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PROBABILISTIC APPROACH AND SIMULATION METHODS FOR RISK ASSESSMENT

CĂLIN I. ANGHEL*, RADU IATAN**

** Department of Chemical Engineering, Faculty of Chemistry and Chemical Engineering, University "Babeş-Bolyai", 400028 Cluj-Napoca;*
*** Department of Technological Equipment, Faculty of Mechanical Engineering, University "Politehnica" Bucuresti,*

ABSTRACT. The purpose of this paper is to present a probabilistic approximation procedure for the risk assessment, suitable for technological equipments. It is reported like a risk of failure for a tank vessel type during its serviceable life. Simulation technique of a threshold value, named the limit state function (LSF), was prefered. It was conducted under the direct simulation method named *Latin Hypercube Sampling* (LHS). The *LSF* was establishing on the base of the growth of a local admissible flaw of the welding seams until the fracture occurs. A professional analysis package *Cristal Ball 2000-free trial version*, was used to perform the simulation. Finally the study reveals the probabilistic assessment of failure for a chlorine tank under technological loads and design parameters. The study estimates the risk of damage as a measure for the safety. Highly values for *LSF* lead to low values for the risk of failure. This threshold value may be a key factor for engineers to decide when the structure can become unsafe. The method can be used to predict the probability of failure, such as the limit-state in risk and reliability analysis. This type of study is suitable for chemical engineers to work out optimal inspection and mentenance schedules

1. Introduction

The behaviour of any technological equipment for process industries under operating conditions are always affected by variability and uncertainties; for example, variations in service loadings, scatters in material properties, chemical degradation and so one. The level of safety of these structures – often estimated by the load carrying capacity diminish with time and the risk of a major technological accident increase. Many studies were developed in order to maintain an acceptable level of safety and avoid technological incidents, achieving improvements in serviceability assessment. Some of the main issues related are: (a) introduction of qualitatively representation of service loads, (b) more consistent analysis of load effect combination and (c) transition to probabilistic reliability assessment concept.

The aim of this paper is to present a procedure suitable for engineers in the stage of a preliminary risk analyse, to avoid major technological incidents. It focuses on the reliability assessment concept. The development of computer technology allows a transition from deterministic structural reliability assessment methods to 'fully' probabilistic concepts using the full potential of the computer. The need to incorporate variability and uncertainties in an engineering design has long been recognized. One of the traditional approach, the so-called ''deterministic design'', makes use of safety coeficients in order to prevent unpredicted failures due to the variability of the data. On the other side, a relatively new trend, named "probabilistic design", allowing the estimation of the reliability of the design, considers the full stochastic variability of the data.

Briefly, the reliability of an engineering system can be defined as its ability to fulfill its design purpose for a time period. The fundamental base to measure this ability is provided by the theory of probability. So the reliability of a structure can be viewed as the probability of its satisfactory performance according to some performance functions under specific service conditions within a stated time period. In estimating this probability, system uncertainties are modeled using random variables with probability distribution functions. Many methods have been proposed for structural reliability assessment purposes, such as: first-order reliability method (FORM), first-order second moment method (FOSM), advanced second moment method (ASM) and computer-based simulation methods [6-8].

In this paper only the computer-based probabilistic simulation method for reliability assessment is presented. It is a direct simulation method, named *Latin Hypercube Sampling* (LHS). Latin hypercube sampling is considered to be a development of simple random sampling or *Monte Carlo Simulation*. It is more efficient than simple random sampling *Monte Carlo Simulation* (MCS). It requires fewer simulations to produce the same level of precision. Latin hypercube sampling is generally recommended over simple random sampling when the model is complex or when time and resource constraints are an issue. Like as *Monte Carlo Simulation*, *Latin Hypercube Sampling* is a computer-based method of analysis that uses statistical sampling techniques in order to obtain a probabilistic approximation to the solution of a mathematical equation or model.

