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# Summation process of monotone and sublinear operators in $\mathfrak{B}-$statistical sense

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**Abstract.** By employing the $\mathcal{A}-$summation process in the $\mathfrak{B}-$statistical sense, where $\mathcal{A}$ and $\mathfrak{B}$ are sequences of infinite matrices, we provide new results on the classical Korovkin theorem for a sequence of monotone and sublinear operators. Reported results essentially extend some theorems existing in the literature.

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## 1. Introduction

Approximation theory is closely related to several fields, including the theory of polynomial approximation, functional analysis, summability theory, probability theory, numerical methods for solving differential and integral equations and measure theory.

The classical Korovkin-type theorems are fundamentally focused on the approximation of real-valued functions using positive linear operators (see, e.g., [1, 25]). In fact, this theorem establishes the conditions under which a given sequence of positive linear operators converges to the identity operator in the space of continuous functions on a compact interval. Specifically, if $(U_n)_n$ is a sequence of positive linear operators that map $C([0,1])$ into itself and the sequence $(U_n(f))_n$ converges uniformly to $f$ on $[0,1]$ for the three test functions $1, x$ and $x^2$, then this sequence also converges to $f$ uniformly on $[0,1]$ for every $f \in C([0,1])$ where $C([0,1]) := \{f : f \text{ is continuous on } [0,1]\}$. If the sequence of positive linear operators does not converge to the identity operator, it may be beneficial to apply certain matrix summability techniques (see, e.g., [3, 24]). A considerable number of

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researchers have examined such summability methods for various operators and sequences defined across different spaces [11, 14, 30, 31, 32, 34, 35, 38, 39, 40, 41, 42]. In the classical Korovkin theorem, many classical operators typically converge to the value of the approximated function. However, at discontinuity points, they often converge to the average of the left and right limits of the function. In such instances, Cesàro-type matrix summability methods are useful for correcting the convergence deficiencies [9]. The primary objective of using summability theory has always been to transform a non-convergent sequence into a convergent one. This motivation led to Féjér's renowned theorem, which demonstrates the effectiveness of the Cesàro method in ensuring the convergence of the Fourier series of a continuous periodic function. Matrix summability methods are also applied in the field of physics [10], in ergodic theory [27, 28, 29]. Non-matrix summability methods, such as statistical convergence (see, [15, 16, 36]) have also been studied within the approximation theory (see, e.g. [17, 18, 22]). By employing the $\mathcal{A}-$summation process, Atlıhan and Orhan [4] also present a Korovkin-type result.

Subsequently, the Korovkin theorem has been extended within the framework of monotone and sublinear operators acting on various function spaces, as discussed in [19, 20, 21]. For results related to approximation by sublinear operators, refer to [2]. Gal and Iancu [18] have investigated results on the Korovkin theorem using the concept of statistical convergence for a sequence of monotone and sublinear operators (see, also [22]). At present, related studies can be found in [12, 13, 43]. In this paper, by employing $\mathcal{A}-$summation process in $\mathfrak{B}-$statistical sense, we present two new versions of the Korovkin-type theorem for a sequence of monotone and sublinear operators.

## 2. Preliminaries

Let $X$ be a metric measure space where $X$ is equipped with the metric $d$ and the measure $m$ defined on the sigma field of Borel subsets of $X$.

If we consider the vector lattice $\mathcal{F}(X)$ of all real-valued functions defined on $X$, endowed with the pointwise ordering, then some significant vector sublattices of $\mathcal{F}(X)$ are

$$\mathcal{C}\left(X\right)=\left\{f\in\mathcal{F}(X): f \text{ is bounded and continuous}\right\},$$

and

$$\mathcal{AC}_b(X)=\left\{f\in\mathcal{F}(X): f \text{ is bounded and almost everywhere continuous}\right\}.$$

These spaces are equipped with the supremum norm, defined as $\|f\|=\sup_{x\in X}|f(x)|.$

