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The influence of the change of the longitudinal modulus of elasticity on the dynamic behavior of a Warren truss

Zeno-Iosif Praisach*, Dan Alexandru Pîrșan

Abstract. *Trusses are commonly used structures and they are present in the most constructions because it use a small amount of material and for the loads they can support. The paper presents the dynamic behavior of a Warren truss with seven elements when for one member of the truss the modulus of elasticity is much different compared to the other elements. The relative frequency shift for the first seven vibration mode are calculated taking into consideration that the slenderness ratio has a constant value and the transverse section of the elements is changed. The conclusions reveal that by maintaining the constant value of a slenderness ratio, the relative frequency shift is not modified for the considered transverse sections of the elements of the Warren truss.*

Keywords: *Warren truss, slenderness ratio, modal analysis, relative frequency shift*

1. Introduction

Trusses are simple structures whose members are subject to axial compression and tension only [1]. Warren truss comprise of a series of equilateral triangles, that means that every member has the same length. In a truss, all the elements are pinned (hinged) at both ends.

A Warren truss is to be designed to achieve a definite set of natural frequencies to avoid resonance, or to provide study materials for certain critical computation of vibration instrumentation [2-3].

To calculate the natural frequency analysis assumes that a structure vibrates in the absence of excitation and damping [4].

The natural frequency depends upon stiffness and mass distributions, boundary conditions [5].

In this paper, the authors approach the numerical modal analysis for a single-span Warren truss with seven elements by calculating the natural frequencies and comparing them for the same structure, but in which for one element the elasticity modulus has been deliberately changed, considering this element as weakened or damaged.

2. Analytic approach

A single-span Warren truss with seven elements is illustrated in figure 1.

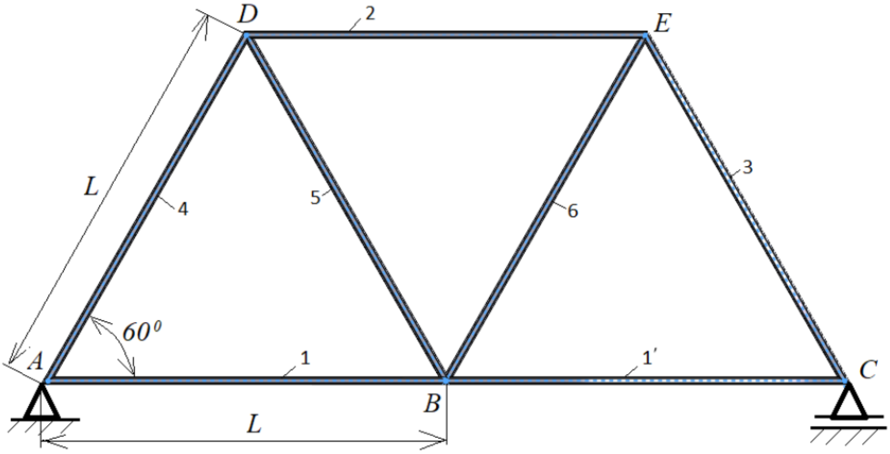


Figure 1. Warren truss with seven elements

It is known from the literature that in order to determine the longitudinal natural frequencies, the structural elements can have the following contour conditions:

- fixed at one end and free to the other with axial displacement *u(x)*:

$$u(x) = B\sin(k_L x) \tag{1}$$

where,

B is the integration coefficient,

k_L (2) are the eigenvalues for the considered natural frequencies (f_n) which can be computed with relation (3):

$$k_L = \frac{(2n-1)\pi}{2L} \tag{2}$$

$$f_n = \frac{(2n-1)\pi}{4L} \cdot \sqrt{\frac{E}{\rho}} \tag{3}$$

with,

L [m] is the length of the element,

E [N/m^2] is the modulus of elasticity,

ρ [kg/m^3] is the mass density of the element.

