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Analysis of certain determinants for a defined subclass of analytic functions using Poisson distribution series in petal-shaped domain

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Abstract. The current study focuses on obtaining the sharp coefficient estimates and Fekete-Szegö inequality for the class $\Psi_\vartheta(m,\lambda)$ and uses the Poisson distribution series to obtain the sharp estimates of coefficient inequalities, Fekete-Szegö inequality, second order Toeplitz determinants and upper bounds of third order Toeplitz determinants and second order Hankel determinants for a certain analytic function $\mathbb{U}(z) = z + \delta_2 z^2 + \delta_3 z^3 + \cdots, \mathbb{U}(z) \neq 0, \quad z \in \Delta$ belonging to the class $\mathbb{P}\Psi_\vartheta(m,\lambda,\Upsilon) = \{\mathbb{U} \in \mathcal{H} : I^k\mathbb{U} \in \Psi_\vartheta(m,\lambda)\}, m \in \mathbb{N}_0 = \{0,1,2,\cdots\}, \lambda, \vartheta \in \mathbb{N} = \{1,2,...\}$, $\Upsilon = \Upsilon_i(k) = \frac{k^{i-1}}{(i-1)!}e^{-k}$, defined on the open unit disc $(z \in \Delta := \{z : |z| < 1\})$. This research could motivate others to delve deeper into the coefficient functional problem related to the Poisson distribution series of analytic functions across different categories of univalent functions.

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1. Introduction

Consider the class $\mathcal{H}$ comprising analytic functions of the form

$$\mathbb{U}(z) = z + \delta_2 z^2 + \delta_3 z^3 + \cdots, \quad (z \in \Delta := \{z : |z| < 1\}), \tag{1.1}$$

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and $\mathcal{S} = \{\mathbb{U}(z) \in \mathcal{H} : \mathbb{U}(z) \ \ is \ \ univalent \ \ in \ \ \Delta\}$. Subordination of $\mathbb{U}$ to $\mathbb{V}$ in Δ is written as $\mathbb{U} \prec \mathbb{V}$, where $\mathbb{U}, \mathbb{V} \in \mathcal{H}$, provided there exists a Schwarz function w such that $w(0) = 0$ and $|w(z)| \leq |z|$ such that $\mathbb{U}(z) = \mathbb{V}(w(z)), \ z \in \Delta$.

Let $\mathcal{P}$ denote the class of all functions $p(z)$ given by

$$p(z) = 1 + \sum_{i=1}^{\infty} c_i z^i, \quad (z \in \Delta), \ \mathrm{Re}\{p(z)\} > 0 \text{ and } \ p(0) = 1. \tag{1.2}$$

Let $h_j(z) = \sum_{n=0}^{\infty} \delta_{n,j} z^n$ $(j = 1, 2)$ be analytic in Δ. The Hadamard (or convolution) product of h_1 and h_2 is defined by $(h_1 * h_2)(z) = \sum_{n=0}^{\infty} \delta_{n,1}\delta_{n,2} z^n, \ z \in \Delta$.
Following [10], define

$$(\mathbb{U}(z))^{\vartheta} = \left(z + \sum_{k=2}^{\infty} \delta_k z^k\right)^{\vartheta}, \tag{1.3}$$

for $\mathbb{U}(z) \neq 0$ for all $z \neq 0$ and $\vartheta \in \mathbb{N} := \{1, 2, ...\}$.
We observe that the power function given by formula (1.3) has an analytic branch in Δ if and only if $\mathbb{U}(z) \neq 0$ for all $z \neq 0$ and $\vartheta \in \mathbb{N}$. Therefore,

$$(\mathbb{U}(z))^{\vartheta} = \left(z + \delta_2 z^2 + \delta_3 z^3 + \delta_4 z^4 + \ldots\right)^{\vartheta}.$$

Applying binomial formula, we observe that

$$\begin{aligned}(\mathbb{U}(z))^{\vartheta} = z^{\vartheta}[1 + \vartheta\left(\delta_2 z + \delta_3 z^2 + \delta_4 z^3 + \ldots\right) \\ + \tfrac{\vartheta(\vartheta-1)}{2!}\left(\delta_2 z + \delta_3 z^2 + \delta_4 z^3 + \ldots\right)^2 + \ldots].\end{aligned}$$

From [20], we have

$$(\mathbb{U}(z))^{\vartheta} = z^{\vartheta} + \sum_{k=2}^{\infty} A_k z^{\vartheta+k-1}, \tag{1.4}$$

where

$$\begin{aligned}&A_2 = \vartheta\delta_2; \ A_3 = \delta_3\vartheta + \frac{\vartheta(\vartheta-1)}{2}\delta_2^2; \ A_4 = \delta_4\vartheta + \vartheta(\vartheta-1)\delta_2\delta_3 + \frac{\vartheta(\vartheta-1)(\vartheta-2)}{6}\delta_2^3; \\ &A_5 = \delta_5\vartheta + \tfrac{\vartheta(\vartheta-1)}{2}[\delta_3^2 + 2\delta_2\delta_4] + \tfrac{\vartheta(\vartheta-1)(\vartheta-2)}{2}\delta_2^2\delta_3 + \tfrac{\vartheta(\vartheta-1)(\vartheta-2)(\vartheta-3)}{24}\delta_2^4, \ \cdots.\end{aligned} \tag{1.5}$$

If $\mathbb{U}(z) \neq 0$ for all $z \neq 0$, define

$$\Lambda(z) = z\left(\frac{\mathbb{U}(z)}{z}\right)^{\vartheta} = z\left[\frac{\left(z^{\vartheta} + \sum_{k=2}^{\infty} A_k z^{\vartheta+k-1}\right)}{z^{\vartheta}}\right] = z + \sum_{k=2}^{\infty} A_k z^k. \tag{1.6}$$

