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Advanced Modal Analysis Technique for Structures with Non-Uniform Mass Distribution

Rusalin Lucian Paun, Gilbert-Rainer Gillich*, Mitsinjo Randrianarisoa

Abstract. *This paper presents a study of how the natural frequencies of bending vibration modes change for a double-clamped beam with a concentrated mass added to the beam mid-span. We compare the beam's response under different loads using a contrived analytical relation and experimental results. The results show that we can estimate a small variation in mass distribution and that the frequency estimation method, especially developed by the authors to capture slight structural, is effective.*

Keywords: *Non-uniform mass distribution, Frequency estimation technique, Modal analysis, Analytical method, Experiment.*

1. Introduction

It is often necessary to know the natural frequencies of mechanical systems [1]-[3] and how they change when the structure changes, for example, by adding mass [4],[5]. There are several ways to find these frequency changes [6]. Impulsive force-based excitation techniques are simple and low-cost; they also produce broadband excitation, but the amplitudes of the higher-order natural frequencies decay quickly [7]. The signal-to-noise ratio (SNR) is low, and repeatability is poor, limiting the method's accuracy. The use of non-impact excitation techniques is often preferred to impact methods [8,9], especially in laboratory conditions. However, this procedure is more complicated and time-consuming.

Methods using sinusoidal excitation easily ensure test repeatability due to the stability of the excitation signal parameters [10]. Each frequency of interest can be analyzed separately; simulations can also be performed around each frequency of interest or in a broadband manner, modifying the excitation signal in various ways [11], thereby reducing the analysis time. The analysis time can also be reduced by using appropriate response analysis methods [12].

If physical contact with the tested structure is not allowed, not efficient, or impossible, non-contact excitation techniques are applied, for example, acoustical excitation. The method is suitable for lightweight structures and allows control of various parameters of the energy transferred to the structure [13]. This controlled energy transfer reduces the effects of noise, especially when the vibration modes are analyzed separately.

The addition of mass modifies the structure's natural frequencies, with changes that depend on the mode number and the position of the added mass. In this paper, we evaluate the sensitivity of the proposed modal analysis technique to added mass.

2. Frequency Changes due to Added Mass

In this paper, we perform experiments on a prismatic fiberglass beam fixed at both ends. The specimen has the following dimensions: length L = 0.78 m, thickness H = 0.003 m, and width B = 0.02 m. For these data, the cross-sectional area is $A = 60\cdot10^{-6}$ m^2 and the second moment of inertia is $I = 4.5\cdot 10^{-11}$ m^4. The Young modulus provided by the producer for the fiberglass is $E = 2.484\cdot10^{10}$ N/m^2, and the mass density is ρ = 1800 kg/m^3. This mass density value was confirmed by experiments conducted as part of this research.

The research focuses on testing the sensitivity of a developed experimental modal analysis technique to slight changes in a structure's mass. To this end, we consider in the study the weight of the accelerometer m_a = 8 g, and three supplementary masses, m_1 = 0.5 g, m_2 = 1 g, and m_3 = 2.33 g.

For the beam without supplementary mass, we calculate the first five natural frequencies of the fiberglass beam with the formula:

$$f_i = \frac{\lambda^2}{2\pi L^2}\sqrt{\frac{EI}{\rho A}} \tag{1}$$

In Eq. (1), we denote with λ the eigenvalues for the double-clamped beam. The first five weak-axis bending vibration modes of the beam are given in Table 1.

