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Comparative Evaluation of Frequency-Based Indicators for Crack Detection

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Abstract. *This paper compares three frequency-based damage indicators: the absolute frequency, the frequency difference, and the relative frequency shift (RFS). The indicators, calculated using mathematical relations developed by the authors, are used to create a database, and an artificial neural network (ANN) is trained on them. This network is used to assess cracks at various locations and with different depths. The frequencies obtained for these cracks are from ANSYS simulations. A comparison of the indicators is made to identify the most sensitive one. It was found that both the relative frequency shift and the frequency difference provide reliable and accurate crack position detection.*

Keywords*: Crack, Natural Frequency, Artificial Neural Network, Simulated Data, Sensitivity.*

1. Introduction

Frequency-based indicators are widely used in vibration-based structural health monitoring (SHM) because they are easy to acquire, robust to noise, and the acquisition equipment is simple and adequate for real-world measurements [1-3]. Still, their ability to detect early-stage cracks depends on how the frequency data is analyzed and compared. Small cracks can cause slight, mode-specific frequency changes that may be hidden by operational or environmental factors, such as shifts in boundary conditions, added mass, or temperature changes [4,5].

Cracks lower the local stiffness of beams and plates, making them more flexible and usually causing the natural frequencies of the affected modes to drop [6,7]. Therefore, the core of vibration-based crack detection is the link between local damage and frequency changes. There are several indicators based on changes in frequency due to damage and derived features, but neither a clear description of these indicators nor a systematic analysis of their effectiveness is provided in the literature.

Using absolute frequency alone is intuitive, but it can be affected by other factors that cause shifts similar to those caused by early damage [1,4]. The frequency difference, which is the deviation of the frequencies measured on a damaged beam from the baseline (i.e., the frequencies of the healthy beam), removes fixed offsets and is easy to interpret but is still sensitive to operational and environmental variability, such as changes in boundary conditions, mass loads, and temperature [2,5]. The relative frequency shift, defined as the normalized frequency difference relative to the baseline frequency, improves cross-mode comparability and ensures a good depiction of crack parameters [8].

This paper comparatively evaluates absolute frequency, frequency difference, and relative frequency shift arising from small to moderate cracks, quantifies performance in crack assessment using sensitivity indices, and discusses mode selection. The goal is to determine which indicator is most suitable for robust crack detection.

2. Typical Damage Indicators

For crack assessment, three closely related indicators are most common: absolute frequency, frequency difference, and relative frequency shift. Below, each indicator is defined, followed by practical strengths, limitations, and usage guidance in the presence of environmental and operational variability and mode-tracking issues.

The absolute frequency f_{iD} is defined as the measured natural frequency of the weak-axis bending vibration mode i in the current state. It is always lower than the baseline frequency f_i, which is the frequency of the healthy beam.

The frequency difference Δf_i is defined as the deviation from a healthy baseline. It is obtained by subtracting from the baseline f_i the frequency f_{iD} acquired after damage events; it highlights the changes accumulated after damage.

$$\Delta f_i = f_i - f_{iD} \tag{1}$$

Besides measurements, the frequency difference can be calculated if the location x and the crack depth a are known [9]. The authors derived the following mathematical relation:

$$\Delta f_i(x,a) = f_i - f_{iD}(x,a) = f_i \gamma(a)[\overline{\phi''}(x)]^2 \tag{2}$$

where $\gamma(a)$ is the damage severity and $\overline{\phi''}(x)$ is the normalized modal curvature. The curvature has a well-known relation for beams with different boundary conditions, while the severity can be calculated using the relation presented in [10] or by applying a fracture mechanics procedure [11].