The study reported as a probabilistic assessment of failure for a chlorine tank, under the technological loads and design parameters, has two distinct objectives. The first is the crude probabilistical estimation of failure of a tank-type reactor. The second is the preliminary assessment of the safety level or the risk level. Based on the general presumptions of the probabilistic simulation methods the formulation of a performance or limit state function (LSF) it is necessary. This function represents the total performance of the structure and includes the main operating and dimensional parameters. Simulation technique based on a professional risk analysis package Cristal Ball 2000-free trial version are used to estimate the probability of failure under operating parameters. The main stages of this approach are:

- establishing the *LSF* on the base of failure pressure function;
- establishing the probability of failure based on the *LSF;*
- assessing the risk level.

The model and the established *LSF* contain some idealizations and assumptions, which can introduce additional uncertainties. Supplementary a sensitive analysis-according variance reduction of the *LSF* to the basic variables on which depends may be performed. This is necessary to identify the most important variables in failure analysis, so the model could be improved by focusing on the most crtitical parameters.

2. Theoretical considerations

To avoid some cumbersome approaches, unnecessary for the purpose of this paper, only general design standard limitations [1,2] are considered. For simplicity, the main idealisations and assumptions are mentioned:

• basic variables: material properties, design parameters, rate of corrosion-erosion, operating parameters, depth of acceptable defect, etc. are assumed to be random variables;

• any estimator is statistic, hence any estimated parameter is a random variable;

• the random variables were assumed to be statistically independent – just for simplicity;

•it will be considered only two constructive areas of the tank: *"I"* for the collecting bottom closure and *"II"* for pipe connections "R7A , R7B";

There are a lot of methods to establish the failure pressure model. Based on general design standards [4,5,8] the failure pressure model choosen in this paper derived from one deterministic, when the damage is done by a pre-existing initial flaw on welding joints that escaped in non-destructive detection. Then the limit state function (*LSF)* is expressed as a failure pressure mode, reported to fracture mechanics, on the base of the critical energy for propagation the admissible flaws of the welding seams.

2.1 Latin Hypercube Sampling (LHS).

Latin hypercube sampling is a stratified sampling scheme designed to ensure that the upper or lower ends of the distributions used in the analysis are well represented. Because direct *Monte Carlo simulation* is a foundation for *Latin hypercube sampling* technique, some general statements is timely. The direct *Monte Carlo simulation* is a process of approximating the output of a model through repetitive random application of a model's algorithm. In the context of the *cumulative distribution function* of a real-valued random variable *X* the output of the model may be generally defined as $P(x) = \Pr ob\{X \le x\}$.

The corresponding notion for a multidimensional random vector $(X_1, X_2, \ldots X_n)$ is the *joint cumulative distribution function*

$$P(x_1, x_2, \ldots x_n) = \Pr ob\{X_i \le x_i \rightarrow for \cdots all \cdots i = 1 \ldots n\}.$$

Considering the *joint density function* we have the relationship :

$$\Pr ob\{x \in D\} = \int_D f(x_1, x_2, \ldots x_n) \cdot dx_1 \cdot dx_2 \cdots dx_n , \tag{1}$$

where $D \subset R^n$ for any Lebesgue measurable subset.

The reliability of a process equipment structure using the computer simulation methods can be estimated based on a performance or limit state function (LSF). In the context of previous statements these functions can be expressed in terms of basic random variables X_i for relevant loads and structural strength. Mathematically, the performance function *Z* can be described as

$$Z = Z(X_1, X_2, \ldots X_n) \tag{2}$$

where *Z* is called the limit state function (LSF) of interest. The unsatisfactory performance limit state of interest can be defined as $Z \le 1$. Accordingly, with $Z < 1$, the structure is in the unsatisfactory performance state and when $Z > 1 \approx D$ it is in the safe state. If the joint probability density function for the basic random variables X_i 's is $f_{X_1, X_2, \ldots X_n}(x_1, x_2, \ldots x_n)$ then the unsatisfactory performance probability P_U of a structure can be given by the integral:

$$P_u = \iint\limits_D \cdots \iint f_{X_1, X_2, \ldots X_n}(x_1, x_2, \ldots x_n) \cdot dx_{x_1} \cdot dx_2 \cdot \ldots dx_n \quad (3)$$

where the integration is performed over the domain *D* in which $Z > 1 \approx D$. In general, the joint probability density function is unknown and the integral is a difficult and cumbersome task.