Since $\left(\frac{\mathbb{U}(z)}{z}\right)^{\vartheta} \neq 0$ for all $z \in \Delta$, $\left(\frac{\mathbb{U}(z)}{z}\right)^{\vartheta}$ is analytic in Δ and so $\Lambda(z)$ is analytic in Δ. Indeed, $\Lambda \in \mathcal{H}$.
Bernardi [5] studied the general operator

$$I_2\mathbb{U}(z) = \frac{1+\gamma}{z^{\gamma}} \int_0^z \mathbb{U}(t) t^{\gamma-1} dt, \quad \gamma \in \mathbb{N}, \tag{1.7}$$

and showed that $I_2(\mathcal{S}^*) \subset \mathcal{S}^*$. The important subfamily of $\mathcal{S}$ is the family $\mathcal{S}^*$ called the starlike function.
By [10] and applying the operator to the function $\Lambda(z)$ defined in (1.6), we define the

integral operator J^m iteratively. To ensure a well-defined sequence, we first establish the base cases J^0 and J^1:

$$J^0(\Lambda(z)) = \Lambda(z) \tag{1.8}$$

The first-order integral operator J^1 is defined as:

$$J^1(\Lambda(z)) = \frac{1+\lambda}{z^\lambda}\int_0^z \Lambda(t)t^{\lambda-1}dt, \quad (\lambda \in \mathbb{N}). \tag{1.9}$$

Substituting the power series expansion $\Lambda(z) = z + \sum_{k=2}^{\infty} A_k z^k$ into the expression above, we obtain:

$$J^1(\Lambda(z)) = z + \sum_{k=2}^{\infty}\left(\frac{1+\lambda}{k+\lambda}\right) A_k z^k. \tag{1.10}$$

For $m \in \{2,3,\dots\}$, the higher-order operators are defined via the following recursive relation:

$$J^m(\Lambda(z)) = J\left(J^{m-1}(\Lambda(z))\right). \tag{1.11}$$

Consequently, by the principle of mathematical induction, the general form of the operator for any $m \in \mathbb{N}_0$ where $\mathbb{N}_0 = \{0,1,2,3,\dots\}$. is given by:

$$J^m(\Lambda(z)) = z + \sum_{k=2}^{\infty}\left(\frac{1+\lambda}{k+\lambda}\right)^m A_k z^k. \tag{1.12}$$

For $k \geq 1$ and $n \geq 1$, Pommerenke [21] introduced the k^{th} Hankel determinant defined as

$$H_k(n) = \begin{vmatrix} \delta_n & \delta_{n+1} & \cdots & \delta_{n+k-1} \\ \delta_{n+1} & \cdots & \cdots & \delta_{n+k} \\ \cdots & \cdots & \cdots & \cdots \\ \delta_{n+k-1} & \delta_{n+k} & \cdots & \delta_{n+2k-2} \end{vmatrix}, \quad (\delta_1 = 1).$$

In particular, the second Hankel determinant takes the form $\mathbb{H}_2(2) = \begin{vmatrix} \delta_2 & \delta_3 \\ \delta_3 & \delta_4 \end{vmatrix}$. It follows that Fekete-Szegö inequality is $\mathbb{H}_2(1)$. Toeplitz determinants and the Hankel determinants are closely connected.

As described in [25], the symmetric Toeplitz determinant $\mathcal{T}_k(n)$ takes the form

$$\mathcal{T}_k(n) = \begin{vmatrix} \delta_n & \delta_{n+1} & \cdots & \delta_{n+k-1} \\ \delta_{n+1} & \delta_n & \cdots & \delta_{n+k} \\ \vdots & & & \\ \delta_{n+k-1} & \delta_{n+k} & \cdots & \delta_n \end{vmatrix}.$$

In particular,

$$\mathcal{T}_2(2) = \begin{vmatrix} \delta_2 & \delta_3 \\ \delta_3 & \delta_2 \end{vmatrix}, \quad \mathcal{T}_3(1) = \begin{vmatrix} 1 & \delta_2 & \delta_3 \\ \delta_2 & 1 & \delta_2 \\ \delta_3 & \delta_2 & 1 \end{vmatrix}.$$

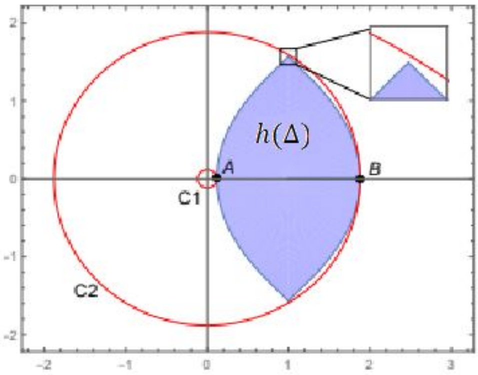

FIGURE 1. Boundary curve: $h(\Delta)$ lies in the annular region bounded between the circles $C1$ and $C2$.

Toeplitz matrices has applications in pure and applied mathematics [28] .
Arora and Kumar [2] considered the petal-shaped region

$$\Delta_p = \{w \in \mathbb{C} : |sinh(w-1)| < 1\}, \tag{1.13}$$

which admits the following functional characterization:

$$h(z) = 1 + sinh^{-1} z. \tag{1.14}$$

Its boundary curve is given in Figure 1.

Although $h(z)$ is multivalued, it becomes holomorphic in Δ upon choosing branch cuts along $(-\infty, -i) \cup (i, \infty)$ on the imaginary axis, and hence is analytic in Δ. Geometrically, the function $h(z)$ maps the unit disc Δ onto the petal-shaped domain Δ_p. This domain has attracted considerable attention from researchers [1, 2, 4, 12, 17].

2. Preliminary results

In [10], the authors studied the class $\Psi_\lambda^n(m, \delta)$ for $m \in \mathbb{N}_0 = \{0, 1, 2, 3 \cdots\}$, $n \in \mathbb{N}$, $\lambda > 0$, $z \in \Delta$ and $0 \leq \delta < 1$. Motivated by this study, we introduce and investigate a subclass of $\mathcal{H}$ defined on a petal-shaped domain, as described below.