The relative frequency shift $\Delta\overline{f_i}$ is defined as the normalized change relative to the baseline. Normalization promotes scale invariance, improving cross-mode comparability. The expression of the relative frequency shift is:

$$\Delta\overline{f_i} = \frac{f_i - f_{iD}}{f_i} \tag{3}$$

Besides measurements, the relative frequency shift can be calculated, for known damage location and depth, using the mathematical relation:

$$\Delta\overline{f_i}(x,a) = \frac{f_i - f_{iD}(x,a)}{f_i} = \gamma(a)[\overline{\phi''}(x)]^2 \tag{4}$$

An example of calculated data a specific input data (x and a) is presented in Table 1. In total, we generated 10,020 scenarios, including 501 locations and 20 severities. The complete dataset used in this study is publicly available in the project repository in [13]. This data is used to train the ANNs for the three indicator types.

Table 1. Example of damage indicators used to train the ANN.

Mode No.	f_i [Hz]	f_{iD} [Hz]	$\Delta f_i(x,a)$ [Hz]	$\Delta\overline{f_i}(x,a)$ [Hz]	x [mm]	a [mm]
1	4.0434	4.03685	$6.56781\cdot10^{-3}$	$1.62432\cdot10^{-3}$	288	1.045
2	25.3397	25.33116	$8.50084\cdot10^{-3}$	$3.35476\cdot10^{-4}$		
3	70.9518	70.81964	$1.32141\cdot10^{-1}$	$1.86240\cdot10^{-3}$		
4	139.0372	138.90376	$1.33415\cdot10^{-1}$	$9.59564\cdot10^{-4}$		
5	229.8384	229.82609	$1.23516\cdot10^{-2}$	$5.37406\cdot10^{-5}$		
6	343.3388	342.76311	$5.75720\cdot10^{-1}$	$1.67683\cdot10^{-3}$		

3. The ANN architecture and training process

3.1. Dataset Preprocessing

For each damage indicator (absolute frequency, frequency difference, and relative frequency shift), the generated database was combined into a single dataset containing all crack locations and severities. Since the objective of the ANN model is to predict only the crack position, the target variable was defined as the spatial coordinate x, while the selected modal quantities served as input features.

To ensure numerical stability and comparable scaling across modes, all input features were standardized using z-score normalization. For each feature z_i, normalization was performed as:

$$z_i^* = \frac{z_i - \mu_i}{\sigma_i} \quad (5)$$

where μ_i and σ_i denote the mean and standard deviation computed exclusively from the training subset. The same normalization parameters were subsequently applied to the validation and test subsets to prevent data leakage.

The dataset was randomly divided into training (70%), validation (15%), and test (15%) subsets. The training set was used for model parameter optimization, the validation set for hyperparameter tuning and early stopping, and the test set exclusively for final performance evaluation.

This preprocessing procedure was applied independently for each indicator type to ensure a fair comparison while maintaining consistent data handling.

3.2. Feature Analysis

Prior to defining the ANN architecture, a feature relevance analysis was conducted to identify the most informative vibration modes for crack localization. Although six bending modes were available for each damage indicator, not all contribute equally to predicting crack position. Since the implemented multilayer perceptron (MLP) model estimates only the crack location (and not the crack severity), the objective of this analysis was to determine which modal features provide the strongest and most robust spatial information.

For each indicator, the first six vibration modes were initially evaluated. Feature relevance was assessed using complementary metrics designed to capture different aspects of dependency between each modal quantity and crack position.

Pearson correlation was used to quantify linear association between each modal feature and crack position, indicating the strength and direction of a global linear relationship. Spearman correlation was computed to assess monotonic dependence, capturing nonlinear but consistently increasing or decreasing trends that Pearson correlation may not detect.

To identify more general nonlinear relationships, mutual information was calculated. This metric measures the reduction in uncertainty of crack position when a specific modal feature is known, regardless of whether the relationship is linear or monotonic.

Predictive relevance was further evaluated using permutation importance derived from a nonlinear ensemble model. This metric quantifies the decrease in prediction accuracy when the values of a given feature are randomly permuted, thereby reflecting how strongly the model relies on that feature.

