

## COMPARISON OF PID TUNING ALGORITHMS FOR PROCESSES WITH TIME DELAY

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**ABSTRACT.** The purpose of this paper is to study by simulation some PID tuning methods for process with time delay. The ZIGLER-NICHOLS (ZN), ALESSANDRO BRAMBILLA (AB) and INTERNAL MODEL CONTROL (IMC) methods are considered. The performances of the close-loop are studied, for 1<sup>st</sup> and 2<sup>nd</sup> order plus time delay processes, in the cases of step set point and load disturbance.

### 1. INTRODUCTION

The main characteristics of the dynamic behaviour of the chemical processes (especially heat and mass transfer units) are the time delay and the long settling time that depends on the inputs and dimensions of the equipment. Usually, the models used are the 1<sup>st</sup> and 2<sup>nd</sup> order with time delay. These mathematical models describe the real processes with uncertainty due to the approximations considered or to the parameters variation in time. Despite the availability of advanced algorithms, conventional controllers (PI and PID type) are still widely used in process industry due to their versatility, effectiveness and easy to use them. Another quality of PID control algorithms is that they are known to be robust, i.e. the performances of the closed-loop do not deteriorate significantly with changing process conditions. But, the popular tuning methods ZIGLER and NICHOLS (ZN) or COHEN and COON (CC) give a too oscillating response of the closed-loop, with long settling time and require continuous plant cycling. Also, it is well known that ZN and CC tuning rules don't lead to good performances of the close-loop in the case of processes with large time delay – the loop becomes unstable if time delay increases.

As a consequence, the subject of tuning conventional controllers has received great attention, with the goal to give tuning algorithms to achieve better performance than from ZN and CC rules, especially for processes with large time delay. A. BRAMBILLA developed a simple method (AB) that obtains a single tuning parameter enabling PID controllers to give a desired response rivaling more advanced algorithms [2].

M. MORARI and E. ZAFIRIOU used Internal Model Control design procedure to derive PID controller parameters for a large variety of models commonly used in the process industry [3]. The IMC tuning method uses also a single adjustable parameter that corresponds approximately to the closed-loop time constant and it is selected by the designer to achieve the appropriate compromise between performance and robustness.

This paper evaluates AB and IMC tuning methods and compares them to ZN rule.

## 2. DESCRIPTION OF THE TUNING METHODS

For a large variety of processes from chemical industry, the models commonly used are:

- the first order plus time delay, with the transfer functions given by eq. (1)

$$G_p(s) = \frac{k_p \cdot e^{-s \cdot T_d}}{T_p \cdot s + 1}, \quad (1)$$

where  $k_p$  is the process steady state gain,  $T_p$  is the time constant and  $T_d$  is the time delay (dead time);

- the second order plus time delay, with the transfer functions given by eq. (2)

$$G_p(s) = \frac{k_p \cdot e^{-s \cdot T_d}}{(T_{p1} \cdot s + 1) \cdot (T_{p2} \cdot s + 1)}, \quad (2)$$

where  $k_p$  is the process steady state gain,  $T_{p1}$  and  $T_{p2}$  are the time constants and  $T_d$  is the time delay (dead time).

In the case of AB tuning method, the dynamic parameters of a PI or PID controller ( $k_C$ ,  $T_I$  and  $T_D$ ) are functions of the process parameters, presented in *Table 1* for different process transfer functions.

**Table 1.**

The parameters of a PID controller for AB tuning method

Process model	$k_C$	$T_I$	$T_D$
first order plus time delay	$\frac{1}{k_p} \cdot \frac{T_p + T_d / 2}{T_d (c + 1)}$	$T_p + T_d / 2$	$\frac{T_p \cdot T_d}{2T_p + T_d}$
second order plus time delay	$\frac{1}{k_p} \cdot \frac{T_{p1} + T_{p2} + T_d / 2}{T_d (2c + 1)}$	$T_{p1} + T_{p2} + T_d / 2$	$\frac{T_{p1} \cdot T_{p2} + (T_{p1} + T_{p2}) \cdot T_d}{T_{p1} + T_{p2} + T_d}$

The only tuning parameter is **c**, appearing in the gain of the controller and affecting the speed of response. It can be noted that controller parameters  $k_C$  and  $T_I$  have the same expressions for PI and PID but the tuning parameter **c** has different values in the two cases as widely presented in [2]. Parameter **c** depends on the ratio  $T_d/T_p$  (first order process) or  $T_d/(T_{p1}+T_{p2})$  (second order process).