Definition 2.1. *Let $\mathbb{U}(z) \neq 0$ for $z \in \Delta$ and let $\vartheta \in \mathbb{N}$. Define $\Lambda(z)$ by (1.6). Then the function $\mathbb{U}$ is said to belong to the class $\Psi_\vartheta(m, \lambda)$, where $m \in \mathbb{N}_0$ and $\lambda, \vartheta \in \mathbb{N}$, if $\Lambda(z)$ satisfies the condition*

$$\frac{(J^m \Lambda(z))}{z} \prec 1 + sinh^{-1} z, \tag{2.1}$$

Lemma 2.2 ([6], p.41, Carathéodory's Lemma)**.** *If $p \in \mathcal{P}$, then*

$$|c_k| \leq 2 \ (k \geq 1), \tag{2.2}$$

and the estimate is sharp.

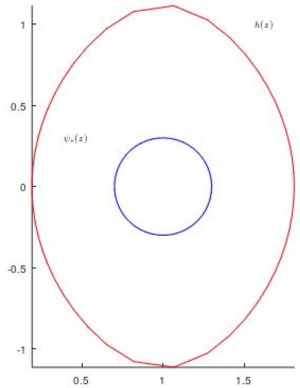

FIGURE 2. The images of $\Psi_*(\Delta)$ (blue color) and $h(\Delta)$(red color).

Lemma 2.3. [13] *Suppose $p \in \mathcal{P}$. Then, for every complex number μ, one finds*

$$|c_2 - \mu c_1^2| \leq 2\ max\{1, |2\mu - 1|\}. \tag{2.3}$$

Lemma 2.4. [13] *Let $p \in \mathcal{P}$ of the form (1.2). Then there exist some $\xi, \zeta \in \mathbb{C}$ with $|\xi| \leq 1$, $|\zeta| \leq 1$, such that*

$$2c_2 = c_1^2 + (4 - |c_1^2|)\xi, \tag{2.4}$$

$$4c_3 = c_1^3 + 2c_1\xi(4 - c_1^2) - (4 - c_1^2)c_1\xi^2 + 2(4 - c_1^2)(1 - |\xi|^2)\zeta. \tag{2.5}$$

Remark 2.5. We shall illustrate with an example that the aforementioned class $\Psi_\vartheta(m, \lambda)$ is non-empty.
With $F_1(z) = z + az^2$, we obtain $\Lambda(z) = z\,(1 + az)^\vartheta$, consequently

$$\Psi_*(z) = \frac{(J^m(\Lambda(z)))}{z} = 1 + t_2A_2z + t_3A_3z^2 + \cdots,$$

where $\vartheta \in \mathbb{N}$, $m \in \mathbb{N}_0$, $\lambda \in \mathbb{N}$.

For the values of $\vartheta = 1$, we get $\Psi_*(z) = 1 + t_2az$, when $m = \lambda = 1, a = 1/2$ and $z = 0.9e^{i\theta}$, we get $t_2 = \frac{2}{3}$ and hence

$$\Psi_*(0.9e^{i\theta}) = 1 + \frac{1}{3}(0.9e^{i\theta}).$$

Thus from the figure (Figure 2) it is clear that $\Psi_*(\Delta) \subset h(\Delta)$ and hence the class $\Psi_\vartheta(m, \lambda)$ is non-empty.

Remark 2.6. The function $g(z)$, given explicitly by $g(z) = \frac{z}{1-z}$, $z \in \Delta$ is in the class $\Psi_\vartheta(m, \lambda)$, since

$$\Psi_1(m, \lambda) = \frac{(J^m(\Lambda(z)))}{z} = 1 + t_2A_2z + t_3A_3z^2 + \cdots,$$

where $A_2 = \vartheta$; $A_3 = \vartheta + \frac{\vartheta(\vartheta-1)}{2}$; $A_4 = \vartheta + \vartheta(\vartheta-1) + \frac{\vartheta(\vartheta-1)(\vartheta-2)}{6}; \cdots$ and $\Lambda(z) = z(\frac{g(z)}{z})^{\vartheta}$ and $\vartheta \in \mathbb{N}$, $m \in \mathbb{N}_0$, $\lambda \in \mathbb{N}$. Proceeding as like previous remark, we have $\Psi_1(\Delta) \subset h(\Delta)$.

Extensive research has been devoted to obtaining the upper bounds of the second and third order Hankel determinants [3, 7, 8, 9, 10, 11, 14, 15, 18, 19, 26, 27]. However, determining the bounds for a subclass of analytic functions utilizing Poisson distribution series in a petal-shaped domain presents numerous applications in science and engineering.
Hence, in the present paper we derive coefficient estimates and Fekete-Szegö inequality for the class $\Psi_\vartheta(m,\lambda)$ and by using the Poisson distribution series, we determine the precise bounds for coefficient inequalities, Fekete-Szegö inequality, second order Toeplitz determinants, upper bounds of third order Toeplitz determinants, second order Hankel determinants for the class $\mathbb{P}\Psi_\vartheta(m,\lambda,\Upsilon)$, $m \in \mathbb{N}_0$, $\lambda \in \mathbb{N}$ and $\vartheta \in \mathbb{N}$ associated with the petal-shaped domain.

3. Coefficient bounds and the Fekete–Szegö inequality for the class $\Psi_\vartheta(m,\lambda)$

Theorem 3.1. *If the function $\mathbb{U}$ given by (1.1) is in the class $\Psi_\vartheta(m,\lambda)$, for $m \in \mathbb{N}_0$, $\lambda \in \mathbb{N}$, $\vartheta \in \mathbb{N}$ then*

$$|\delta_2| \le \frac{1}{\vartheta t_2}, \ |\delta_3| \le 2\sigma_2 \ max\left\{1, \left|\frac{2\sigma_1}{\sigma_2} - 1\right|\right\},$$

where $t_i = \left(\frac{\lambda+1}{\lambda+i}\right)^m$, $\sigma_1 = \frac{1}{4t_3\vartheta} + \frac{(\vartheta-1)}{8t_2^2\vartheta^2}$, $\sigma_2 = \frac{1}{2t_3\vartheta}$. The inequalities are sharp.