Finally, the standalone predictive capability of each mode was assessed using cross-validated R^2 values obtained from single-feature regression models. The R^2 metric represents the proportion of variance in crack position explained by one modal feature alone and serves as a direct indicator of identifiability.

These metrics were selected to ensure that feature selection was not biased toward a single modeling assumption, combining linear, monotonic, nonlinear, and predictive perspectives. All analyses were performed exclusively on the training subset to avoid data leakage.

The results consistently showed that the lower-order modes dominate crack localization performance for all three indicators. Mode 1 exhibited the strongest association and predictive power, followed by Modes 2 and 3. Mode 4 provided additional but smaller contributions, while Modes 5 and 6 showed limited standalone relevance and marginal incremental benefit. This behavior is consistent with beam theory, as lower bending modes are more sensitive to global stiffness reductions caused by cracks.

Based on this analysis, the four most significant modes (Modes 1-4) were selected as input features for the ANN model for each damage indicator. This reduced feature set preserves the dominant spatial information while limiting model complexity and reducing the risk of overfitting. The subsequent section presents the ANN architecture and training procedure used for crack position prediction.

3.3. ANN Architecture and Training Procedure

A multilayer perceptron (MLP) architecture was adopted for crack position prediction [14]. The network consisted of an input layer corresponding to the selected modal features (Modes 1-4), a single hidden layer with a fixed number of neurons, and one output neuron representing the predicted crack position $\hat{\boldsymbol{x}}$.

The hidden layer employed the Rectified Linear Unit (ReLU) activation function:

$$ReLU(u) = \max(0, u) \tag{6}$$

The output layer was linear, as crack position prediction represents a regression problem.

Model parameters were optimized using the AdamW optimizer with L2 regularization (weight decay) to reduce overfitting. The training objective was to minimize the Mean Squared Error (MSE):

$$MSE = \frac{1}{N}\sum_{j=1}^{N}\left(x_j - \hat{x}_j\right)^2 \tag{7}$$

where x_j is the true crack position and $\hat{x}_j$ is the predicted position.

Training was performed for a predefined maximum number of epochs. Early stopping was implemented based on validation RMSE; training was terminated if validation performance did not improve for a specified number of consecutive epochs. To ensure reproducibility and a fair comparison between indicators, the network architecture and training hyperparameters were kept identical across all experiments.

3.4. Evaluation Metrics and Results

Model performance was evaluated using three standard regression metrics [15]. A brief description of the metrics is given below.

Mean Absolute Error (MAE) represents the average absolute deviation between predicted and true crack positions and has a direct physical interpretation in units of beam length.

$$MAE = \frac{1}{N}\sum_{j=1}^{N}\left|x_j - \widehat{x_j}\right| \tag{8}$$

Root Mean Squared Error (RMSE) penalizes larger errors more strongly than MAE and provides an overall measure of prediction accuracy.

$$RMSE = \sqrt{\frac{1}{N}\sum_{j=1}^{N}\left(x_j - \widehat{x_j}\right)^2} \tag{9}$$

Coefficient of Determination (R^2) quantifies the proportion of variance in crack position explained by the model. $\bar{x}$ is the mean crack position in the dataset

$$R^2 = 1 - \frac{\sum_{j=1}^{N}\left(x_j - \widehat{x_j}\right)^2}{\sum_{j=1}^{N}\left(x_j - \bar{x}\right)^2} \tag{10}$$

3.5. Training Results and Interpretation

The comparative training results indicate that all three indicators enable accurate crack position prediction under simulated conditions. Among them, models trained using absolute frequencies achieved the lowest RMSE and the highest R^2 values. The relative frequency shift and frequency difference exhibited nearly identical performance, with slightly higher prediction errors but still strong explanatory capability. These findings suggest that, within a controlled simulation framework where environmental and operational variability are absent, all three frequency-based indicators contain sufficient spatial information for reliable crack localization.

