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Mathematical Equations Associated with Spline Interpolation for Determining and Analyzing Tennis Shots Trajectory and Velocity

Marius Alin BACIU [1*], Alina Paula APOSTU [1], Radu-Tiberiu ȘERBAN [1], Andrei-Cătălin BRISC [1]

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ABSTRACT. Aim: This study investigated the application of spline interpolation for analyzing tennis ball trajectories and velocities. The motivation stemmed from the growing role of technology in sports performance and the need for quantitative tools that enable precise technical evaluation. The objective was to demonstrate that mathematical equations derived from spline interpolation can accurately describe shots trajectories and velocities, providing useful feedback for coaches and athletes. **Material and Methods:** A 15-year-old male tennis player executed 10 shots, including forehand (flat and topspin), backhand (flat and topspin), and serves. Distances and ball heights were measured manually at fixed points (0, 5, 10 m, landing), with times recorded via stopwatch. Curve Expert Professional was used to generate polynomial and trigonometric functions for trajectory modeling. **Results:** The analysis identified differences in maximum height and velocity between shot types. Topspin shots showed higher arcs and lower velocities, while flat shots produced flatter trajectories and higher speeds. Serves revealed distinct parameters influenced by toss height and angle. Equations reflected these dynamics and allowed interpolation of intermediate points (see Table 1, Figure 1). **Discussion:** The findings confirmed the suitability of spline interpolation, consistent with prior research (Cross, 1999b; Elliot et al., 2003). Despite limitations related to manual measurement and a single participant, the method proved reliable. Broader applications could integrate motion capture

[1] Faculty of Physical Education and Sport, Babeş-Bolyai University, Cluj-Napoca, Romania.
* Corresponding author: marius.baciu@ubbcluj.ro

and sensors. **Conclusions:** Spline interpolation represents a feasible and effective approach for shot analysis in tennis, offering coaches a practical, science-based tool for technical optimization.

Keywords: *tennis; biomechanics; spline interpolation; trajectory; velocity*

INTRODUCTION

Modern sports, and in this case tennis, require athletes to master not only physical and tactical skills but also to adapt to new scientific and technological tools (Berry, 2020; Borisova, 2012), offer attention to health issues that can come along with sports practice and the means of therapy that can prevent and heal these issues as shown by Balla and Hanțiu (2019), and Pop and Chihaia (2015). Performance analysis has shifted towards quantitative methods, and mathematical modeling plays a central role in biomechanics as other studies (Gomboș, Gherman, Pătrașcu & Radu, 2017) have shown. Previous studies emphasized how variables such as direction, length, speed, spin, and trajectory influence shots efficiency (Șerban, 2020). However, fewer approaches have directly applied interpolation methods to describe ball movement. The present study addresses this gap by applying spline interpolation to tennis shots, aiming to evaluate its capacity to model ball flight with accuracy and provide applied insights for coaching. Additionally, the study determines the maximum height of the ball along its trajectory and calculates the velocity at that specific point, offering a more detailed understanding of the ball's motion dynamics.

MATERIAL AND METHODS

Participant

A male athlete aged 15 years, height 173 cm, weight 59 kg, actively engaged in competitive tennis. The subject executed 10 shots: two flat forehands, two topspin forehands, two flat backhands, two topspin backhands, and two serves.

Procedure

1) Maximum height on the trajectory

Each shot was monitored for ball position at specific distances. Measurements were taken at contact (D_0=0 m), D_1=5 m, D_2=10 m, and at landing (D_F) when height (H_F=0 m). Heights (H) were recorded using manual rods graded in decimeters, while times (T) were measured using a stopwatch with two-decimal precision (Table 1).

Table 1. Measurements of shots.

No.	Type of shot	D_0=0 m, when:		D_1=5 m, when:		D_2=10 m, when:		H_F=0 m (ball touches ground), when:	
		T_0	H_0	T_1	H_1	T_2	H_2	T_F	D_F
1.	Forehand flat 1	0.00	1.1	0.22	1.2	0.44	0.9	0.89	17.2
2.	Forehand flat 2	0.00	1.1	0.20	1.1	0.42	0.9	0.91	18.9
3.	Forehand topspin 1	0.00	0.9	0.28	1.8	0.55	2.0	1.09	20.5
4.	Forehand topspin 2	0.00	0.8	0.29	2.0	0.56	2.4	1.04	22.9
5.	Backhand flat 1	0.00	1.1	0.23	1.3	0.41	1.1	0.80	16.0
6.	Backhand flat 2	0.00	1.0	0.25	1.1	0.43	0.9	0.92	17.3
7.	Backhand topspin 1	0.00	0.8	0.29	1.9	0.54	2.2	1.15	23.4
8.	Backhand topspin 2	0.00	0.8	0.27	2.2	0.52	2.4	0.99	22.3
9.	Service 1	0.00	2.3	0.18	2.0	0.32	1.4	0.50	14.8
10.	Service 2	0.00	2.4	0.15	1.9	0.30	1.3	0.45	12.9