Proof. Let $\mathbb{U}(z) \in \Psi_\vartheta(m,\lambda)$, according to the subordination relationship, there exists a Schwarz function $w(z)$ with $w(0) = 0$ and $|w(z)| < 1$, satisfying

$$\frac{J^m(\Lambda(z))}{z} = 1 + sinh^{-1}(w(z)) \quad (z \in \Delta). \tag{3.1}$$

Consider the function

$$p(z) = \frac{1+w(z)}{1-w(z)} = 1 + c_1 z + c_2 z^2 + \cdots, \ p \in \mathbb{P}. \tag{3.2}$$

$$w(z) = \frac{p(z)-1}{1+p(z)} = \frac{c_1}{2}z + \left(\frac{c_2}{2} - \frac{c_1^2}{4}\right)z^2 + \left(\frac{c_3}{2} - \frac{c_1c_2}{2} + \frac{c_1^3}{8}\right)z^3 + \cdots. \tag{3.3}$$

After simple computation, we get

$$1 + sinh^{-1}(w(z)) = 1 + \frac{1}{2}c_1 z + \left(-\frac{c_1^2}{4} + \frac{c_2}{2}\right)z^2 + \left(\frac{5c_1^3}{48} - \frac{c_1c_2}{2} + \frac{c_3}{2}\right)z^3 + \cdots. \tag{3.4}$$

Using series expansion, we obtain

$$\frac{J^m(\Lambda(z))}{z} = 1 + \sum_{i=2}^{\infty} t_i A_i z^{i-1} = 1 + t_2A_2 z + t_3A_3 z^2 + t_4A_4 z^3 + \cdots, \tag{3.5}$$

where $t_i = \left(\frac{1+\lambda}{i+\lambda}\right)^m$.
Equating the coefficients between (3.4) and (3.5), we determine that

$$A_2 = \frac{c_1}{2t_2} \;\Rightarrow \delta_2 = \frac{c_1}{2t_2\vartheta} = c_1\Lambda_1, \tag{3.6}$$

$$\delta_3 = -c_1^2\sigma_1 + c_2\sigma_2, \tag{3.7}$$

$$\delta_4 = \gamma_1 c_1^3 - \gamma_2 c_1 c_2 + \gamma_3 c_3, \tag{3.8}$$

where,

$$\Lambda_1 = \frac{1}{2t_2\vartheta}, \sigma_1 = \frac{1}{4t_3\vartheta} + \frac{(\vartheta-1)}{8t_2^2\vartheta^2}, \sigma_2 = \frac{1}{2t_3\vartheta},$$

$$\gamma_1 = \frac{5}{48t_4\vartheta} + \frac{\vartheta-1}{2t_2\vartheta}\left[\frac{1}{4t_3\vartheta} + \frac{\vartheta-1}{8t_2^2\vartheta^2} - \frac{(\vartheta-2)}{24t_2^2\vartheta^2}\right], \gamma_2 = \frac{1}{2t_4\vartheta} + \frac{(\vartheta-1)}{4\vartheta^2 t_2 t_3}, \gamma_3 = \frac{1}{2\vartheta t_4}.$$

Applying Lemma 2.2 in (3.6), we get

$$|\delta_2| \le 2\Lambda_1 \le \frac{1}{\vartheta t_2}.$$

Sharpness is achieved by the function (3.2) with $p(z) = 1 + 2z$.
From (3.7), we have

$$|\delta_3| = |-c_1^2\sigma_1 + c_2\sigma_2| = \sigma_2\left|c_2 - c_1^2\frac{\sigma_1}{\sigma_2}\right|.$$

By implementing Lemma 2.3, we obtain

$$|\delta_3| \le 2\sigma_2\ max\left\{1, \left|\frac{2\sigma_1}{\sigma_2} - 1\right|\right\}.$$

Equality is attained for the function defined by (3.2) with $p(z) = 1 + 2z^2$, when $max\ \left\{1, \left|\frac{2\sigma_1}{\sigma_2} - 1\right|\right\} = 1$ and the inequality is sharp for the function (3.2) with $p(z) = 1 + 2z + 2z^2$, when $max\ \left\{1, \left|\frac{2\sigma_1}{\sigma_2} - 1\right|\right\} = \left|\frac{2\sigma_1}{\sigma_2} - 1\right|$. □

Corollary 3.2. *If* $\vartheta = 1$, *then* $|\delta_2| \le \frac{1}{t_2}$, $|\delta_3| \le \frac{2}{t_3}$.

Theorem 3.3. *Suppose the function* $\mathbb{U}$ *defined in* (1.1) *lies in the class* $\Psi_\vartheta(m, \lambda)$, *for* $m \in \mathbb{N}_0$, $\lambda \in \mathbb{N}$, $\vartheta \in \mathbb{N}$, *Consequently, for any* $\nu \in \mathbb{C}$

$$|\delta_3 - \nu\delta_2^2| \le 2\sigma_2 max\left\{1, \left|2\left(\frac{\sigma_1 + \nu\Lambda_1^2}{\sigma_2}\right) - 1\right|\right\}.$$

The inequality is sharp.

Proof. From (3.6) and (3.7), we get

$$|\delta_3 - \nu\delta_2^2| = |c_2\sigma_2 - c_1^2\sigma_1 - \nu(c_1^2\Lambda_1^2)| = \sigma_2\left|c_2 - c_1^2\left(\frac{\sigma_1 + \nu\Lambda_1^2}{\sigma_2}\right)\right|.$$

Applying (2.3), we get

$$|\delta_3 - \nu\delta_2^2| \le 2\ \sigma_2\ max\ \left\{1, \left|2\left(\frac{\sigma_1 + \nu\Lambda_1^2}{\sigma_2}\right) - 1\right|\right\}.$$

Equality is attained for the function given by (3.2) with $p(z) = 1 + 2z^2$, when $max \left\{1, \left|2\left(\frac{\sigma_1+\nu\Lambda_1^2}{\sigma_2}\right) - 1\right|\right\} = 1$ and when $max \left\{1, \left|2\left(\frac{\sigma_1+\nu\Lambda_1^2}{\sigma_2}\right) - 1\right|\right\} = \left|2\left(\frac{\sigma_1+\nu\Lambda_1^2}{\sigma_2}\right) - 1\right|$, sharpness is attained for the function given by (3.2) with $p(z) = 1 + 2z + 2z^2$. □

Corollary 3.4. *If* $\vartheta = 1$, *then* $|\delta_3 - \nu\delta_2^2| \leq \frac{1}{t_3}$.