The slight performance differences observed among indicators reflect variations in their sensitivity characteristics but do not significantly affect predictive capability in the considered scenario.

4. Sensitivity Analysis

To further evaluate the robustness of the trained ANN models, a sensitivity analysis was conducted using new frequency measurements obtained from finite element simulations. Unlike the datasets used for training, these scenarios were not included in the learning process and therefore represent an independent validation of the predictive capability of each damage indicator.

The analyzed scenarios correspond to a fixed crack depth of $a = 1\ mm$ while the crack location was varied along the beam length. For each scenario, the first four natural frequencies of the damaged beam f_{iD} (Modes 1-4) were extracted from the modal analysis module performed using ANSYS. These frequencies were used to compute the associated damage indicators. The previously trained ANN models were then used to predict the crack position. The results are summarized in Table 2.

Table 2. Numerical simulation data used to assess the indicators' sensitivity.

f_{1D} [Hz]	f_{2D} [Hz]	f_{3D} [Hz]	f_{4D} [Hz]	x [mm]	$\hat{x}$ with f_{iD} [mm]	$\hat{x}$ with $\Delta f_i(x, a)$ [mm]	$\hat{x}$ with $\Delta \bar{f}_i(x, a)$ [mm]
4.0452	25.4108	71.2916	139.9644	34	34476.63	37.33	37.12
4.07181	25.6031	71.3911	140.2604	288	202382.05	300.79	300.07
4.07553	25.5679	71.4045	140.5964	338	208919.74	343.85	344.69
4.08367	25.4733	71.7518	139.8021	486	245474.33	490.52	490.34
4.08468	25.4680	71.7482	139.7996	512	248596.15	501.65	501.49
4.08518	25.4670	71.7316	139.8536	526	249491.18	535.06	535.38
4.08570	25.4675	71.7009	139.9555	542	249887.60	548.35	548.52
4.08921	25.5570	71.2662	140.0709	724	265808.90	739.27	740.10
4.08963	25.5932	71.4019	139.6646	788	283383.30	762.77	763.35
4.08979	25.6123	71.5634	139.8527	838	295376.83	831.06	831.11
4.08988	25.6260	71.7461	140.5731	938	303909.18	913.72	913.59

The natural frequencies for the intact beam are:
$f_1 = 4.0899$ Hz, $f_2 = 25.6265$ Hz, $f_3 = 71.7547$ Hz, $f_4 = 140.6276$ Hz.

Table 3. ANN performance metrics for FEM scenarios.

Damage Indicator	**MAE**	**RMSE**	$\boldsymbol{R^2}$
f_{iD}	234.699263	244.936230	-934330.128734
$\Delta f_i(x, a)$	0.011270	0.013370	0.997216
$\Delta \overline{f_i}(x, a)$	0.011336	0.013384	0.997210

5. Discussion

The results clearly indicate a fundamental difference in robustness among the considered indicators when applied to new frequency measurements.

The models trained using the relative frequency shift and the frequency difference maintain excellent predictive accuracy, with very small absolute errors and near-perfect R^2 values. The predicted crack locations closely follow the true spatial positions along the entire beam length.

In contrast, the model trained using absolute damaged frequencies fails under the same conditions. The predicted crack positions become physically unrealistic, leading to extremely large errors and a strongly negative R^2. This behavior indicates that absolute frequency is highly sensitive to variations between the training dataset and the new measurement scenarios.

The underlying reason lies in the scale dependence of absolute frequency. While absolute frequencies performed well within the controlled training dataset, they are strongly influenced by global system properties such as stiffness scaling, modeling assumptions, or boundary condition variations. In contrast, both the frequency difference Δf_i and the relative frequency shift normalize the frequency variation with respect to the healthy baseline, thereby reducing sensitivity to global offsets and improving cross-scenario generalization.

