcome of his research activity does not only reduces to his publications but is also visible through the PhD students he supervised who are currently occupying important positions both

in Romania and abroad. Gabi has had an extensive coordination activity. Indeed, he coordinated 3 exploratory and research projects (IDEAS) obtained by competition at the national level, all with significant scientific output e.g. [21, 22, 19]. Gabi was also the coordinator of the Analysis and Optimization Research Group within our Faculty of Mathematics and Computer Science, a group with important scientific production. Last, but not least, Gabi had an extensive editorial activity, being a member of the editorial board of 6 international journals.”

Szilárd Csaba László, Technical University of Cluj-Napoca, Romania:

”Professor Gábor Kassay was my PhD supervisor, mentor and, last but not least, my good friend. He was full of zest and enthusiasm for living, he was driven by curiosity about new things. In his mathematical proofs he was characterized by strict logic and consistency, but at the same time he was able to pass on even the newly acquired knowledge to his students or colleagues.

During my doctoral studies, I had the opportunity to observe his attitude towards science and mathematics. I always listened to his scientific lectures and refined explanations with great interest. He taught that not all mathematical results are worth publishing and that we should distinguish between really valuable and negligible mathematical results. He also showed me the importance of examples and counterexamples in a mathematical study. He shared the open questions and obstacles that arose during his research with his colleagues and friends. He was happy when someone could give a counterexample or an explanation. In such cases, he gladly involved the given person in his current research, he made no difference whether he was a student or a professor.

Personally, I can thank Gábor a lot. He introduced me into the world of research and taught me how to write a scientific article [22]. Later, he was also my mentor in a postdoctoral project. He kept track of my scientific work, and I often held presentations at the research seminar he led. The loss of Gábor left a huge space behind, but his memory continues to live for us, those who knew him and respected his consciousness, helpfulness and optimism.”

3. Concluding remarks

Gábor Kassay was driven by a desire to learn and discover new things. He also reached several places on each continent of the world and he shared many stories and experiences with his friends and colleagues. The presented memories show that Gábor Kassay was an excellent researcher, instructor, a good colleague and a great friend, whose loss leaves a hole in our hearts.

We would like to express our thanks to all who contributed to the realization of this article through memories and useful recommendations.

References

- [1] Kassay, G., *A fixed point theorem for generalized contractive mappings*, Babeş-Bolyai University Cluj, Seminar on Mathematical Analysis, **7**(1985), 93–100.

- [2] Kassay, G., *On solvability of nonlinear Hammerstein equations*, Babeş-Bolyai University Cluj, Seminar on Mathematical Analysis, **7**(1985), 93–100.
- [3] Kassay, G., Kolumbán, I., *Implicit functions and variational inequalities for monotone mappings*, Babeş-Bolyai University Cluj, Seminar on Mathematical Analysis, **7**(1989), 79–92.
- [4] Kassay, G., *On Brézis-Nirenberg-Stampacchia's minimax principle*, Babeş-Bolyai University Cluj, Seminar on Mathematical Analysis, **7**(1991), 101–106.
- [5] Illés, T., Kassay, G., *Farkas type theorems for generalized convexities*, Pure Math. Appl., **5**(1994), no. 2, 225–239.
- [6] Kassay, G., *A simple proof for König's minimax theorem*, Acta Math. Hungar., **63**(1994), no. 4, 371–374.
- [7] Frenk, J.B.G., Kassay, G., *On classes of generalized convex functions, Gordan-Farkas type theorems and Lagrangian duality*, J. Optim. Theory Appl., **102**(1999), no. 2, 315–343.
- [8] Kassay, G., Kolumbán, J., Páles, Zs., *On Nash stationary points*, Publ. Math. Debrecen, **54**(1999), no. 3-4, 267–279.
- [9] Kassay, G., Páles, Zs., *A localized version of Ky Fan's minimax inequality*, Nonlinear Anal.: Theory Methods and Appl., **35**(1999), 505–515.
- [10] Kassay, G., *The Equilibrium Problem and Related Topics*, Risoprint, Cluj, Romania, 2000.
- [11] Kassay, G., Kolumbán, J., Páles, Zs., *Factorization of Minty and Stampacchia variational inequality systems*, European J. Oper. Res., **143**(2002), no. 2, 377–389.
- [12] Bianchi, M., Kassay, G., Pini, R., *Existence of equilibria via Ekeland's principle*, J. Math. Anal. Appl., **305**(2005), no. 2, 502–512.
- [13] Boţ, R.I., Kassay, G., Wanka G., *Strong duality for generalized convex optimization problems*, J. Optim. Theory Appl., **127**(2005), no. 1, 45–70.
- [14] Frenk, J.B.G., Kassay, G., *Introduction to convex and quasiconvex analysis*, in: Handbook of Generalized Convexity and Monotonicity, Series: Nonconvex Optim. Appl., Vol. 76, Eds. N. Hadjisavvas, S. Komlósi, S. Schaible, Springer, Berlin-Heidelberg-New York, 2005, pp. 3–87.
- [15] Bianchi, M., Kassay, G., Pini, R., *Ekeland's principle for vector equilibrium problems*, Nonlinear Anal. **66**(2007), 1454–1464.
- [16] Frenk, J.B.G., Kassay, G., *Lagrangian duality and cone convexlike functions*, J. Optim. Theory Appl., **134**(2007), no. 2, 207–222.
- [17] Frenk, J.B.G., Kassay, G., *On noncooperative games, minimax theorems and equilibrium problems*, in: Pareto Optimality, Game Theory and Equilibria, Series: Springer Optim. Appl., Athanasios Migdalas (Crete), Panos Pardalos (Florida), Leonidas Pitsoulis (London) and Altannar Chinchuluun (Florida) (Eds.), Vol. 17, 2008. XXII, pp. 53–94.
- [18] Bianchi, M., Kassay, G., Pini, R., *Well-posedness for vector equilibrium problems*, Math. Methods Oper. Res. **70**(2009), 171–182.
- [19] Kassay, G., Pintea, C., Szenkovits, F., *On convexity of preimages of monotone operators*, Taiwanese J. Math., **13**(2009), 675–686.
- [20] Bianchi, M., Kassay, G., Pini, R., *Well-posed equilibrium problems*, Nonlinear Anal. **72**(2010), 460–468.