As in [18], let $X$ and $Y$ be two metric spaces, and $E$ and $F$ two ordered vector subspaces (or the positive cones) of $\mathcal{F}(X)$ and $\mathcal{F}(Y)$, respectively, that contain the unity. An operator $U:E\longrightarrow F$ is called a weakly nonlinear operator (or a weakly nonlinear functional when $F=\mathbb{R}$) if it satisfies the following three conditions:

• (Sublinearity) $U$ is subadditive and positively homogeneous, i.e.,

$$U\left(f+g\right)\leq U\left(f\right)+U\left(g\right) \text{ and } U\left(\alpha f\right)=\alpha U\left(f\right),$$

for all $f, g$ in $E$ and $\alpha \geq 0$.
• (Monotonicity) $f \leq g$ in $E$ implies $U(f) \leq U(g)$.
• (Translatability) $U(f + \alpha.1) \leq U(f) + \alpha U(1)$ for all $f \in E$ and $\alpha \geq 0$.
• (Subunital property) $U(1) \leq 1$

If $E$ and $F$ are closed vector sublattices of the Banach lattices $\mathcal{C}(X)$ and $\mathcal{C}(Y)$, respectively, then any monotone and subadditive operator (or functional when $F = \mathbb{R}$) $U : E \longrightarrow F$ satisfies the following inequality

$$|U(f) - U(g)| \leq U(|f - g|) \text{ for all } f, g.$$

Before proceeding to our main results we present below certain basic definitions, notations and results which will be needed from $\mathfrak{B}-$summability [8, 37].

Let $\mathfrak{B} = (\mathfrak{B}_i)$ be a sequence of infinite matrices with $\mathfrak{B}_i = (b_{nk}^{(i)})$. Then the sequence $x = (x_k)$ is $\mathfrak{B}-$summable to the value $L$ if

$$\lim_n (\mathfrak{B}_i x)_n = \lim_n \sum_k b_{nk}^{(i)} x_k = L, \text{ uniformly in } i.$$

The method $\mathfrak{B}$ is called $m-$multiplicative if each convergent sequence $x$ is $\mathfrak{B}-$summable and there exists a $m \in \mathbb{R}$ such that $\lim_n (\mathfrak{B}_i x)_n = m \lim_n x_n$, uniformly in $i$ [7, 8, 37].
Let us recall the following theorem from Bell's Ph.D. Thesis [7] and from [37] that characterizes $m-$multiplicativity and regularity.

**Theorem 2.1** ([7, 37]). *The method $\mathfrak{B}$ is $m-$multiplicative if and only if*
*(i) $\lim_n b_{nk}^{(i)} = 0$ for all $k \in \mathbb{N}$, uniformly in $i$,*
*(ii) $\lim_n \sum_k b_{nk}^{(i)} = m$, uniformly in $i$,*
*(iii) $\sum_k \left| b_{nk}^{(i)} \right| < \infty$ for all $n, i \in \mathbb{N}$, and there exists $N \in \mathbb{N}$ such that $\sup_{i \in \mathbb{N}, n > N} \sum_k \left| b_{nk}^{(i)} \right| < \infty$.*

When $m = 1$, the method $\mathfrak{B}$ is called regular [7, 8, 37].

A set $D := \{k_1 < k_2 < ... < k_p < ...\} \subset \mathbb{N}$ is said to have $\mathfrak{B}-$density $\delta_{\mathfrak{B}}(D)$ equal to $d$, if the characteristic sequence $\chi_D$ of $D$ is $\mathfrak{B}-$summable to $d$, i.e.

$$\lim_n \sum_k b_{nk}^{(i)} \chi_D(k) = \lim_n \sum_{k \in D} b_{nk}^{(i)} = d, \text{ uniformly in } i,$$

[8, 23].

By $\mathcal{R}^+$ we denote the set of all regular methods $\mathfrak{B}$ with $b_{nk}^{(i)} \geq 0$ for all $n, k$ and $i$.

Let $\mathfrak{B} \in \mathcal{R}^+$. A sequence $x = (x_k)$ is called $\mathfrak{B}-$statistically convergent to $L$ provided that, for every $\varepsilon > 0$, $\delta_{\mathfrak{B}}(\{k : |x_k - L| \geq \varepsilon\}) = 0$ holds [23]. In this case we write $st_{\mathfrak{B}} - \lim x_k = L$.