4. Functions generated from the Poisson distribution

A random variable $\mathcal{X}$ is said to follow a Poisson distribution with parameter $k > 0$ if it assumes the values $0, 1, 2, 3, \ldots$ with respective probabilities

$$P(\mathcal{X} = i) = \frac{k^i e^{-k}}{i!}, \quad i = 0, 1, 2, 3, \ldots$$

Thus $P(\mathcal{X} = \tau) = k^\tau \frac{e^{-k}}{\tau!}, \quad \tau = 0, 1, 2, \cdots$.

Porwal [22] investigated a power series whose coefficients correspond to the probabilities of a Poisson distribution.

$$I(k, z) = z + \sum_{i=2}^{\infty} \frac{k^{i-1}}{(i-1)!} e^{-k} z^i, \quad z \in \Delta, \tag{4.1}$$

where $k > 0$ also by applying ratio test one can find the radius of convergence of the above series is infinity and $J^m(\Lambda(z))$ is defined in (1.12).
Following recent research [16, 24, 23], consider the linear operator $I^k(z) : \mathcal{H} \to \mathcal{H}$ defined by

$$I^k(\Lambda(z)) = I(k, z) * (\Lambda(z)) = z + \sum_{i=2}^{\infty} \frac{k^{i-1}}{(i-1)!} e^{-k} A_i z^i \tag{4.2}$$

$$= z + \sum_{i=2}^{\infty} \Upsilon_i(k) A_i z^i, \tag{4.3}$$

where $A_i's$ are defined in (1.5) and

$$\Upsilon = \Upsilon_i(k) = \frac{k^{i-1}}{(i-1)!} e^{-k} \tag{4.4}$$

and $*$ denotes the convolution of two series. In particular, $\Upsilon_2 = ke^{-k}$ and $\Upsilon_3 = \frac{k^2}{2} e^{-k}$. Implementing (1.12), we have

$$J^m(I^k \Lambda(z)) = z + \sum_{i=2}^{\infty} \Upsilon_i t_i A_i z^i, \tag{4.5}$$

where $t_i = \left(\frac{1+\lambda}{\lambda+i}\right)^m$.

Definition 4.1. *Let $\mathbb{U}(z) \neq 0$ for $z \in \Delta$ and let $\vartheta \in \mathbb{N}$. If the function $\Lambda(z)$ is defined by* (1.6) *then, the function $\mathbb{U}(z)$ is said to belong to the class $\mathbb{P}\Psi_\vartheta(m, \lambda, \Upsilon)$, where $m \in \mathbb{N}_0$, $\lambda \in \mathbb{N}$ and $\vartheta \in \mathbb{N}$, if the function $\Lambda(z)$ satisfies the following condition:*

$$\frac{(J^m(I^k\Lambda(z)))}{z} \prec 1 + sinh^{-1}z, \tag{4.6}$$

or equivalently, $\mathbb{P}\Psi_\vartheta(m, \lambda, \Upsilon) = \{\mathbb{U}(z) \in \mathcal{H} : \quad I^k\mathbb{U}(z) \in \Psi_\vartheta(m, \lambda, \Upsilon)\}$, where $J^m(I^k\Lambda(z))$ is given in (4.5) and $\Upsilon_i,\ i = 2, 3, \cdots$ is defined in (4.4).

By employing the corresponding coefficient estimates together with the Fekete–Szegö inequality for functions belonging to the class $\Psi_\vartheta(m, \lambda)$, we derive the coefficient estimates and the associated Fekete–Szegö inequality for functions in the class $\mathbb{P}\Psi_\vartheta(m, \lambda, \Upsilon)$. Applying Theorems (3.1) and (3.3) yields the following theorems for the function $I^k\Lambda(z)$.

5. Fekete–Szegö inequality and related estimates for the class $\mathbb{P}\Psi_\vartheta(m, \lambda, \Upsilon)$

Theorem 5.1. *Let $\mathbb{U}$ be the function defined in* (1.1). *If $\mathbb{U}$ is in the class $\mathbb{P}\Psi_\vartheta(m, \lambda, \Upsilon)$, for $m \in \mathbb{N}_0, \quad \lambda \in \mathbb{N}, \quad \vartheta \in \mathbb{N}$ and $\Upsilon = \Upsilon_i(k) = \frac{k^{i-1}}{(i-1)!}e^{-k}$, then*

$$|\delta_2| \leq \frac{1}{\vartheta t_2 \Upsilon_2}, \ |\delta_3| \leq 2\beta_2 \ max\left\{1, \left|\frac{2\beta_1}{\beta_2} - 1\right|\right\},$$

where $t_i = \left(\frac{\lambda+1}{\lambda+i}\right)^m$, $\beta_1 = \frac{1}{4t_3\Upsilon_3\vartheta} + \frac{(\vartheta-1)}{8t_2^2\Upsilon_2^2\vartheta^2}$, $\beta_2 = \frac{1}{2t_3\Upsilon_3\vartheta}$. The inequalities are sharp.

Proof. Since $\mathbb{U}(z) \in \mathbb{P}\Psi_\vartheta(m, \lambda, \Upsilon)$, the subordination condition ensures the existence of a Schwarz function $w(z)$ for which $w(0) = 0$ and $|w(z)| < 1$, satisfying

$$\frac{J^m(I^k\Lambda(z))}{z} = 1 + sinh^{-1}(w(z)) \quad (z \in \Delta). \tag{5.1}$$

Consider the function

$$p(z) = \frac{1 + w(z)}{1 - w(z)} = 1 + c_1 z + c_2 z^2 + \cdots, \tag{5.2}$$

where $p \in \mathbb{P}$. Consequently,

$$w(z) = \frac{p(z) - 1}{1 + p(z)} = \frac{c_1}{2}z + \left(\frac{c_2}{2} - \frac{c_1^2}{4}\right)z^2 + \left(\frac{c_3}{2} - \frac{c_1 c_2}{2} + \frac{c_1^3}{8}\right)z^3 + \cdots \tag{5.3}$$