Because Internal Model Control (IMC) is very general and powerful M. MORARI and E. ZAFIRIOU explored the relationships between IMC, PI and PID in order to gain insight into the tuning of these simpler controllers, their performance, robustness and limitations. First of all, for a process with time delay, the pure delay ( $e^{-T_d \cdot s}$ ) is rationalised by a Padé polynomial of zeroth or first order (eq. 3):

First order Padé approximation

$$e^{-T_d \cdot s} \approx 1$$

Second order Padé approximation

$$e^{-T_m s} = \frac{1 - T_m / 2 \cdot s}{1 + T_m / 2 \cdot s} \quad (3)$$

In this manner it is obtained a nominal model  $\tilde{G}_p(s)$  for the real process. It was found that for virtually all nominal models common in industrial practice, IMC leads to PID controllers for which the parameters are calculated with expressions from Table 2. This problem is discussed in more details in [3]. In Table 2,  $\lambda$  is the single adjustable parameter that corresponds approximately to the closed-loop time constant and it is selected to achieve the appropriate compromise between robustness and performances. There are practical recommendations for choosing the value of  $\lambda$ . For a PID controller the ratio  $\lambda/T_d > 0.25$  and for a PI controller  $\lambda/T_d > 1.7$ . For some nominal models, the PID controller is augmented by a first order filter with time constant  $T_F$ .

**Table 2.**

*IMC controllers for nominal models, interpreted as PID controllers with filter*

Type	Nominal model $\tilde{G}_p$	$k_c k_p$	$T_I$	$T_D$	$T_F$
A	$\frac{k_p}{T_p s + 1}$	$\frac{T_p}{\lambda}$	$T_p$	-	-
B	$\frac{k_p}{(T_{p1} s + 1)(T_{p2} s + 1)}$	$\frac{T_{p1} + T_{p2}}{\lambda}$	$T_{p1} + T_{p2}$	$\frac{T_{p1} T_{p2}}{T_{p1} + T_{p2}}$	-
C	$\frac{k_p}{T_p^2 s^2 + 2\zeta T_p s + 1}$	$\frac{2\zeta T_p}{\lambda}$	$2\zeta T_p$	$\frac{T_p}{2\zeta}$	-
D	$k_p \frac{-\beta s + 1}{T_p^2 s^2 + 2\zeta T_p s + 1}$	$\frac{2\zeta T_p}{2\beta + \lambda}$	$2\zeta T_p$	$\frac{T_p}{2\zeta}$	$\frac{\beta \lambda}{2\beta + \lambda}$
E	$\frac{k_p}{s(T_p s + 1)}$	$\frac{2\lambda + T_p}{\lambda^2}$	$2\lambda + T_p$	$\frac{2\lambda T_p}{2\lambda + T_p}$	-
F	$k_p \frac{-\beta s + 1}{s(T_p s + 1)}$	$\frac{1}{2\beta + \lambda}$	-	$T_p$	$\frac{\beta \lambda}{2\beta + \lambda}$

### 3. SIMULATION

*First order with time delay process*

It is considered a 1<sup>st</sup> order plus time delay process described by equation (1) with the following nominal values of the parameters:  $k_p = 0.08$ ,  $T_d = 2.5$  min and  $T_p = 9$  min.

ZN tuning rule leads to the following controller parameters:  $k_C = 60.24$ ,  $T_I = 5.7$  and  $T_D = 0.95$ .

By using AB tuning method, the controller parameters obtained are:  $k_C = 38.825$ ,  $T_I = 10.25$  and  $T_D = 0.56$  with adjustable parameter  $c = 0.32$ . For the process considered, a first-order Padé approximations for the time delay yields the following transfer function for the nominal process:

$$\tilde{G}_p(s) = \frac{-0.1s + 0.08}{11.25s^2 + 10.25s + 1} \quad (4)$$

This nominal process has the general form corresponding to case D from Table 2 and the PID controller parameters for IMC tuning rule have the values  $k_C = 41$ ,  $T_I = 10.25$  and  $T_D = 1.09$ . A first-order filter with time constant  $T_F$  augments the PID controller. For  $\lambda = 0.7$  the transfer function of the filter is

$$G_f(s) = \frac{1}{0.25s + 1} \quad (5)$$

As it can be observed, the controller parameters for AB and IMC are very different from those of ZN, especially  $k_C$  and  $T_I$ .

All the simulation work in this paper was performed using the package SIMULINK from MATLAB 5.0.