Now, let $\mathcal{A} := \left(A^{(n)}\right) = \left(a_{kj}^{(n)}\right)$ be a sequence of infinite matrices with nonnegative real entries.

A sequence $(U_j)$ of monotone and sublinear operators of $\mathcal{C}(X)$ into itself is called an $\mathcal{A}-$Summation Process in $\mathfrak{B}-$statistical sense on $\mathcal{C}(X)$ if $(U_j(f))$ is $\mathcal{A}-$summable to $f$ in $\mathfrak{B}-$statistical sense for every $f \in \mathcal{C}(X)$, i.e.,

$$st_{\mathfrak{B}} - \lim_{k} \left\| \sum_{j=1}^{\infty} a_{k,j}^{(n)} U_j(f) - f \right\| = 0, \text{ uniformly in } n, \tag{2.1}$$

for every $f \in \mathcal{C}(X)$ where it is assumed that the series in (2.1) converges for each $k, n$ and $f$.

In particular, when $A^{(n)} = A$ for some matrix $A$, $\mathcal{A}-$summability coincides with the ordinary matrix summability by $A$.

Consider the matrices $(a_{kj}^{(n)})$ by

$$(a_{kj}^{(n)}) = \begin{cases} \frac{1}{k}, & \text{if } 1+n \leq j \leq k+n, \\ 0, & \text{otherwise.} \end{cases}$$

In this special case, $\mathcal{A}-$summability coincides with almost convergence (see [26]) (or uniform statistical convergence [5, 33]) which is a well-known example of a non-matrix method of summability.

Let $\{U_j\}$ be a sequence of monotone and sublinear operators such that for each $k, n \in \mathbb{N}$

$$\sum_{j=1}^{\infty} a_{k,j}^{(n)} |U_j(1)| < \infty. \tag{2.2}$$

For each $k, n \in \mathbb{N}$ and $f \in E \cap \mathcal{AC}_b(X)$, let

$$B_k^{(n)}(f)(x) := \sum_{j=1}^{\infty} a_{k,j}^{(n)} U_j(f)(x),$$

which is well defined by (2.2).

**Remark 2.2.** If each operator $U_j$ is monotone, sublinear and translatable, then $B_k^{(n)}$ is also monotone, sublinear and translatable for each $n$ and $k$, respectively.

## 3. Main results

In recent studies, Gal and Iancu [18] have investigated certain Korovkin-type results for sequences of monotone and sublinear operators within the framework of statistical convergence. Motivated by their work, we employ the $\mathcal{A}-$summation process in $\mathfrak{B}-$statistical sense to establish two novel versions of the Korovkin-type theorem for these operators.

**Theorem 3.1.** *Let* $\mathfrak{B} \in \mathcal{R}^+$. *Assume that* $X$ *is a locally compact subset of* $\mathbb{R}$, *and* $E$ *is a vector sublattice of* $\mathcal{F}(X)$ *that contains the four test functions:* $1, x, -x, x^2$. *Let* $\mathcal{A} := \{A^{(n)}\}$ *be a sequence of infinite matrices with nonnegative real entries.*

$(i)$ If $(U_j)$ is a sequence of monotone and sublinear operators from $E$ into itself such that

$$st_{\mathfrak{B}} - \lim_k B_k^{(n)}(f) = f \text{ a.e. and uniformly in } n, \tag{3.1}$$

for each of the four test functions: $1, x, -x, x^2$, and (2.2) holds, then $(U_j)$ is an $\mathcal{A}-$summation process in $\mathfrak{B}-$statistical sense on $E \cap \mathcal{AC}_b(X)$, i.e., (3.1) holds for all nonnegative functions $f$ in $E \cap \mathcal{AC}_b(X)$.

$(ii)$ If, in addition to the hypotheses of $(i)$, each operator $U_j$ is translatable, then $st_{\mathfrak{B}} - \lim_k B_k^{(n)}(f) = f$ a.e. and uniformly in $n$ for all $f \in E \cap \mathcal{AC}_b(X)$.