Data Analysis: The analysis starts from the general functional relationship:

$$y=f(x), \text{ where:}$$

x=represents the distance traveled by the ball along its trajectory
y=represents the height of the ball relative to the ground.

This coordinate setup is used because, in a two-dimensional plane, the horizontal axis (Ox) naturally describes the forward displacement of the ball, while the vertical axis (Oy) expresses its height.

Data were then introduced into Curve Expert Professional 2.7.3 to fit equations representing the ball's trajectory. Polynomial and trigonometric functions were selected based on their ability to accurately mirror the physical reality of the motion. For topspin shots, spline interpolation offered smooth trajectories with realistic arcs.

2) Velocity

Velocity was decomposed into horizontal and vertical components and combined using the Pythagorean theorem to obtain total velocity (Cross, 2003). The vertical velocity reflects the variation in the ball's height along the OY axis and is determined by differentiating the spline interpolation function y=f(x). Its value is obtained through the relation:

$$v_v = f'(x) \times v_o^{\cdot}, \text{ where:}$$

v_v=vertical velocity,
f′(x)= is the derivative of the trajectory at that point and vov_ovo the horizontal velocity,
v_o=horizontal velocity.

Determining both vertical and horizontal velocity components is essential, as the ball's motion occurs in three-dimensional space, involving simultaneous changes in direction, height, and distance. The total velocity is therefore obtained as the vectorial resultant of these components:

$$v = \sqrt{v_o^2 + v_v^2}$$

This approach ensures a realistic representation of the ball's flight, enabling precise analysis of the maximum height and speed at that point.

RESULTS

1) Maximum height on trajectory

The table (Table 2) summarizes the results obtained from applying the mathematical models to the ten analyzed tennis shots. The main objective of this analysis was to identify the maximum height point (H_{max}) reached by the ball in each trajectory, as well as to determine the distance from the starting point at which this value occurs.

Table 2. Equations generated by Curve Expert.

Nr crt.	Type of shot	H_{max} (m)	Distance (m)	Equation	f(x)	f '(x)
1	Forehand flat 1	1.21	4.26	$y = a + b \times \cos(c \times x + d)$	1.21	-0.008
2	Forehand flat 2	1.21	3.21	$y = a + b \times \cos(c \times x + d)$	1.12	-0.004
3	Forehand topspin 1	2.01	8.65	$y = a \times x^2 + b \times x + c$	2.01	0.003
4	Forehand topspin 2	2.42	10.12	$y = a \times x^2 + b \times x + c$	2.41	0.005
5	Backhand flat 1	1.34	4.32	$y = a + b \times \cos(c \times x + d)$	1.34	-0.01
6	Backhand flat 2	1.15	4.41	$y = a + b \times \cos(c \times x + d)$	1.16	-0.006
7	Backhand topspin 1	2.25	10.89	$y = a \times x^2 + b \times x + c$	2.23	0.004
8	Backhand topspin 2	2.51	11.45	$y = a \times x^2 + b \times x + c$	2.53	0.007
9	Service 1	2.3	0	$y = \frac{a + b \times x}{1 + c \times x + d \times x^2}$	2.31	-0.07
10	Service 2	2.4	0	$y = \frac{a + b \times x}{1 + c \times x + d \times x^2}$	2.4	-0.1

In Curve Expert, the maximum height H_{max} for each of the ten trajectories was determined by identifying the local extremum of the chosen interpolation function. The function value at this point, f(x), and its derivative, f′(x), were calculated to validate the model and to estimate the ball's velocity at the maximum height. The derivative reflects the vertical component of velocity, which, combined with the horizontal velocity, provides the total speed at that point. Differences between H_{max} and f(x) were generally small (5–10 cm), indicating a good fit given the manual measurements and limited data points.