- free at both ends with axial displacement $u(x)$:

$$u(x) = A\cos(k_L x) \tag{4}$$

where,

B is the integration coefficient,

$$k_L = \frac{n \cdot \pi}{L} \tag{5}$$

are the eigenvalues for the considered natural frequencies (f_n) which can be computed with relation (6):

$$f_n = \frac{n}{2L} \cdot \sqrt{\frac{E}{\rho}} \tag{6}$$

- fixed at both ends with axial displacement given with (1) and k_L with (4);
- fixed at one end and free to the other with a mass. The axial displacement is given with (1) and the characteristic equation which permit us to calculate the eigenvalues is:

$$\cos(k_L \cdot L) - \frac{M}{m}(k_L \cdot L)\sin(k_L \cdot L) = 0 \tag{7}$$

here,

M [kg] is the mass at the free end,

m [kg] is the mass of the structural element.

For this case, the natural frequencies are given with (8):

$$f_n = \frac{k_L}{2\pi} \cdot \sqrt{\frac{E}{\rho}} \tag{8}$$

3. Numerical investigations

For the numerical investigations it was considered the Warren truss presented in figure 1. All the elements have the same cross section. By using FEM analysis it was obtained the first seven natural frequencies ($n=1, 2, \ldots, 7$). The SolidWorks software were used to calculate the natural frequencies. By using this software and analyzing the structure as a truss, structural elements 1 and 1' will have the same material type.

The following cases were analyzed:

- all the elements are of the same material, structural steel with $E=2.1\cdot10^{11}$ N/m^2 and all the elements are the same. The natural frequencies f_U were determined;

- for each structural elements only the modulus of elasticity was changed to value of $E_1=1.6\cdot10^{11}$ N/m^2 considering it as a weakened or damaged structural element, and the natural frequencies f_D were determined;
- the natural frequencies obtained by modifying each structural element were compared with the natural frequencies obtained in the case of the structure made of the same material. The relative frequencies shift $\varepsilon = \frac{f_U - f_D}{f_U} \cdot 100$ [%] were calculated;
- for the cases mentioned above, three types of structural elements were considered, as follows: rectangular tube 120x80x8 mm; pipe 33,7x4 mm and square tube 80x80x5 mm;
- for these three types of structural elements, the length of the structural element was calculated so that the slenderness coefficient has the same value of: $i = L \cdot \sqrt{\frac{A}{I}} = 0.01$, where A [m^2] is the cross section area and I [m^4] is the moment of inertia.

The results are presented in the tables below. In the first column, are indicated the truss bars for which the modulus of elasticity was reduced to E_1.

- Warren truss with truss bars of rectangular tube 120x80x8 mm with:
    - $L=4265$ mm,
    - $A=2779.19$ mm^2,
    - $I=4957718.55$ mm^4.

The natural frequencies for the first seven vibration modes ($f_1, f_2, \ldots, f_7$) are presented in table 1.

Table 1. Natural frequencies for Warren truss with rectangular tube 120x80x8 mm.

Weakened element	f_1 [Hz]	f_2 [Hz]	f_3 [Hz]	f_4 [Hz]		f_5 [Hz]	f_6 [Hz]	f_7 [Hz]
-	67.69	90.502	158.67	177.26		227.76	263.57	284.78
1	61.595	89.193	153.900	169.12		226.43	261.45	265.13
2	67.142	88.236	157.64	176.55		218.49	250.91	284.43
3	65.569	89.817	151.05	175.09		224.09	263.54	279.48
4	67.568	84.713	154.62	174.4		222.02	263.46	284.76
5	67.645	87.289	157.59	173.81		217.37	258.01	283.43
6	66.632	90.501	152.52	171.4		226.36	254.77	282.81

- Warren truss with structural elements of pipe 33.7x4 mm with:
    - o $L=1070$ mm,
    - o $A=373.22$ mm^2,
    - o $I=41898.28$ mm^4.

Table 2 contains the natural frequencies for the first seven vibration modes.

Table 2. Natural frequencies for Warren truss with pipe 33.7x4 mm.