*Proof.* $(i)$ Let $f \in E \cap \mathcal{AC}_b(X)$ such that $f \geq 0$. Suppose that $s$ is a continuity point of $f$ and also satisfies the condition

$$st_{\mathfrak{B}} - \lim_k B_k^{(n)}(h)(s) = h\left(s\right), \text{ uniformly in } n, \tag{3.2}$$

for each of the test functions $h \in \left\{1, x, -x, x^2\right\}$.

Then, for an arbitrarily fixed $\varepsilon > 0$, there exists $\delta > 0$ such that

$$\left|f\left(x\right) - f\left(s\right)\right| \leq \varepsilon \text{ for every } x \in X \text{ satisfying } \left|x - s\right| \leq \delta.$$

Moreover, if $|x - s| \geq \delta$, then

$$\left|f\left(x\right) - f\left(s\right)\right| \leq \frac{2\left\|f\right\|}{\delta^2}\left|x - s\right|^2.$$

As a result, we obtain

$$\left|f\left(x\right) - f\left(s\right)\right| \leq \varepsilon + \frac{2\left\|f\right\|}{\delta^2}\left|x - s\right|^2, \tag{3.3}$$

for all $x \in X$.

Putting $M := \max\left\{0, s\right\}$, we can write (3.3) as

$$\left|f\left(x\right) - f\left(s\right)\right| \leq \varepsilon + \frac{2\left\|f\right\|}{\delta^2}\left[x^2 + 2x(M - s) + 2M(-x) + \left|s\right|^2\right].$$

Utilizing this inequality along with the monotonicity and sublinearity of the operators $U_j$, we deduce that

$$\begin{aligned}
\left|B_k^{(n)}(f)(s) - f(s)\right| &\leq \left|B_k^{(n)}(f)(s) - B_k^{(n)}\left(f(s)\right)(s) + f(s)B_k^{(n)}(1)(s) - f(s)\right| \\
&\leq B_k^{(n)}(|f - f(s)|)(s) + f(s)\left|B_k^{(n)}(1)(s) - 1\right| \\
&\leq \varepsilon + \frac{2\left\|f\right\|}{\delta^2}\left[B_k^{(n)}(t^2)(s) + 2(M - s)B_k^{(n)}(t)(s)\right. \qquad (3.4) \\
&\quad \left. + 2MB_k^{(n)}(-t)(s) + \left|s\right|^2 B_k^{(n)}(1)(s)\right] + f(s)\left|B_k^{(n)}(1) - 1\right| \\
&\leq \varepsilon + C\left[\left|B_k^{(n)}(t^2)(s) + 2(M - s)B_k^{(n)}(t)(s)\right.\right. \\
&\quad \left.\left. + 2MB_k^{(n)}(-t)(s) + \left|s\right|^2 B_k^{(n)}(1)(s)\right| + \left|B_k^{(n)}(1)(s) - 1\right|\right],
\end{aligned}$$

for each $n, k \in \mathbb{N}$, where $C = \max\left\{\frac{2\|f\|}{\delta^2}, f(s)\right\}$.

Given $r > 0$, choose $\varepsilon > 0$ such that $\varepsilon < r$. We now define the following sets:

$D := \left\{k \in \mathbb{N} : \sup_n \left|B_k^{(n)}(f)(s) - f(s)\right| \geq \frac{r-\varepsilon}{C} \right\},$
$D_1 := \Big\{k \in \mathbb{N} : \sup_n \Big|B_k^{(n)}(t^2)(s) + 2(M-s)B_k^{(n)}(t)(s)$
$+2MB_k^{(n)}(-t)(s) + |s|^2 B_k^{(n)}(1)(s)\Big| \geq \frac{r-\varepsilon}{2C}\Big\},$
and
$D_2 := \left\{k \in \mathbb{N} : \sup_n \left|B_k^{(n)}(1)(s) - 1\right| \geq \frac{r-\varepsilon}{2C} \right\}.$
From (3.4), it follows that

$$D \subset D_1 \cup D_2,$$

which implies

$$\sum_{k\in D} b_{mk}^{(i)} \leq \sum_{k\in D_1} b_{mk}^{(i)} + \sum_{k\in D_2} b_{mk}^{(i)},$$

for all $m \in \mathbb{N}$.