To calculate the frequencies of the beam with an added mass, f_{iR}, we used mathematical relation deduced by the authors [14].

$$f_{Ri} = f_i\sqrt{\frac{\varphi_i^{0-L}}{\varphi_i^{0-a} + \varphi_i^{a-b}\frac{\bar{m}_R}{\bar{m}} + \varphi_i^{b-L}}} \tag{2}$$

where

$$\varphi_i^{0-a} = \int_0^a \left[\bar{\varphi}_i(x)\right]^2 dx\,,\quad \varphi_i^{a-b} = \int_a^b \left[\bar{\varphi}_i(x)\right]^2 dx\,, \ldots \varphi_i^{b-L} = \int_b^L \left[\bar{\varphi}_i(x)\right]^2 dx \tag{3}$$

are mass participation coefficients, and represent the square of the normalized mode shapes on segments 0-*a*, *a*-*b*, and *b*-*L*. With $\bar{m}$ we denoted the distributed mass of the beam, and with $\bar{m}_R$ the distributed mass of the segment where the supplementary loads act.

The analyzed beam is shown on Fig. 1. It has both ends fixed and on the mid-span, on a segment of 10 mm length, is applied the supplementary mass. On one face it is mounted an accelerometer, on the other the masses. The frequency of the beam is calculated with the mounted accelerometer and afterward adding different weights. The results are presented in Table 1. One can observe that the frequency drops significantly due to the accelerometer.

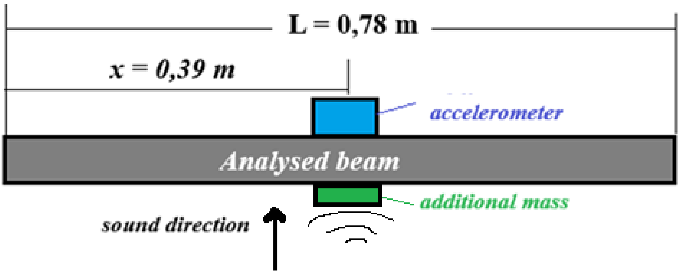

Figure 1. The text of figure caption. (TNR, 11, Normal)

Table 1. The calculated frequencies for the first five bending vibration modes.

Frequency / Mass	f_1 [Hz]	f_2 [Hz]	f_3 [Hz]	f_4 [Hz]	f_5 [Hz]
No mass	18.83	51.91	101.75	168.25	251.35
m_a = 8 g	16.85	51.90	93.13	168.20	229.88
$m_a + m_1$ = 8.5 g	16.76	51.90	92.66	168.19	228.72
$m_a + m_2$ = 9 g	16.65	51.90	92.20	168.19	227.57
$m_a + m_3$ = 10.33 g	16.39	51.90	91.01	168.18	224.60
$m_a+m_1+m_2+m_3$ = 11.83 g	16.11	51.90	89.71	168.17	221.38

3. Experimental validation

In this section we will focus on vibration mode three. The experimental setup is presented in Figure 2 and 3. The excitation is realized with a sinusoidal sound wave signal of constant amplitude, with a frequency near the calculated frequencies for the beam with the accelerometer and additional masses placed in the middle. The signal is 0.1 Vpp. The excitation signal is generated by a signal generator, amplified, and transmitted to the speaker. Acoustic vibrations are induced in the beam using a low-frequency speaker with an impedance of 4 ohms and a power of 50 watts. The speaker is positioned directly in front of the beam, centered on its midpoint, approximately 0.04...0.05 m away, to minimize vibration energy losses. The speaker's position can be adjusted. Measurement is realized in LabVIEW using a Kistler 8772A5 one-axis accelerometer, connected to the four-channel NI cDAQ-9172 module. The received signal is converted and sent to a laptop, where it is stored, processed, and displayed.

Figure 2. The experimental stand – control and measurement system

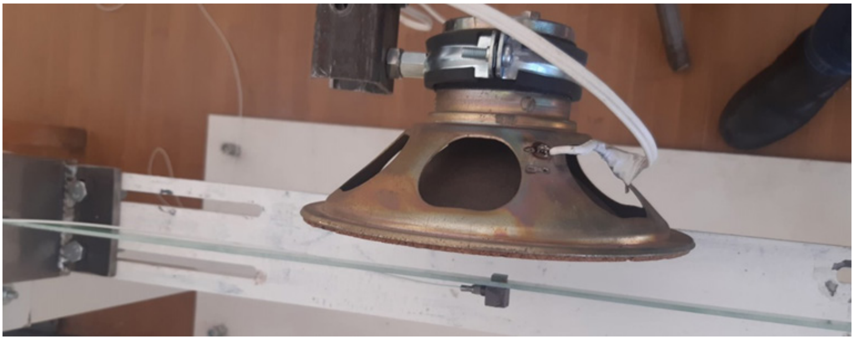

Figure 3. The beam with accelerometer and weight and the excitation system (upper view)