- [21] Kassay, G., Pintea, C., *On preimages of a class of generalized monotone operators*, *Nonlinear Anal.*, **73**(2010), 3537–3545.
- [22] Kassay, G., Pintea, C., László, Sz., *Monotone operators and closed countable sets*, *Optimization*, **60**(2011), 1059–1069.
- [23] Bianchi, M., Kassay, G., Pini, R., *Conditioning for optimization problems under general perturbations*, *Nonlinear Anal.* **75**(2012), 37–45.
- [24] Bianchi, M., Kassay, G., Pini, R., *An inverse map result and some applications to sensitivity of generalized equations*, *J. Math. Anal. Appl.* **399**(2013), 279–290.
- [25] Bianchi, M., Kassay, G., Pini, R., *Stability results of variational systems under openness with respect to fixed sets*, *J. Optim. Theory Appl.* **164**(2015), 92–108.
- [26] Bianchi, M., Kassay, G., Pini, R., *Linear openness of the composition of set-valued maps and applications to variational systems*, *Set-Valued Var. Anal.* **24**(2016), 581–595.
- [27] Bianchi, M., Kassay, G., Pini, R., *Stability of equilibria via regularity of the diagonal subdifferential operator*, *Set-Valued Var. Anal.* **25**(2017), 789–805.
- [28] Bianchi, M., Kassay, G., Pini, R., *On a sufficient condition for weak sharp efficiency in multiobjective optimization*, *J. Optim. Theory Appl.* **178**(2018), 78–93.
- [29] Al-Homidan, S., Ansari, Q. H., Kassay, G., *Takahashi's Minimization Theorem and Some Related Results in Quasi-metric Spaces*, *J. Fixed Point Theory Appl.*, **21**(2019), Article ID 38, 20 pp.
- [30] Al-Homidan, S., Ansari, Q. H., Kassay, G., *On Sensitivity of Vector Equilibria by Means of the Diagonal Subdifferential Operator*, *J. Nonlinear Convex Anal.* **20**(2019), 527–537.
- [31] Kassay, G., Rădulescu, V.D., *Equilibrium Problems and Applications*, Series: Mathematics in Science and Engineering, Elsevier/Academic Press, London, 2019.
- [32] Al-Homidan, S., Ansari, Q.H., Kassay, G., *Vectorial form of Ekeland variational principle with applications to vector equilibrium problems*, *Optimization*, **69**(2020), no. 3, 415–436.
- [33] Bianchi, M., Kassay, G., Pini, R., *Regularization of Brézis pseudomonotone variational inequalities*, *Set-Valued Var. Anal.*, **29**(2021), 175–190.
- [34] Al-Homidan, S., Ansari, Q.H., Kassay, G., *Bregman type regularization of variational inequalities with Mosco approximation of the constraint set*, *Positivity*, **26**(2022), no. 3.
- [35] Bianchi, M., Kassay, G., Pini, R., *Brezis pseudomonotone bifunctions and quasi equilibrium problems via penalization*, to appear in *J. of Global Optim.*
- [36] Kása, Z., Kolumbán, J., Rigó, R. P., Szenkovits, F., *Gábor Kassay (1956–2021) – In Memoriam and List of Publication*, Preprint, 2022.
[<https://www.cs.ubbcluj.ro/wp-content/uploads/Kassay-Gabor1956-2021.pdf>]

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