After simple computation, we get

$$1 + sinh^{-1}(w(z)) = 1 + \frac{1}{2}c_1 z + \left(-\frac{c_1^2}{4} + \frac{c_2}{2}\right)z^2 + \left(\frac{5c_1^3}{48} - \frac{c_1 c_2}{2} + \frac{c_3}{2}\right)z^3 + \cdots. \tag{5.4}$$

Using series expansion, we obtain

$$\frac{J^m(I^k\Lambda(z))}{z} = 1 + \sum_{i=2}^{\infty} t_i \Upsilon_i(k) A_i z^{i-1} = 1 + t_2\Upsilon_2 A_2 z + t_3 \Upsilon_3 A_3 z^2 + \cdots, \tag{5.5}$$

where $t_i = \left(\frac{1+\lambda}{i+\lambda}\right)^m$.
Equating the coefficients between (5.4) and (5.5), we determine that $A_2 = \frac{c_1}{2t_2\Upsilon_2}$.
Hence,

$$\delta_2 = \frac{c_1}{2t_2\vartheta\Upsilon_2} = c_1\alpha_1, \tag{5.6}$$

$$\delta_3 = -c_1^2\beta_1 + c_2\beta_2, \tag{5.7}$$

$$\delta_4 = \eta_1 c_1^3 - \eta_2 c_1 c_2 + \eta_3 c_3, \tag{5.8}$$

where,

$$\alpha_1 = \frac{1}{2t_2\vartheta\Upsilon_2},\ \beta_1 = \frac{1}{4t_3\Upsilon_3\vartheta} + \frac{(\vartheta-1)}{8t_2^2\Upsilon_2^2\vartheta^2},\ \beta_2 = \frac{1}{2t_3\Upsilon_3\vartheta},$$

$$\eta_1 = \frac{5}{48t_4\Upsilon_4\vartheta} + \frac{\vartheta-1}{2t_2\vartheta\Upsilon_2}\left[\frac{1}{4t_3\Upsilon_3\vartheta} + \frac{\vartheta-1}{8t_2^2\Upsilon_2^2\vartheta^2} - \frac{(\vartheta-2)}{24t_2^2\vartheta^2\Upsilon_2^2}\right],$$

$$\eta_2 = \frac{1}{2t_4\Upsilon_4\vartheta} + \frac{(\vartheta-1)}{4\vartheta^2 t_2 t_3\Upsilon_2\Upsilon_3},\ \eta_3 = \frac{1}{2\vartheta t_4\Upsilon_4}.$$

Applying (2.2) in (5.6), we get

$$|\delta_2| \leq 2\alpha_1 \leq \frac{1}{\vartheta t_2\Upsilon_2}.$$

The bound is attained by the function defined by (5.2) with $p(z) = 1 + 2z$.
From (5.7), we have

$$|\delta_3| = |-c_1^2\beta_1 + c_2\beta_2| = \beta_2|c_2 - c_1^2\frac{\beta_1}{\beta_2}|.$$

By implementing (2.3), we obtain

$$|\delta_3| \leq 2\beta_2\ max\left\{1, \left|\frac{2\beta_1}{\beta_2} - 1\right|\right\}.$$

The inequality is sharp for the function given by (5.2) with $p(z) = 1 + 2z^2$, when $max\ \left\{1, \left|\frac{2\beta_1}{\beta_2} - 1\right|\right\} = 1$ and it is sharp for the function given by (5.2) with $p(z) = 1 + 2z + 2z^2$, when $max\ \left\{1, \left|\frac{2\beta_1}{\beta_2} - 1\right|\right\} = \left|\frac{2\beta_1}{\beta_2} - 1\right|$. □

Corollary 5.2. *If $\vartheta = 1$, then*

$$|\delta_2| \leq \frac{1}{t_2\Upsilon_2},\ |\delta_3| \leq \frac{1}{t_3\Upsilon_3}.$$

In view of Theorem (3.3), let us prove the following theorem.

Theorem 5.3. *Suppose that the function $\mathbb{U}$ given by (1.1) belongs to the class $\mathbb{P}\Psi_\vartheta(m,\lambda,\Upsilon)$, for $m \in \mathbb{N}_0$, $\lambda \in \mathbb{N}$, $\vartheta \in \mathbb{N}$ and $\Upsilon = \Upsilon_i(k) = \frac{k^{i-1}}{(i-1)!}e^{-k}$, then for any $\nu \in \mathbb{C}$,*

$$|\delta_3 - \nu\delta_2^2| \leq 2\beta_2 max\left\{1, \left|2\left(\frac{\beta_1 + \nu\alpha_1^2}{\beta_2}\right) - 1\right|\right\}.$$

The inequality is sharp.

Proof. From (5.6) and (5.7), we get

$$|\delta_3-\nu\delta_2^2| = |c_2\beta_2 - c_1^2[\alpha_1^2\nu+\beta_1]| = \beta_2\left|c_2 - c_1^2\left(\frac{\alpha_1^2\nu+\beta_1}{\beta_2}\right)\right|.$$

Applying (2.3), we get

$$|\delta_3-\nu\delta_2^2| \le 2\ \beta_2\ max\ \left\{1, \left|2\left(\frac{\alpha_1^2\nu+\beta_1}{\beta_2}\right)-1\right|\right\}.$$

The estimate is attained by the function defined in (5.2) with $p(z)=1+2z^2$, whenever $\max\left\{1,\left|2\left(\frac{\beta_1+\nu\alpha_1^2}{\beta_2}\right)-1\right|\right\}=1$ and the bound is attained for the function defined in (5.2) with $p(z)=1+2z+2z^2$, whenever $\max\left\{1,\left|2\left(\frac{\beta_1+\nu\alpha_1^2}{\beta_2}\right)-1\right|\right\}=\left|2\left(\frac{\beta_1+\nu\alpha_1^2}{\beta_2}\right)-1\right|.$

□

6. Toeplitz determinant $\mathcal{T}_2(2)$ *and* $\mathcal{T}_3(1)$ for the class $\mathbb{P}\Psi_\vartheta(m,\lambda,\Upsilon)$