Due to the difficulties in solving this integral Eq. (3) for practical purposes alternate methods of evaluating P_U are required. One of these methods is the direct simulation method named *Latin Hypercube Sampling* (LHS). This sampling method is a stratified sampling scheme designed to ensure that tails of the distributions used in the analysis are well represented. This simple and intuitive method consists in calculating Eq. (2) for a great number of combinations of X_i . The combinations, called "trials", are randomly sampled from the probability distribution of each X_i, by means of the standard random-generator functions implemented on any modern computer.

The probability P_U according to the performance function of Eq. (2) is provided by the integral of Eq. (3). The larger the margin of safety *Z* and the smaller its variance, the larger the needed simulation effort to obtain sufficient simulation runs with unsatisfactory performances. In other words, smaller unsatisfactory-performance probabilities require larger numbers of simulation cycles. Assuming N_U to be the number of simulation cycles for which $Z < 0$ in a total *N* simulation cycles, unsatisfactory performance probability P_U of a structure given by the integral Eq. (2) can be expressed as:

$$P_U = N/N_U \, . \quad (4)$$

2.2 Crack propagation model

To avoid cumbersome approaches only general design standard and several already published papers [1-6] limitations are considered. Based on general design standards settlements, the critical load model used in this paper derived from one deterministic model. Accordingly some scenarios of failure, mentioned in several already published works [3-6,8], the admissible value of a pre-existing initial flaw on welding joints that escaped in non-destructive detection is consider a threshold value for fracture. These flaws are approximated with a curved crack in a plane stress state. Based on the general statements of fracture mechanic, the *LSF* is established on the base of the spreading the local admissible flaws of the welding seams, until the fracture occurs. The flaws increasings are assumed to be linearly directly along the crack direction. The analyze is reported to the welding seam in the two areas of interest, on the base of critical energy of spreading the admissible flaws of the welding seams.

When the amplitude of stress intensity factor reaches the value for the impact ductility $KCV \approx K_{IC,}$ the crack propagation will become unstable. According to some previous papers [3,4] related to fracture for a ductile elastic-plastic steel type R52 STAS 2883/2-80, the *LSF* is reported as the matching of the fracture toughness of the materials K_{IC} and the stress intensity factor K_I. Based on the previous statements, the general form of the *LSF* in this stage is:

$$\mathrm{LSB} = K_{IC} / K_I \geq 1; \quad (5)$$

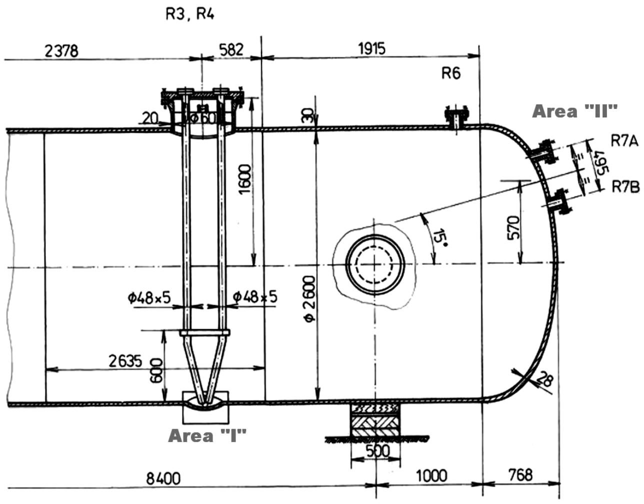

Fig. 1. *The main design and constructive dimensions of the tank.*

On these presumptions, two distinct *LSF* may be defined for the areas of interest:

"Area I"

$$LS1B = \frac{[1{,}034 \times E \times KCV]^{0{,}5}}{\left[1{,}77KA1 \times R_{S1}^{0{,}5}\left[\frac{Pe \times (D + Sp - Vc \times Tu)}{Ks \times (Sp - Vc \times Tu)} + \frac{10Mi \times De}{[De^4 - (D - 2Vc \times Tu)^4]}\right]\right]} \geq 1 \quad (6)$$

"Area II"

$$LS2B = [1{,}034 \times E \times KCV]^{0{,}5} \Big/ \left[1{,}77KA2 \times R_{F1}^{0{,}5}\left[\frac{Pe \times (D + Sp)}{Ks \times Sp}\right]\right] \geq 1. \quad (7)$$

Table 1.