The performances of the three methods are compared in Figs. 1-4. The variations in time of the controlled variable ( $y$ ), for the nominal values of the process parameters, are plotted in Fig. 1. As it was expected, the time responses of the closed loop are much better for AB and IMC methods. This fact is also pointed out by the IAE criteria presented in Fig. 3. In Fig. 2 there are shown the time responses for a +40% error in time delay. This figure illustrates the robustness of AB and IMC methods. ZN method is at the instability limit.

Fig. 4 shows the responses of the three tuning methods to a unit step in load disturbance, for the nominal value of the time delay. AB and IMC methods produce satisfactory tuning, while ZN rule is too oscillatory. In the case of load disturbance, for all the tuning rules considered, the settling time is longer than in the case of step set point.

#### *Second order with time delay process*

It is considered a 2<sup>nd</sup> order plus time delay process described by equation (2) with the following nominal values of the parameters:  $k_p = 1$ ,  $T_d = 2$  min,  $T_{p1} = 1$  min and  $T_{p2} = 2$  min.

For ZN tuning rule the controller parameters are:  $k_C = 1.92$ ,  $T_I = 4.8$  and  $T_D = 0.8$ .

If AB tuning rule is used, the controller parameters are  $k_C = 1.1$ ,  $T_I = 4$  and  $T_D = 1.25$  with adjustable parameter  $c = 0.4$ .

For IMC tuning of the process considered, a zeroth order Padé approximations for the time delay yields the following transfer function for the nominal process:

$$\tilde{G}_p(s) = \frac{1}{(s+1) \cdot (2s+1)} \quad (6)$$

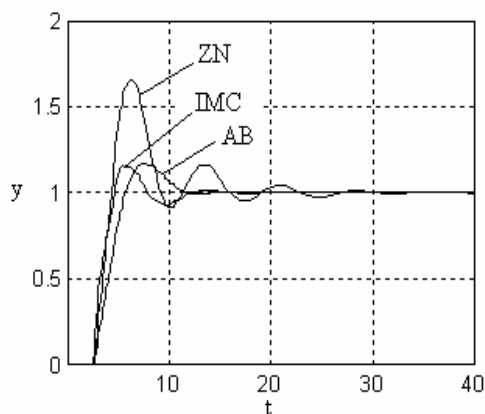


Fig.1. Time responses for  $T_d=2.5$  min, in the case of step set point

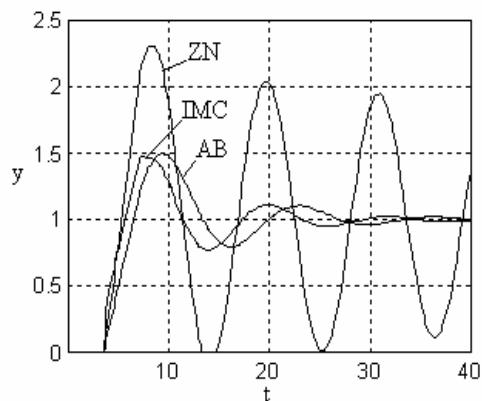


Fig.2. Time responses for  $T_d=3.5$  min, in the case of step set point

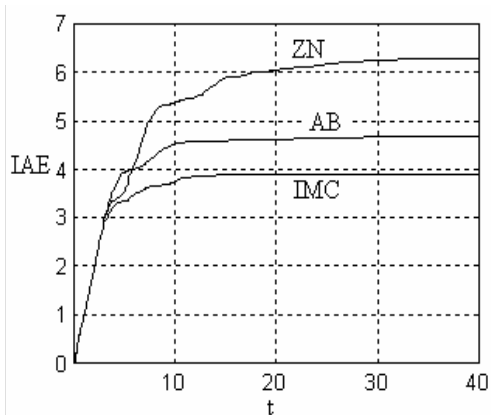


Fig.3. IAE criterion in the case of step set point

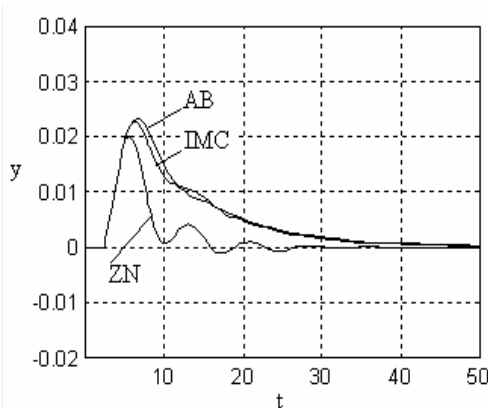


Fig.4. Time responses for  $T_d=2.5$  min, in the case of load disturbance

This nominal process corresponds to the case B from *Table 2* and the PID controller parameters for IMC tuning method have the values  $k_C = 1$ ,  $T_I = 3$  and  $T_D = 0.66$  for  $\lambda = 3$ .