Theorem 6.1. *Consider the function* $\mathbb{U}$*, as defined in* (1.1)*, to be a member of the class* $\mathbb{P}\Psi_\vartheta(m,\lambda,\Upsilon)$*, for* $m\in\mathbb{N}_0,\quad \lambda\in\mathbb{N},\quad \vartheta\in\mathbb{N}$ *and* $\Upsilon=\Upsilon_i(k)=\frac{k^{i-1}}{(i-1)!}e^{-k}$*. Then,*

$$|\mathcal{T}_2(2)| \le 16\beta_1^2+36\beta_2^2+48\beta_1\beta_2+4\alpha_1^2,$$

where $\alpha_1,\quad \beta_1,\quad \beta_2$ *are presented in theorem (5.1), and it is the best estimate.*

Proof. The bound of $\mathcal{T}_2(2)$ is denoted by $|\mathcal{T}_2(2)|=|\delta_3^2-\delta_2^2|$.
Applying (5.6) and (5.7)

$$|\mathcal{T}_2(2)| = |c_1^4\beta_1^2 - 2c_1^2c_2\beta_1\beta_2 - c_1^2\alpha_1^2 + c_2^2\beta_2^2|. \tag{6.1}$$

Using the values of c_2 from Lemma 2.4, yields

$$|\mathcal{T}_2(2)| = \left|c_1^4\beta_1^2 - c_1^2\beta_1\beta_2\left[c_1^2+(4-c_1^2)\xi\right] - c_1^2\alpha_1^2 + \frac{1}{4}\left[c_1^2+(4-c_1^2)\xi\right]^2\beta_2^2\right|. \tag{6.2}$$

Denoting $|c_1|=c$ and $|\xi|=\rho$, then $c\in[0,2]$ and $\rho\in[0,1]$, and applying triangle inequality, we get

$$|\mathcal{T}_2(2)| \le h_1c^4 + h_2\rho c^2|4-c_1^2| + \alpha_1^2c^2 + \frac{\beta_2^2}{4}\rho^2|4-c_1^2|^2,$$

where $h_1=\frac{\beta_2^2}{4}+\beta_1^2+\beta_1\beta_2,\quad h_2=\beta_2\beta_1+\frac{\beta_2^2}{2}$. Again, applying triangle inequality for this terms $|4-c_1^2|\le 4+|c_1^2| = 4+c^2\ge 0$ similarly,$|(4-c_1^2)|^2\le(4+c^2)^2$, we get

$$|\mathcal{T}_2(2)| \le h_1c^4 + h_2\rho c^2(4+c^2) + c^2\alpha_1^2 + \frac{\beta_2^2}{4}\rho^2(4+c^2)^2 = A(c,\rho).$$

Next, we maximize the function $A(c,\rho)$ for $(c,\rho)\in[0,2]\times[0,1]$. Partially differentiating $A(c,\rho)$ with respect to ρ, we obtain $\frac{\partial A}{\partial\rho} = h_2c^2(4+c^2)+\frac{\beta_2^2}{2}\rho(4+c^2)^2\ge 0$, for $0\le c\le 2$ and $0\le\rho\le 1$. Hence $A(c,\rho)$ is increasing in ρ and attains its maximum at $\rho=1$. Therefore, $A(c,\rho)\le A(c,1)=B(c)$, where $B(c)=$

$|h_1|c^4 + |h_2|c^2(4+c^2) + \alpha_1^2 c^2 + \frac{\beta_2^2}{4}(4+c^2)^2$. We note that $B'(c) \geq 0$ implies $B(c)$ increases with c and thus attains its maximum value at $c = 2$. Therefore,

$$\begin{aligned}|\mathcal{T}_2(2)| &\leq 16|h_1| + 32|h_2| + 4\alpha_1^2 + 16\beta_2^2 \\ &\leq 16\beta_1^2 + 36\beta_2^2 + 48\beta_1\beta_2 + 4\alpha_1^2.\end{aligned}$$

The obtained bound is sharp. Equality holds for the extremal Carathéodory function $p(z) = \frac{1+e^{i\theta}z}{1-e^{i\theta}z}, \quad \theta \in \mathbb{R}$, for which $c_1 = 2e^{i\theta}, \quad c_2 = 2e^{2i\theta}$.
Substituting these values into the expression (6.1), $\mathcal{T}_2(2)$ yields equality. Hence, the result is sharp.

□

Corollary 6.2. *If* $\vartheta = 1$, *then* $|\mathcal{T}_2(2)| \leq 16B_1^2 + 36B_2^2 + 48B_1B_2 + 4A_1^2$, *where* $A_1 = \frac{1}{2t_2\Upsilon_2}, \ B_1 = \frac{1}{4t_3\Upsilon_3}, \ B_2 = \frac{1}{2t_3\Upsilon_3}$.

Theorem 6.3. *Suppose the function* $\mathbb{U}$, *defined by* (1.1), *belongs to the class* $\mathbb{P}\Psi_\vartheta(m,\lambda,\Upsilon)$, *for* $m \in \mathbb{N}_0, \quad \lambda \in \mathbb{N}, \quad \vartheta \in \mathbb{N}$ *and* $\Upsilon = \Upsilon_i(k) = \frac{k^{i-1}}{(i-1)!}e^{-k}$, *then*

$$|\mathcal{T}_3(1)| \leq 1 + 8\alpha_1^2 + 16(\beta_1^2 + 2\alpha_1^2\beta_1) + 36\beta_2^2 + 48(\alpha_1^2\beta_2 + \beta_1\beta_2).$$

Proof. The bounds of $\mathcal{T}_3(1)$ is denoted by $|\mathcal{T}_3(1)| = |1 - 2\delta_2^2 + 2\delta_2^2\delta_3 - \delta_3^2|$.
Proceeding on the similar lines of theorem (6.1), the upper bound is obtained as

$$|\mathcal{T}_3(1)| \leq 1 + 8\alpha_1^2 + 16(\beta_1^2 + 2\alpha_1^2\beta_1) + 36\beta_2^2 + 48(\alpha_1^2\beta_2 + \beta_1\beta_2).$$