The main values of design and simulation parameters

Parameters	Values	Statistical distribution
Elasticity modulus, E[MPa]	210000	N
Corrosion rate, Vc[mm/an]	0.5	N
Working elapsed time, Tu[year]	10	W
Fracture toughness, K_{IC}[N/mm$^{3/2}$]	2115	N
Yielding strength, $\sigma_{0,2}$[MPa]	340	N
Fracture strength, σ_r[MPa]	510	N
Inner diameter, D[mm]	2600	N
Design body thickness, Spc[mm]	25.5	N
Design bottom thickness, Spf[mm]	30	N

Parameters	Values	Statistical distribution
Operating pressure, Pe[Mpa]	1.6	W
Welding seam's radius, Rs1[mm]	200	N
Maximum bending moment, Mi[N*mm]	340000000	N
Hole's pipe diameter, d[mm]	40	N
Bottom's radius, Rif[mm]	1300	N
Welding seam's radius, Rs2[mm]	20	N
Welding factor *Ks*	1.8	N
*** According to technical book of the tank "Ch. 441.437 – 7" and Fig. 1		

The geometrical shape factors of the flaws in the areas of interest ***"I"-"II"*** were defined according related statement [3] by the following form:

$$\mathrm{KA1} = \left[0{,}5\sin\alpha_1 \times (1+\cos\alpha_1)^{0{,}5}\right] \Big/ \left[1-\sin^2\frac{\alpha_1}{2}\right] \tag{8}$$

$$\mathrm{KA2} = \left[0{,}5\sin\alpha_2 \times (1+\cos\alpha_2)^{0{,}5}\right] \Big/ \left[1-\sin^2\frac{\alpha_2}{2}\right]. \tag{9}$$

where the angular rates for the spreading of the flaws, α_1 and α_2, may be calculated by an approximate expression :

$$\alpha = \frac{2a}{2R_{C,F}} \times \frac{180}{\pi} \text{ [degree]}, \tag{10}$$

where *"2a"* is the initial or current size of the flaws.

Based on some general assumptions presented in some related papers [4,6,8], the crack propagation model to catastrophic failure is evaluated under the Paris-Erdogan's law,. in the following basic form:

$$\frac{da}{dn} = \frac{C \cdot (\Delta K_I)^m}{(1-R)\cdot[K_{IC} - \Delta K_I]^s} \tag{11}$$

where *da/dn* is the crack propagation rate, ΔK_I is the change in the stress intensity factor K_I at the crack tip for the "*nth"* cycle; *C*, *m*, s, ΔK_I, K_{IC} are material constants and *R* is the cyclic amplitude ratio. Finally the characteristic forms of the *LSF* in the areas of interest is given by the following expressions [4]:

"Area I"

$$N_{CI} = 8{,}17\times 10^{12} \times \int_{a_0}^{a_m} \frac{da}{K_{I1}^{2{,}93}} + 3{,}07 \div 10^{8} \times \int_{a_m}^{a_{critic}} \frac{2115 - K_{I1}}{K_{I1}^{2{,}61}} da\,; \tag{12}$$

"Area II"

$$N_{CII} = 8{,}17\times 10^{12} \times \int_{a_0}^{a_{critic}} \frac{da}{K_{I2}^{2{,}93}}. \tag{13}$$

The *LSF* defined by the previous Eq.(5-10) state the safety margins during tank's service. Reaches or overfulfilment the limit state values of these is evaluate probabilistic from *Latin Hypercube Sampling*. Thus the probability of *LSF***'**s values under the limit state range is defined as the safety domain. According to the basic probability statements, the probability of failure will be stated as the reverse. Finally, considering a comparative assessment of these scenarios, risk assessment of failure for the chlorine tank is judged on the base of the values of this limit state functions, yielding the probability of failure.