To accurately find the natural frequencies, a technique presented in [13] is used. It presumes exciting the structure with frequencies around the presumed natural frequency and targeting a response that does not mimic the beetle phenomenon. Figure 4 shows a vibration signal acquired for an excitation frequency that does not match the natural frequency of the beam. One can observe that the amplitude increase and decrease, which means the resonance frequency is not achieved.

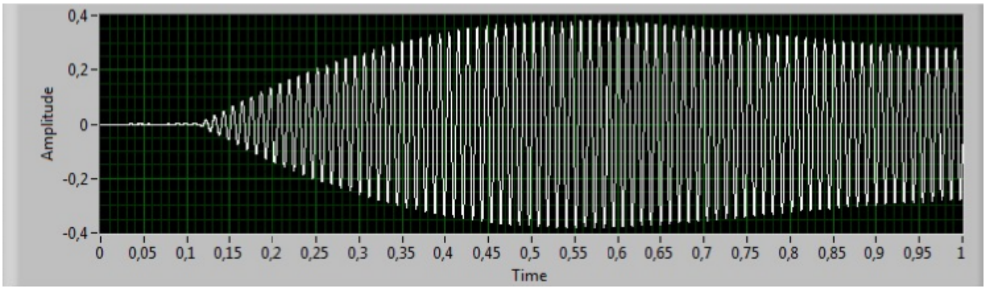

Figure 4. Beam response to an excitation with the frequency that does not fit the resonance frequency.

In contrast, Figure 5 shows a response acquired when the excitation and natural frequency fit. One can observe that the amplitude increases constantly and is significantly higher than that in Figure 4.

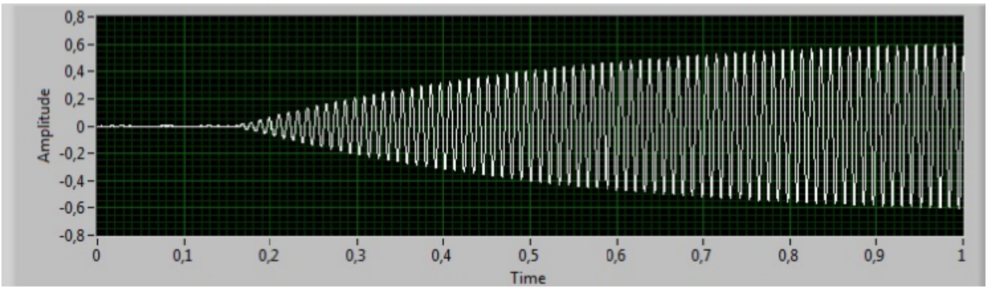

Figure 5. Beam response to an excitation with the frequency that fits the resonance frequency.

The spectrum of the vibration signal depicted above is plotted in Figure 6. With blue color we represent the spectrum on a larger range around the natural frequency domain, and a zoom on the peak is shown with a red color in the small window. One can observe that the neighbors of the maximizer have equal amplitudes, which means the frequency is correctly estimated.