□

Corollary 6.4. *If* $\vartheta = 1$, *then*

$$|\mathcal{T}_3(1)| \leq 1 + 8A_1^2 + 16(B_1^2 + 2A_1^2B_1) + 36B_2^2 + 48(A_1^2B_2 + B_1B_2),$$

where A_1, B_1, B_2 *are defined in corollary (6.2).*

7. Second Hankel determinant associated with the class $\mathbb{P}\Psi_\vartheta(m,\lambda,\Upsilon)$

Theorem 7.1. *If the function* $\mathbb{U}$ *of the form (1.1) is in the class* $\mathbb{P}\Psi_\vartheta(m,\lambda,\Upsilon)$, *for* $m \in \mathbb{N}_0, \quad \vartheta \in \mathbb{N}$ *and* $\Upsilon = \Upsilon_i(k) = \frac{k^{i-1}}{(i-1)!}e^{-k}$, *then*

$$\mathbb{H}_2(2) = |\delta_2\delta_4 - \delta_3^2| \leq 16(\eta_1\alpha_1 - \beta_1^2) + 24(2\beta_1\beta_2 - \eta_2\alpha_1) + 44\eta_3\alpha_1 + 36\beta_2^2.$$

Proof. Using (5.6) to (5.8), we have

$$|\delta_2\delta_4 - \delta_3^2| = |[\eta_1\alpha_1 - \beta_1^2]c_1^4 + [2\beta_1\beta_2 - \eta_2\alpha_1]c_1^2c_2 + \eta_3\alpha_1c_1c_3 - c_2^2\beta_2^2|. \tag{7.1}$$

Using c_2 and c_3 values from Lemma 2.4, yields

$$\begin{aligned}|\delta_2\delta_4 - \delta_3^2| = \Bigg|&[\eta_1\alpha_1 - \beta_1^2]c_1^4 + [2\beta_1\beta_2 - \eta_2\alpha_1]c_1^2\frac{1}{2}[c_1^2 + \xi(4-c_1^2)] + \eta_3\alpha_1c_1\frac{1}{4}[c_1^3 \\ &+2\xi c_1(4-c_1^2) - \xi^2c_1(4-c_1^2) + 2(4-c_1^2)(1-|\xi|^2)\zeta] - \frac{\beta_2^2}{4}[c_1^2 + \xi(4-c_1^2)]^2\Bigg|.\end{aligned}$$

Denoting $|c_1| = c$ and $|\xi| = \rho$, then $c \in [0,2]$ and $\rho \in [0,1]$ also using the fact that $|\zeta| \leq 1$ and implementing triangle inequality, we arrive

$$|\delta_2\delta_4 - \delta_3^2| \leq |q_1|c^4 + q_2\rho c^2|4 - c_1^2| + 2q_3c|4 - c_1^2| + q_3c\rho^2|4 - c_1^2|(c+2) + q_4\rho^2|4 - c_1^2|^2$$

where $q_1 = \frac{1}{4}[4\eta_1\alpha_1 - 4\beta_1^2 - 2\eta_2\alpha_1 + 4\beta_1\beta_2 + \eta_3\alpha_1 + \beta_2^2]$, $q_2 = \frac{1}{2}[\eta_3\alpha_1 + \beta_2^2 - \eta_2\alpha_1 + 2\beta_1\beta_2]$, $q_3 = \frac{1}{4}[\eta_3\alpha_1]$, $q_4 = \frac{1}{4}\beta_2^2$.

For $c \in [0,2]$, applying triangle inequality again, we have

$|4 - c_1^2| \leq 4 + |c_1^2| = (4 + c^2) \geq 0$, and $|4 - c_1^2|^2 \leq (4 + c^2)^2$, thus

$$\begin{aligned}|\delta_2\delta_4 - \delta_3^2| &\leq |q_1|c^4 + |q_2|\rho c^2(4+c^2) + 2|q_3|c(4+c^2) + |q_3|c\rho^2(4+c^2)(c+2) + |q_4|\rho^2(4+c^2)^2 \\ &= V(c,\rho),\end{aligned}$$

Proceeding as in Theorem 6.1, we get $V'(\rho) > 0$. Thus $V(c,\rho)$ attains the maximum value at $\rho = 1$. Hence

$$V(c,\rho) \leq V(c,1) = U(c) = |q_1|c^4 + |q_2|c^2(4+c^2) + 2|q_3|c(4+c^2) + |q_3|c(4+c^2)(c+2) + |q_4|(4+c^2)^2.$$

$U'(c) \geq 0$ implies $U(c)$ is an increasing function and it attains the maximum value at $c = 2$. Therefore,

$$\begin{aligned}|\delta_2\delta_4 - \delta_3^2| &\leq 16|q_1| + 32|q_2| + 96|q_3| + 64|q_4| \\ &\leq 16(\eta_1\alpha_1 - \beta_1^2) + 24(2\beta_1\beta_2 - \eta_2\alpha_1) + 44\eta_3\alpha_1 + 36\beta_2^2.\end{aligned}$$

□

Corollary 7.2. *If* $\vartheta = 1$, *then* $|\mathbb{H}_2(2)| \leq 16(D_1A_1 - B_1^2) + 48B_1B_2 + 20D_2A_1 + 36B_2^2$, *where* A_1, B_1, B_2 *are defined in Corollary 6.2 and* $D_1 = \frac{5}{48t_4\Upsilon_4}$, $D_2 = \frac{1}{2t_4\Upsilon_4}$.

8. Conclusion

Toeplitz and Hankel matrices are open to a broad variety of diverse algorithms and offer some of the most appealing computational features. For a given analytic function $\mathbb{U}(z)$ belonging to the class $\Psi_\vartheta(m,\lambda)$, for $\mathbb{U}(z) \neq 0$ for all $z \in \Delta$, $m \in \mathbb{N}_0$, $\lambda \in \mathbb{N}$, $\vartheta \in \mathbb{N}$ defined on the open unit disc Δ, we have obtained sharp estimates of coefficient inequalities and the Fekete-Szegő inequality. Also, using Poisson distribution series, we have obtained sharp coefficient inequalities, the Fekete-Szegő inequality, sharp second-order Toeplitz determinants, upper bounds of third-order Toeplitz determinants, and second-order Hankel determinants in a petal-shaped domain. We anticipate that these findings will have a significant impact on many different types of science, technology, engineering, and mathematics-related sectors.

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