By letting $m \to \infty$ and using (3.2), we conclude that $\delta_{\mathfrak{B}}(D) = 0$, i.e.,

$$st_{\mathfrak{B}} - \lim B_k^{(n)}(f)(s) = f(s), \text{ uniformly in } n,$$

as desired.

$(ii)$ In addition to the hypotheses of $(i)$, assume that each operator $U_j$ is translatable. Then $B_k^{(n)}$ is also translatable. From $(i)$, we obtain that

$$st_{\mathfrak{B}} - \lim B_k^{(n)}(f + \|f\|) = f + \|f\|\,, \text{ a.e. uniformly in } n.$$

Since $B_k^{(n)}$ is translatable, we have $B_k^{(n)}(f + \|f\|) = B_k^{(n)}(f) + \|f\|\, B_k^{(n)}(1)$, which yields that $st_{\mathfrak{B}} - \lim B_k^{(n)}(f) = f$, a.e. and uniformly in $n$. □

Before presenting the next result, let us first introduce the concept of $\mathfrak{B}-$statistical convergence in the measure (see also [6]).

**Definition 3.2.** *Let* $\mathfrak{B}\in \mathcal{R}^+$ *and let* $g_n$, $g$ *be measurable functions on* $[0,1]$. *We say that the sequence* $(g_n)$ *converges* $\mathfrak{B}-$*statistically in the measure* $m$ *to* $g$ *if and only if for all* $\varepsilon, \eta > 0$,

$$\delta_{\mathfrak{B}}\left(\{n \in \mathbb{N} : m\left(\{x \in [0,1] : |g_n(x) - g(x)| \geq \varepsilon\}\right) \geq \eta\}\right) = 0.$$

We are now ready to provide the other main result.

**Theorem 3.3.** *Let* $\mathfrak{B}\in \mathcal{R}^+$, *and let* $\mathcal{A} := \left\{A^{(n)}\right\}$ *be a sequence of infinite matrices with nonnegative real entries, and let* $C([0,1])$ *represent the vector lattice of all continuous functions on* $[0,1]$. *Let* $(U_j)$ *be a sequence of monotone, subunital and sublinear operators from* $C([0,1])$ *into itself such that the following condition holds*

$$st_{\mathfrak{B}} - \lim B_k^{(n)}(f) = f \textit{ in measure and uniformly in } n,$$

*for each of the four test functions* $h \in \left\{1, x, -x, x^2\right\}$, *and let (2.2) hold. Then,* $\mathfrak{B}-$*statistical convergence holds for all nonnegative functions* $h \in C([0,1])$. *If all the operators* $U_j$ *are also translatable, then* $\mathfrak{B}-$*statistical convergence holds for all* $f \in C([0,1])$.

*Proof.* Let $\varepsilon > 0$ and let $f$ be non-negative. Due to the uniform continuity of $f$, we can choose a sufficiently small $\varepsilon_0 > 0$ such that $\varepsilon_0 < \varepsilon$. Then, there exists a $\delta_0 > 0$, such that the following condition holds

$$\left|f(t) - f(x)\right| \leq \varepsilon_0 + 2\left\|f\right\| \frac{(t-x)^2}{\delta_0^2}, \text{ for all } t, x \in [0,1],$$

where $\|f\| = \sup_{x\in[0,1]} |f(x)|$. Therefore, we have

$$\begin{aligned}\left|B_k^{(n)}(f)(x) - f(x)\right| &\leq \varepsilon_0 + \frac{2\|f\|}{\delta_0^2} B_k^{(n)}((t-x)^2)(x) \\ &\leq \varepsilon_0 + \frac{2\|f\|}{\delta_0^2}(B_k^{(n)}(e_2)(x) + 2xB_k^{(n)}(-e_1)(x) + x^2).\end{aligned}$$