3. Numerical applications and discussions

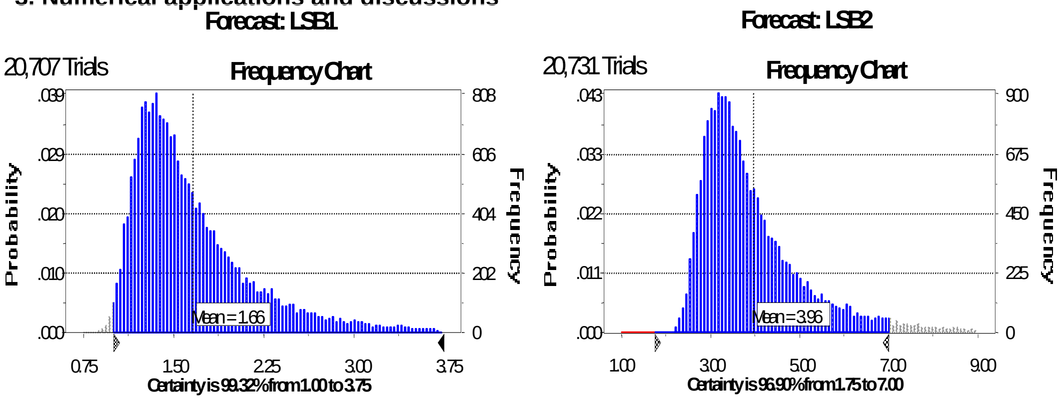

Fig.2. *Probability forecast for LSF on the base of the fracture propagation.*

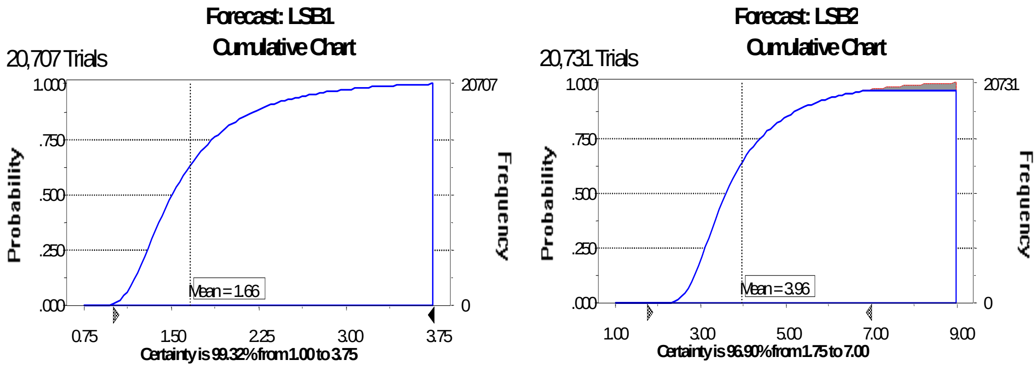

Fig. 3. *Cumulative probability forecast for LSF on the base of the fracture propagation.*

Numerical simulations based on two alternative principles, a maximum number of 100.000 trials or a control of precision defined by the minimum confidence parameters of 95% reported to standard deviation, were conducted under the *Latin Hypercube Sampling*. The probability trends for *LSF* reported as the safety coefficients and the matching of both fracture toughness K_{IC} and stress intensity factor K_I are significant for the *LSB1* in the *"Area I"*(Fig. 4). Thus on the basis of probability histograms (Fig.2-3.) the probability of crack progression to failure is meaningful only in the *"Area I***"**, when***:***

Probability of failure = 1- Certainty $\Rightarrow$1 – 0,9932 = 0,0068.

According to Mc.Leods and Plewes's scale, this probability of failure suits on the scale of risk in the range between $10^{-2}...10^{-3}$. This risk is characterized as a reduced one. Due to the variations in service load during service elapsed time, a fatigue damage can occur. Hence, the containment of this damage becomes essential for the safety working life of the tank.