Weakened element	f_1 [Hz]	f_2 [Hz]	f_3 [Hz]	f_4 [Hz]	f_5 [Hz]	f_6 [Hz]	f_7 [Hz]
-	269.48	360.44	632.67	705.58	907.74	1050.4	1133.7
1-1’	245.21	355.12	613.75	673.07	902.42	1041.1	1056.6
2	267.27	351.45	628.55	702.74	870.97	999.87	1132.2
3	261.02	357.73	602.26	697.08	893.17	1050.3	1112.5
4	268.97	337.44	616.46	694.28	884.82	1050.0	1133.6
5	269.31	347.63	628.39	692.0	866.09	1028.4	1128.2
6	265.3	360.44	608.04	682.31	902.16	1015.2	1126.0

- Warren truss with structural elements of square tube 80x80x5 mm with:
    - o $L=3055$ mm,
    - o $A=1435.62$ mm^2,
    - o $I=1314420.61$ mm^4.

The natural frequencies for the first seven vibration modes are presented in table 3.

Table 3. Natural frequencies for Warren truss with square tube 80x80x5 mm.

Weakened element	f_1 [Hz]	f_2 [Hz]	f_3 [Hz]	f_4 [Hz]	f_5 [Hz]	f_6 [Hz]	f_7 [Hz]
-	94.501	126.35	221.51	247.46	317.97	369.96	397.57
1-1’	85.991	124.51	214.86	236.1	316.11	365.01	370.14
2	93.735	123.18	220.08	246.48	305.02	350.29	397.09
3	91.539	125.39	210.88	244.45	312.85	367.93	390.18
4	94.329	118.27	215.86	243.48	309.95	367.84	397.55
5	94.439	121.86	220.01	242.65	303.47	360.21	395.69
6	93.023	126.35	212.93	239.28	316.01	355.68	394.82

Table 4. The relative frequencies shift (ε_n) for n = 1, 2, .. 7 vibration modes.

Weakened element	ε_1 [%]	ε_2 [%]	ε_3 [%]	ε_4 [%]	ε_5 [%]	ε_6 [%]	ε_7 [%]
1-1’	9.0	1.46	3.0	4.6	0.58	0.8	6.9
2	0.8	2.5	0.7	0.4	4.1	4.8	0.1
3	3.1	0.8	4.8	1.2	1.6	0.0	1.9
4	0.2	6.4	2.6	1.6	2.5	0.0	0.0
5	0.1	3.6	0.7	1.9	4.6	2.1	0.5
6	1.6	0.0	3.9	3.3	0.6	3.3	0.7

The relative frequencies shift (ε) for the cases analyzed above are presented in the table 4 and illustrated in figure 2 for the weakened truss bar 1, 2, ..., 6.