In *Figs. 5-8* there are presented the results of simulations, leading to the same conclusions like those obtained in the case of a 1<sup>st</sup> order process with time delay. For ZN algorithm, the settling time is too long and the open loop becomes unstable for a +50% variation of the time delay (*Fig. 6*). For the process considered the IMC algorithm leads to better controller settings than AB algorithm: the overshoot is smaller (*Fig.5*), the robustness is greater (*Fig.6*) and the IAE value is less (*Fig.7*)

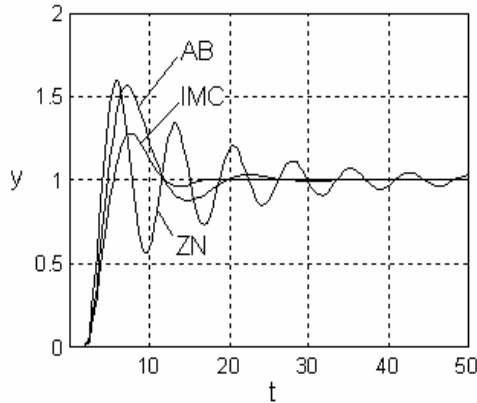


Fig.5. Time responses for  $T_d=2$  min, in the case of step set point

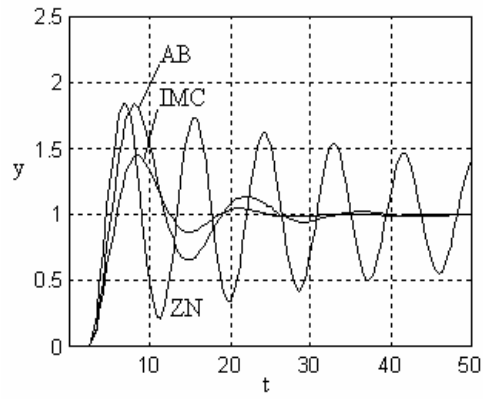


Fig.6. Time responses for  $T_d=3$  min, in the case of step set point

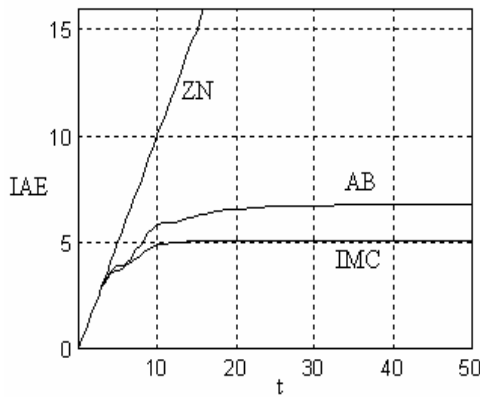


Fig.7. IAE criterion in the case of step set point

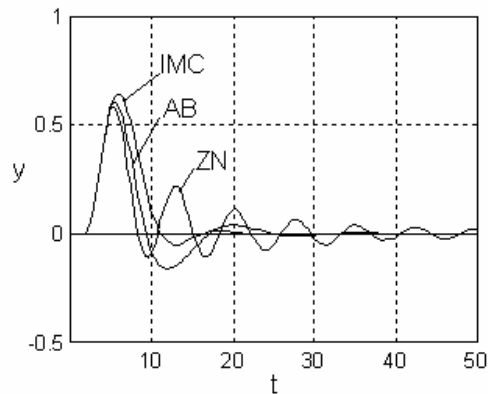


Fig.8. Time responses for  $T_d=2$  min, in the case of load disturbance

#### 4. CONCLUSIONS

This paper studied the performances of the closed loop for PID controllers tuned with ZN, AB and IMC algorithms. The simulations have shown that AB and IMC tuning methods lead to better controller settings than ZN method, both for 1<sup>st</sup> and 2<sup>nd</sup> order process with time delay, in the case of set point changes and disturbance. The IAE criteria pointed out this fact. Both IMC and AB methods are very robust in comparison with ZN method which is very sensitive to time delay changes. For ZN method, the close loop becomes unstable if a variation of +30-50% in time delay occurs.

The performances of AB and IMC are very close. These two methods require good knowledge about the process model (steady state gain, time constants and dead time) and this seems to be a disadvantage. But, due to their high robustness, the closed loop performances are not significantly affected by model uncertainty or by process conditions changes.

## REFERENCES

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