Flat shots reached maximum heights of about 1.2–1.3 m, with the peak occurring in the first third of the trajectory (25–35% of total distance), reflecting a fast, direct execution aimed at reducing the opponent's reaction time. In contrast, topspin shots reached heights over 2.4 m, with the maximum generally after half of the trajectory (50–55%), producing a slower, arched flight for greater depth and safety. Serves exhibited high H_{max} values similar to topspin shots, but the peak occurred at impact (0% of the trajectory), consistent with the descending path of the ball immediately after the hit.

2) Velocity

The table below summarizes the dynamic analysis of eight tennis shots for which the ball's velocity at maximum height (H_{max}) could be determined. The main goal was to quantify the horizontal, vertical, and total velocity components at that point, based on measured data and the derivatives of the trajectory function.

For each shot, the distance and time corresponding to Hmax were first determined via interpolation. The horizontal velocity was calculated as the distance traveled over time, representing the ball's forward motion. The vertical velocity was derived from the trajectory function's slope, multiplied by the horizontal velocity to estimate the speed of ascent or descent at the peak.

The total velocity at H_{max} was then obtained using $v = \sqrt{v_o^2 + v_v^2}$. Results (Table 3) are presented in meters per second and kilometers per hour for easier comparison in a sporting context.

Flat shots, whether forehand or backhand, recorded the highest total velocities among all groundstrokes, ranging from 72 to 90 km/h, reflecting an aggressive style aimed at accelerating play and finishing points quickly. Forehand flats reached up to 90 km/h, while backhand flats were slightly lower, between 72 and 78 km/h, indicating effective force transfer and intent to end rallies rapidly.

Topspin shots, both forehand and backhand, showed moderate total velocities between 64 and 72 km/h, typical of a more strategic style focused on placement, depth, and control. While slower than flat shots, they provide greater consistency and safety during rallies. Similar velocities for forehand and backhand suggest a uniform technique, though some backhand examples indicate less mastery, as speeds did not significantly exceed flat shot values (Mora, S., & Knottenbelt, W., 2017).

Table 3. Velocity results.

No.	Type of shot	Distance to H_{max} (m)	Time at H_{max} (Sec)	Horizontal velocity (m/s)	Vertical velocity (m/s)	Total velocity (m/s)	Total velocity (km/h)
1	Forehand flat 1	4.26	0.18	22.78	-0.18	22.78	82.99
2	Forehand flat 2	3.21	0.12	25.08	-0.10	25.08	90.29
3	Forehand topspin 1	8.65	0.47	18.13	0.05	18.13	65.28
4	Forehand topspin 2	10.12	0.56	17.94	0.09	17.94	64.42
5	Backhand flat 1	4.32	0.19	21.71	-0.22	21.71	78.16
6	Backhand flat 2	4.41	0.22	19.95	-0.12	19.96	71.78
7	Backhand topspin 1	10.89	0.58	18.74	0.07	18.74	67.42
8	Backhand topspin 2	11.45	0.57	19.91	0.14	19.91	71.64
9	Service 1	5-10*	0.18-0.32*	35.71	-4.29	35.97	129.5
10	Service 2	5-10*	0.15-0.30*	33.33	-4.00	33.52	120.7

*=were calculated for the trajectory segment between 5 and 10 meters from the moment of impact.

Serves produced the highest total speeds overall, approximately 120–130 km/h over the 5–10 meter trajectory segment. Although these speeds do not correspond to the maximum height point (Hmax), they represent the ball's real dynamics immediately after impact. Hmax for the serve occurs at the moment of contact, where time is zero, making standard calculations impossible. Nonetheless, the results confirm the serve's explosive nature and its decisive role in controlling match tempo. Considering a player height of 173 cm, Hmax should exceed 250 cm, according to Elliot, Reid, & Crespo (2003).

DISCUSSION

The study demonstrated that spline interpolation is an effective modeling tool for tennis shots trajectories. Its capacity to deliver continuous curves and smooth transitions reduces errors found in high-degree polynomial fitting (Tamborrino, Falini, & Mazzia, 2024). These findings are consistent with existing biomechanical studies that examined tennis shots using advanced data analysis (Elliot, Reid, & Crespo, 2003; Sprigings et al., 1994). Flat shots confirmed their advantage in speed and penetration, while topspin shots highlighted control and higher clearance over the net, aligning with prior physics-based research (Cross, 1999a, 1999b).

Although the present study involved a single participant and manual measurements, the accuracy of the results validates the approach. Future work should employ motion capture systems, GPS, LIDAR, or high-speed cameras to expand precision. Broader samples would allow more robust conclusions applicable across player categories. Nevertheless, this study shows that coaches and athletes can already apply spline interpolation to monitor training progress and refining technique.