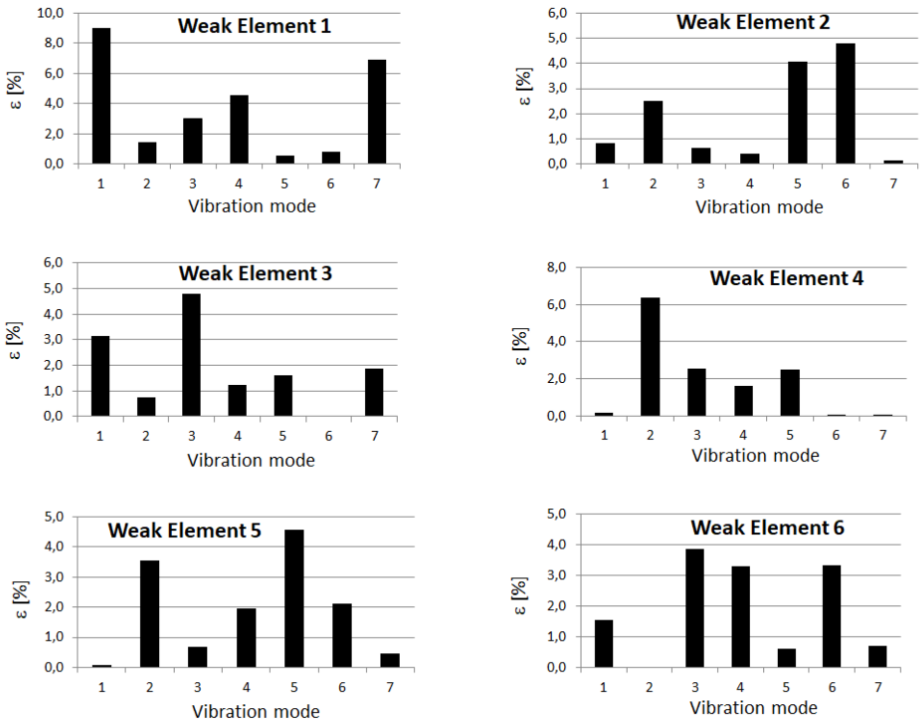
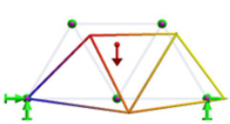
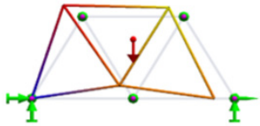
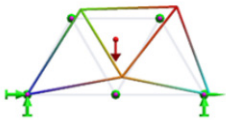
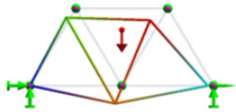
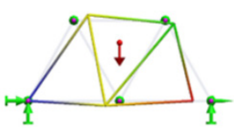
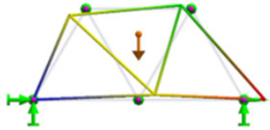
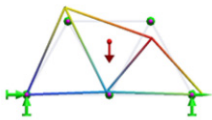
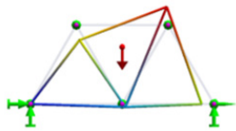
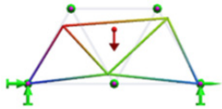
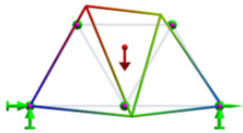
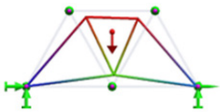
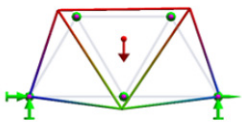
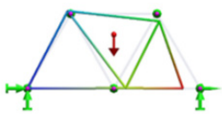
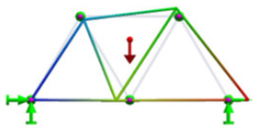


Figure 2. Relative frequencies shift for the first 7 vibration modes.

The mode shapes for the first seven vibration modes are illustrated in table 5.

Table 5. The first seven mode shapes for a single-span Warren truss

Vibration mode	Mode shape
1	 
2	 
3	 
4	 
5	 
6	 
7	 

4. Conclusions

The paper analyzed the dynamic behavior for a Warren truss in case an element is weakened by the forced modification of the modulus of elasticity. The material considered in the modal analysis is structural steel with $E=2.1\cdot10^{11}$ N/m^2, and the weakened structural element has the modulus of elasticity $E_1=1.6\cdot10^{11}$ N/m^2.

The modal analysis for three different cross-sections of the structural element, respectively rectangular tube 120x80x8 mm with *L=4265* mm, pipe 33.7x4 mm with *L=1070* mm and square tube 80x80x5 mm with *L=3055* mm, with the constant slenderness ratio *i=0.01*, revealed the fact that the relative frequency shift ε (table 4), for each individual vibration mode, does not change.

This observation leads to the conclusion that if an element of the Warren truss is weakened, from the point of view of dynamic behavior, respectively by monitoring the natural frequencies, recognition patterns and identification of the weakened element can be obtained by drawing diagrams according to figure 2.

The mode shapes (table 5) obtained from the modal analysis have the same representation and do not take into account the change in the modulus of elasticity of the structural element in the Warren truss component.

References

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Addresses:

- Lect. Dr. Eng. Habil. Zeno-Iosif Praisach, Babeș-Bolyai University, Department of Engineering Science, Cluj-Napoca, Romania, zeno.praisach@ubbcluj.ro (**corresponding author*)
- PhD. Stud. Dan Alexandru Pirsan, Babeș-Bolyai University, Department of Engineering Science, Cluj-Napoca, Romania, dan.pirsan@ubbcluj.ro