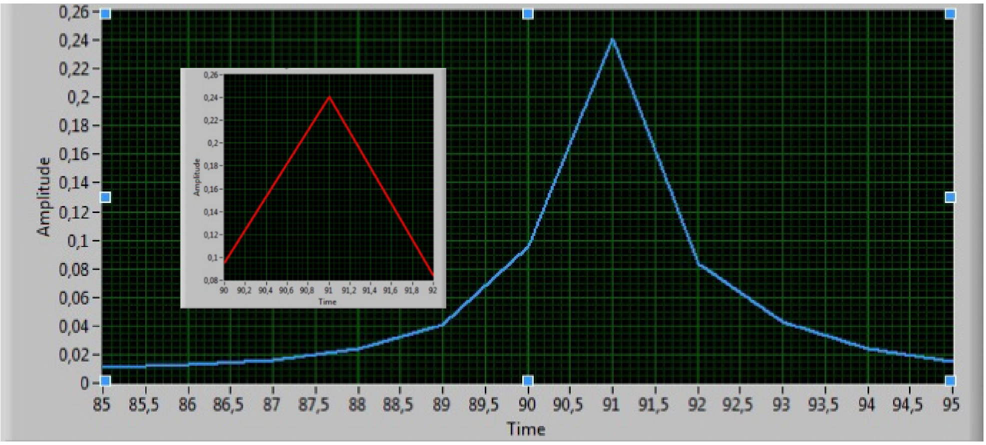

Figure 6. Detail on the frequency spectrum and a zoom on the peak frequency.

The frequency estimates made for the signals acquired when the beam was excited with the correct frequencies are presented in Table 2.

Table 2. The estimated frequencies for the first five bending vibration modes.

Frequency / Mass	f_1 [Hz]	f_2 [Hz]	f_3 [Hz]	f_4 [Hz]	f_5 [Hz]
No mass	-	-	-	-	-
m_a = 8 g	17.24	53.09	92.10	172.05	235.14
$m_a + m_1$ = 8.5 g	17.14	53.09	91.20	172.04	233.95
$m_a + m_2$ = 9 g	17.03	53.09	90.90	172.04	232.78
$m_a + m_3$ = 10.33 g	16.77	53.09	89.90	172.03	229.74
m_a+m_1+m_2+m_3 = 11.83 g	16.48	53.09	88.50	172.02	226.45

To test if small frequency changes can be identified using the advanced technique proposed by the authors, we represent the calculated frequencies and those obtained from measurements. One can observe that the frequency evolution with the mass increase is similar, which demonstrates the mathematical relation (2) is correct and the frequency estimation technique is reliable and allows obtaining estimations with sufficient accuracy.

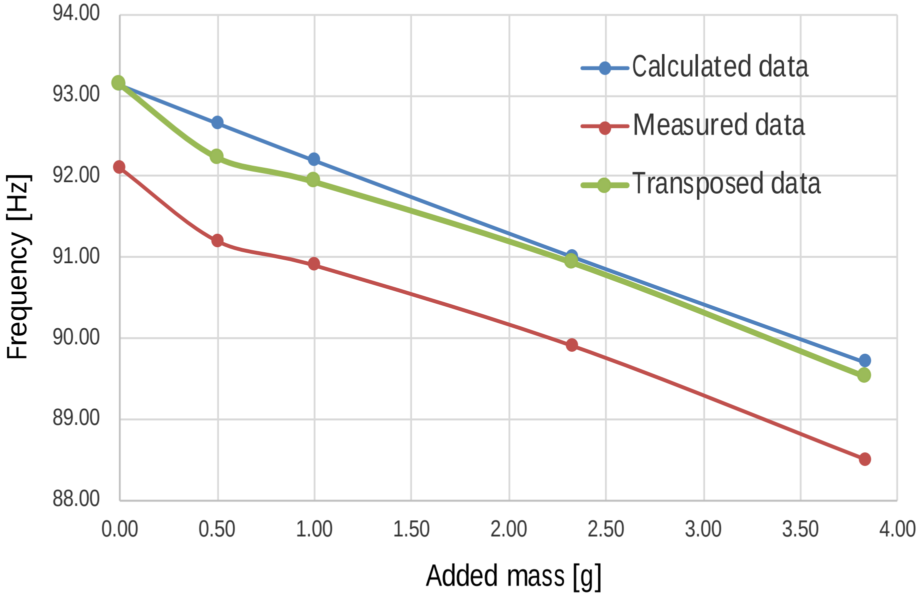

Figure 7. Comparison of calculated and measured frequencies.