This leads to the following inclusions:

$$\begin{aligned} &\left\{x \in [0,1] : \left|B_k^{(n)}(f)(x) - f(x)\right| \geq \varepsilon\right\} \\ \subset\ &\left\{x \in [0,1] : \varepsilon_0 + \tfrac{2\|f\|}{\delta_0^2}(B_k^{(n)}(e_2)(x) + 2xB_k^{(n)}(-e_1)(x) + x^2) \geq \varepsilon\right\} \\ =\ &\left\{x \in [0,1] : B_k^{(n)}(e_2)(x) + 2xB_k^{(n)}(-e_1)(x) + x^2 \geq (\varepsilon - \varepsilon_0)\tfrac{\delta_0^2}{2\|f\|}\right\} \\ =\ &\left\{x \in [0,1] : \left(B_k^{(n)}(e_2)(x) - x^2\right) + 2x\left(B_k^{(n)}(-e_1)(x) + x\right) \geq (\varepsilon - \varepsilon_0)\tfrac{\delta_0^2}{2\|f\|}\right\} \\ \subset\ &\left\{x \in [0,1] : \left|B_k^{(n)}(e_2)(x) - x^2\right| \geq (\varepsilon - \varepsilon_0)\tfrac{\delta_0^2}{4\|f\|}\right\} \\ &\cup\left\{x \in [0,1] : \left|B_k^{(n)}(-e_1)(x) + x\right| \geq (\varepsilon - \varepsilon_0)\tfrac{\delta_0^2}{4\|f\|}\right\}.\end{aligned}$$

From this, we deduce the following inequality:

$$\begin{aligned} &m\left(\left\{x \in [0,1] : \left|B_k^{(n)}(f)(x) - f(x)\right| \geq \varepsilon\right\}\right) \\ \leq\ &m\left(\left\{x \in [0,1] : \left|B_k^{(n)}(e_2)(x) - x^2\right| \geq (\varepsilon - \varepsilon_0)\tfrac{\delta_0^2}{4\|f\|}\right\}\right) \\ &+m\left(\left\{x \in [0,1] : \left|B_k^{(n)}(-e_1)(x) + x\right| \geq (\varepsilon - \varepsilon_0)\tfrac{\delta_0^2}{4\|f\|}\right\}\right).\end{aligned}$$

For any $\eta > 0$, define the sets as

$$D := \left\{k \in \mathbb{N} : m\left(\left\{x \in [0,1] : \sup_n \left|B_k^{(n)}(f)(x) - f(x)\right| \geq \varepsilon\right\}\right) \geq \eta\right\}$$

$$D_1 := \left\{k \in \mathbb{N} : m\left(\left\{x \in [0,1] : \sup_n \left|B_k^{(n)}(e_2)(x) - x^2\right| \geq (\varepsilon - \varepsilon_0)\tfrac{\delta_0^2}{4\|f\|}\right\}\right) \geq \tfrac{\eta}{2}\right\}$$

and

$$D_2 := \left\{ k \in \mathbb{N} : m \left( \left\{ x \in [0,1] : \sup_n \left| B_k^{(n)}(-e_1)(x) + x \right| \geq (\varepsilon - \varepsilon_0) \frac{\delta_0^2}{4\|f\|} \right\} \right) \geq \frac{\eta}{2} \right\}.$$

It is evident that $D \subset D_1 \cup D_2$.

Recall that $\varepsilon > \varepsilon_0$. Now, let $\gamma = (\varepsilon - \varepsilon_0)\frac{\delta_0^2}{4\|f\|}$, and since $\varepsilon - \varepsilon_0$ can be chosen arbitrarily close to 0, $\gamma > 0$ can also be made arbitrarily close to 0.

The remainder of the proof proceeds as in the proof of Theorem 3.1, and we omit it here.

Finally, assume the operators $U_j$ are translatable too and $f \in C([0,1])$. Since $f + \|f\| \geq 0$, by the first part of the proof, we conclude that $st_{\mathfrak{B}} - \lim B_k^{(n)}(f + \|f\|) = f + \|f\|$ in measure and uniformly in $n$. Additionally, since $B_k^{(n)}(f + \|f\|) = B_k^{(n)}(f) + \|f\|$, it follows that $st_{\mathfrak{B}} - \lim B_k^{(n)}(f) = f$ in measure and uniformly in $n$. □

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