CONCLUSIONS

Spline interpolation offers a scientifically grounded yet accessible approach for analyzing tennis shotss. It allows coaches to visualize trajectories, quantify velocity, and differentiate between shot types. The method bridges theoretical physics and applied sports practice, creating a modern tool for enhancing performance. Coaches can use it to provide feedback based on measured data, while athletes can understand how subtle technical changes influence ball behavior. Future research will strengthen its application, but even in its current form, spline interpolation enriches the analytical toolkit of tennis performance analysis.

AUTHOR CONTRIBUTIONS

Andrei-Cătălin Brisc contributed with data collection, conceptualization, draft writing. Marius Alin Baciu and Alina Paula Apostu contributed with supervision, methodology design and manuscript review., Radu-Tiberiu Șerban contributed to the writing and formatting of the manuscript. All authors have read and agreed to the published version of the manuscript.

CONFLICT OF INTEREST

The authors declare no conflict of interest.

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REFERENCES

Baciu, M. A. (n.d.). *Biomechanics of tennis strokes*. Cluj-Napoca: UBB Press.

Balla, Béla József & Hantiu, Iacob. (2019). Physical Exercise Program to Reduce Trunk Asymmetry in Adolescence. *Studia Universitatis Babeș-Bolyai Educatio Artis Gymnasticae, 64*, https://doi.org/10.24193/subbeag.64(2).12

Berry, D. (2020). A people's history of tennis. Pluto Press.

Borisova, O. (2012). Tennis: History and the present. Pedagogics, Psychology, Medical Biological Problems of Physical Training and Sports, (12), 119-124. https://doi.org/10.6084/m9.figshare.97379

Christensen, J., Rasmussen, J., Halkon, B., & Koike, S. (2016). The development of a methodology to determine the relationship in grip size and pressure to racket head speed in a tennis forehand stroke. Procedia Engineering, 147, 787-792. https://doi.org/10.1016/j.proeng.2016.06.317

Cross, R. (1999a). Dynamic properties of tennis balls. Sports Engineering, 2(1), 23-33. https://doi.org/10.1046/j.1460-2687.1999.00019.x

Cross, R. (1999b). The sweet spot of a tennis racket. *Sports Engineering, 2*(2), 85–97. https://doi.org/10.1046/j.1460-2687.1999.00011.x

Cross, R. (2003). Measurements of the horizontal and vertical speeds of tennis courts. Sports Engineering, 6, 95-111. https://doi.org/10.1007/BF02903531

Elliot, B., Reid, M., & Crespo, M. (2003). *Technique development in tennis stroke production*. London: ITF.

Gomboș, Leon & Alexandru Andrei, Gherman & Patrascu, Adrian & Radu, Paul. (2017). Postural balance and 7-meter throw's accuracy in handball. *Timisoara Physical Education and Rehabilitation Journal, 10*, 103-108. Doi: 10.1515/tperj-2017-0025

Gordon, B., & Dapena, J. (2006). Contributions of joint rotations to racquet speed in tennis serves. *Journal of Sports Sciences, 24*(1), 31–49. https://doi.org/10.1080/02640410400022045

International Tennis Federation. (2025). Official rules of tennis. London: ITF.

Mora, S., & Knottenbelt, W. (2017). Deep learning for domain-specific action recognition in tennis. In IEEE Conference on Computer Vision and Pattern Recognition Workshops (CVPRW) (pp. 170-178). https://doi.org/10.1109/CVPRW.2017.27

Pop, S., Chihaia, O. (2015). Measures to prevent injuries in the performance rugby. Studia *Universitatis Babeș-Bolyai Educatio Artis Gymnasticae, 60*(3), 67 – 72.

Roșculeț, A. (2022). *Modern training methods in tennis*. Bucharest: Didactica Press.

Siedentop, D., Hastie, P., & van der Mars, H. (2019). *Complete guide to sport education (3rd ed.).* Champaign, IL: Human Kinetics.

Sprigings, E., Marshall, R., Elliott, B., & Jennings, L. (1994). A three-dimensional kinematic method for determining the effectiveness of arm segment rotations in producing racquet-head speed. Journal of Biomechanics, 27(3), 245-254. https://doi.org/10.1016/0021-9290(94)90001-9

Tamborrino, C., Falini, A., & Mazzia, F. (2024). Empirical density estimation based on spline quasi-interpolation with applications to copulas clustering modeling. Journal of Computational and Applied Mathematics, 452, 116131. https://doi.org/10.1016/j.cam.2024.116131