It can be also observed in Figure 7 that it is a discrepancy between the frequencies obtained from calculus and by measurements. This is caused by the differences between the physical-mechanical parameters for the virtual and real beam. By translating the measured data to fit the calculated data for the beam with the mounted accelerometer, we observe that for small mass changes, the difference between the frequency drops is bigger for the measured case. However, the difference is small and can be even reduced when using more digits for the excitation signal.

A comparison of the estimated frequency drops is given Table 3. Here we can better observe the method's efficiency in assessing small mass changes.

Table 3. The frequency drops due to added mass identified vibration mode three.

Mass difference	Calculated Δf_3 [Hz]	Measured Δf_3 [Hz]
m_1 = 0.5 g	0.47	0.9
m_2 = 1 g	0.93	1.2
m_3 = 2.33 g	2.12	2.2
m_1+m_2+m_3 = 3.83 g	3.42	3.6

4. Conclusion

This study investigated several key aspects of experimental modal analysis performed on a double-clamped fiberglass beam, with a particular focus on the influence of added mass and the precision required for accurate frequency estimation. The work provides both methodological insights and practical considerations relevant to lightweight structures, where even minimal mass perturbations can significantly affect dynamic properties.

A central contribution of the research is the mathematical relation derived by the authors, which quantifies the effect of the accelerometer's added mass on the measured natural frequencies. The results demonstrate that, for lightweight structures such as the fiberglass beam examined here, this corrective relation is not merely useful but mandatory for obtaining the true, unaltered natural frequencies. By compensating for mass-loading effects, the method ensures that the modal parameters reflect the actual behavior of the structure rather than artifacts introduced by the measurement system.

Another significant finding is the high sensitivity of the proposed frequency estimation technique. The method successfully identifies frequency shifts on the order of hundredths, highlighting its suitability for applications requiring elevated precision, such as early-stage damage detection, structural health monitoring, and quality control in manufacturing processes. Such fine resolution is essential when working with structures where small changes in stiffness, mass, or boundary conditions must be detected reliably.

Moreover, the study shows that slight variations in mass are not only observable but can be quantified directly through the resulting frequency changes. The experimental setup enabled the clear detection of a 0.5-gram increase in mass, underscoring the technique's capability to capture minute alterations in structural properties. This level of sensitivity strengthens the method's relevance for monitoring lightweight composite components or any system where minimal mass fluctuations may signify degradation or external influence.

Overall, the findings confirm that accurate modal characterization of lightweight structures requires careful consideration of measurement influences, refined signal-processing techniques, and high-resolution frequency estimation. The methods developed and validated in this study provide a robust foundation for further advancements in non-destructive evaluation and precision modal analysis.

Our next research will focus on using smaller masses and finer steps for the mass increase, in order to find the method's limitations. On the other hand, we will extend the analysis for a larger domain, by adding bigger masses, aiming to observe when nonlinearities occur and how these influence the measurement of natural frequencies.

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Addresses:

- Ph.D. Stud. Rusalin Lucian Paun, Babeș-Bolyai University, Doctoral School of Engineering, P-ta Traian Vuia 1–4, 320085 Reșița, Romania rusalin.paun@ubbcluj.ro
- Prof. Gilbert-Rainer Gillich, Babeș-Bolyai University, Doctoral School of Engineering, P-ta Traian Vuia 1–4, 320085 Reșița, Romania gilbert.gillich@ubbcluj.ro (*corresponding author)
- Assoc. prof. Mitsinjo Randrianarisoa, Institut Supérieur de Technologie d'Antananarivo, Antananarivo 101, Madagascar ramitsinjo@yahoo.fr