6. Conclusion

This study comparatively evaluates three frequency-based damage indicators, namely the absolute frequency, the frequency difference, and the relative frequency shift, for ANN-based crack position detection in beams. A feature relevance analysis demonstrated that the lower bending modes contain the most significant spatial information, and the first four modes were selected as inputs to a baseline multilayer perceptron model.

Under controlled simulation conditions, all indicators enabled accurate crack localization. However, when applied to new frequency measurements obtained from finite element simulations, clear differences in robustness were observed. The relative frequency shift and the frequency difference maintained excellent predictive capability, whereas the model trained on absolute frequencies exhibited large errors and poor generalization.

The baseline MLP predicted the crack position with an average error of approximately 1 cm when using either the relative frequency shifts or the frequency differences as input features. These results confirm that normalized frequency-based indicators provide superior robustness and reliability for crack localization.

As future work, the influence of neural network topology should be further investigated to reduce the prediction error. In particular, alternative architectures and connectivity structures may improve the representation of nonlinear relationships and enhance generalization performance.

References

1 Salawu O.S., Detection of structural damage through changes in frequency: A review, *Engineering Structures*, 19(9), 1997, pp. 718–723.

2 Doebling S.W., Farrar C.R., Prime M.B., A summary review of vibration-based damage identification methods, *Shock and Vibration Digest*, 30(2), 1998, pp. 91–105.

3 Farrar C.R., Worden K., Structural Health Monitoring: A Machine Learning Perspective, *Wiley*, 2012.
(Example for Internet sources)

4 Peeters B., De Roeck G., One-year monitoring of the Z24-Bridge: Environmental effects versus damage events, *Earthquake Engineering & Structural Dynamics*, 30(2), 2001, pp. 149–171.

5 Deraemaeker A., Reynders E., De Roeck G., Kullaa J., Vibration-based structural health monitoring using output-only measurements under changing environment, *Mechanical Systems and Signal Processing*, 22(1), 2008, pp. 34–56.

6 Dimarogonas A.D., Vibration of cracked structures: A state of the art review, *Engineering Fracture Mechanics*, 55(5), 1996, pp. 831–857.

7 Cawley P., Adams R.D., The location of defects in structures from measurements of natural frequencies, *Journal of Strain Analysis for Engineering Design*, 14(2), 1979, pp. 49–57.

8 Gillich G.R., Maia N.M.M., Wahab M.A., Tufisi C., Korka Z.I., Gillich N., Pop M.V., Damage detection on a beam with multiple cracks: A simplified method based on relative frequency shifts, *Sensors*, 21(15), 2021, paper ID 5215.
9 Gillich G.R., Tufoi M., Korka Z.I., Stanciu E., Petrica A., The Relations between Deflection, Stored Energy and Natural Frequencies, with Application in Damage Detection, *Romanian Journal of Acoustics and Vibration*, 13(2), 2016, pp. 87–93.
10 Gillich G.R., Maia N., Mituletu I.C., Tufoi M., Iancu V., Korka Z., A new approach for severity estimation of transversal cracks in multi-layered beams, *Latin American Journal of Solids and Structures*, 13 (8), 2016, pp. 1526–1544.
11 Ostachowicz W.M., Krawczuk C., Analysis of the effect of cracks on the natural frequencies of a cantilever beam, *Journal of Sound and Vibration*, 150(2), 1991, pp. 191–201.
12 Praisach Z.I., Minda F.P., Gillich G.R., Minda A.A., Relative frequency shift curves fitting using FEM modal analyses, *Proceedings of the 4th WSEAS international conference on Finite Differences – Finite Elements – Finite Volumes – Boundary Elements*, 2021, pp. 82–87.
13 https://github.com/IoanaTincu/Studia-Universitatis-Babes-Bolyai-Engineering
14 Goodfellow I., Bengio Y., Courville A., Deep Learning, MIT Press, Cambridge, MA, 2016.
15 Haykin S., Neural Networks and Learning Machines, 3rd ed., Pearson Education, Upper Saddle River, NJ, 2009.

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