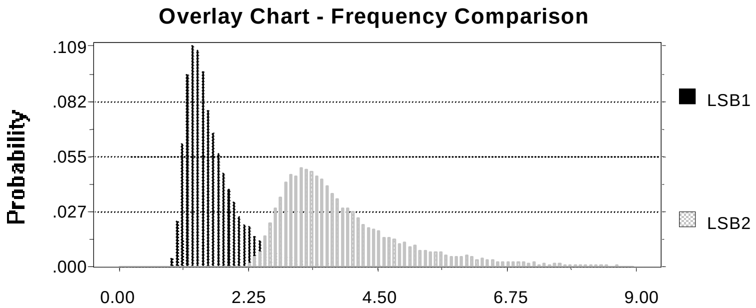

Fig. 4. *Overlay trend probability forecast for LSF*

Based on general crack propagation model, evaluated under the Paris-Erdogan's law Eq. (11-13), when serviceable elapsed lifetime is assimilated to one variable cyclic pressure low rate, numerical simulated values for the crack growth were obtained (Fig.5). Numerical simulated results and the linear crack propagation are in reasonable agreement with probability histograms.

On the base of these simulations, feasible N load cycles or the assessment of safety serviceable life may be evaluated (Table 2).

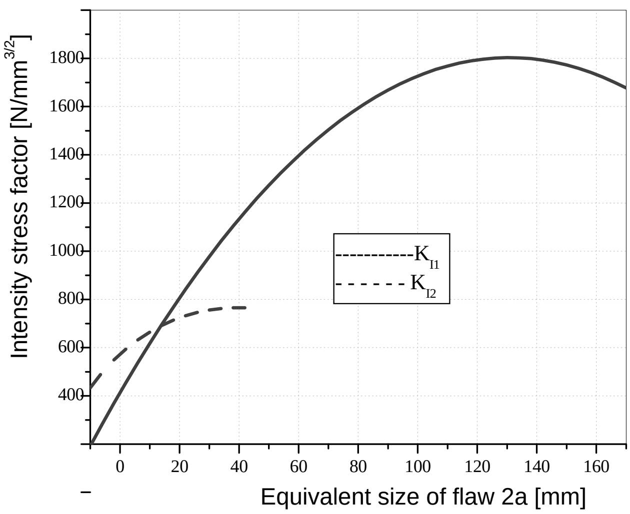

Fig. 5. *Crack growth considering the fracture propagation in the areas of interest.*

Table 2.

Crack propagation scenario for admissible flows

Area "I"		Area "II"	
Initial flaw 2a=3mm	Initial stress concentration factor K_{I1} =327[N/mm$^{3/2}$]	Initial flaw 2a=3mm	Initial stress concentration factor K_{I2} =433[N/mm$^{3/2}$]
Final flaw 2a=155mm	Final stress concentration factor K_{I1} =1815[N/mm$^{3/2}$]	Final flaw 2a=16mm	Final stress concentration factor K_{I2} =765[N/mm$^{3/2}$]
Predicted service lifetime cycles N_{CI} = 2.112×10^5 cycles		Predicted service lifetime cycles N_{CI} = 1,89×10^5 cycles	

Additional assumptions:

- the crack propagation will become unstable when $KCV \approx K_{IC,}$ $K_I \approx K_{IC}$.
- $K_{IC} \cong 2115$ [N/mm$^{3/2}$], the impact ductility of material;
- Assuming N≈100 load cycles/day during a serviceable life $T_U \approx 300$ days/year, the safety probabilistically serviceable life in the areas of interest are:
  - In the *"Area-I"* approximate 2,3 years;
  - In the *"Area-II"* approximate 2,1 years

4. Conclusions

The paper introduces a probabilistic approximation procedure for calculating the risk of failure, named the risk assessment for tank vessel during its serviceable life. Numerical results demonstrate a significant dependence on the operational and design state of the tank vessel. These approaches reduce the need for excessive safety margins in design and more cumbersome experimental and analytical approaches. High values for *LSF* lead to low values for the risk of failure. This threshold value may be a key factor suitable for engineers to decide when the structure can become unsafe. The method can be used to predict the probability of failure, such as the limit-state in risk and reliability analysis. This type of study is recommended for engineers, specially for chemical engineers to work out optimal inspection and maintenance schedules.

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