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Dedicated to Professor Valer Fărcăşan at his 85th anniversary

STRUCTURAL SAFETY, RELIABILITY AND SENSITIVITY

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ABSTRACT. The purpose of this paper is to present some methods available for performing reliability analysis. There are three main objectives. The first focuses on the fundamental link between safety and risk managing or evaluation. The second present the importance of sensitivity and probabilistic uncertainty analysis on complex technological or structural systems. The last introduces one's own procedure, hybrid cyclic recursive method, suitable for engineers in the stage of a preliminary risk analyze. It focuses on the reliability assessment concept.

Finally the study reveals a comparative assessment for the probability of failure implicit for reliability of a tank under technological loads and design parameters. The study estimates the risk of damage as a measure for the safety. Highly values for *most probable point* (*MPP*) lead to low values for the risk of failure. This threshold value is a key factor for engineers to decide when the structures become unsafe. The method is established to predict the probability of failure such as the limit-state in risk and reliability analysis. This type of study is suitable for structural or chemical engineers to work out optimal inspection and maintenance schedules, to avoid major technological incidents.

1. INTRODUCTION

Probabilistic methods and risk assessment also, started to be applied in the process industry in the late 1970s. It is now a well established tool for assessing most types of planned and existing chemical and hazardous materials installations, i.e. major accident hazard installations. Methodologies for both reliability and risk analysis advanced significantly in the 1980s. Since the early 1990s, probabilistic and quantified risk assessment is routinely applied to designs in many areas – structural failure represents one failure event, often the most important, in such risk assessments. The basic key features of the approach of risk assessment are hazard identification and risk analysis. Generally in engineering and particularly in structural engineering safety is perceived as a state or a quality - something to be achieved or assured it is a question of threats. Often safety, can be roughly defined as the state in which:

♦ a structure will not fail under some foreseeable conditions and it is unlikely to fail under some extreme circumstances;

♦ the probability of failure of structures during its serviceable life, are less than specified values.

Because the safety is a state or a quality, it can be achieved in different ways, but safety generally is not quantifiable. However there is a variety of ways in which safety can be assessed quantitatively when necessary. Despite the fact that "safety" and “risk” are very different in nature, “safety and risk” are closely related concepts; generally “risk” is quantifiable. A general principle is that whenever is possible the focus must change from measures of safety to managing risk. An important distinction between risk and safety is that risk is a quantity that can become very small indeed, but can never be zero. Because the ”risk” relates to a future event, it is an estimate, and is therefore uncertain. A number of definitions of risk may be found in the literature. The most usually definitions for “risk” are:

- ♦ a combination of the likelihood and the consequences of a future event;
- ♦ the failure probabilities for a number of different scenarios;
- ♦ the product between the probability of occurrence and the quantified consequence of a future event [2,3];

$$Risk = function(p_f \times Consequences) \approx p_f \times Consequences; \quad (1)$$

For technological or offshore structures, the probability of partial or complete failure of the structural integrity during the service life is one input, often the key input, into a risk assessment. When considering risk, structural engineers generally focus on probability of failure rather than on the consequence of a future event. Despite the fact that probability of failure by itself is limited as a measure of safety, this assumption is generally agreed as the right measure of safety. The last definition tends to be more useful in the process industry, in environmental contexts risk management, rather than in structural engineering. In Probabilistic Risk Assessment (*PRA*) the strategies of achieving safety are related closely to risk management ideas.

The assessment can be both direct, using various analytic procedures or it could be indirect using for example an indicator method. In any case, a two-tier approach is preferable: ensure the structure does not fail, but if it does, then minimize the consequences. In any possible strategies, both direct and indirect, dealing with risk management the problems of complexity and uncertainty must be in the center.

The behavior of any technological equipment for process industries under operating conditions are always affected by variations and uncertainties: fluctuations and variations in service loading, scatters in material properties, uncertainties regarding the analytical models, chemical degradation and so on. The level of safety of these structures diminishes with time and the risk of a major technological accident increases. Many studies [1-7] were developed in order to maintain an acceptable level of safety and avoid technological incidents, in operating conditions affected by variations and uncertainties.

Sensitivity analysis, as it is applied to risk assessment, is a common technique used to understand how risk estimates and in particular, risk-based decisions are dependent on variability and uncertainty in the factors contributing to risk. In short, sensitivity analysis identifies what is “driving” the risk estimates. It is used in both point estimate and probabilistic approaches to identify and rank important sources of variability as well as important sources of uncertainty. The quantitative information provided by sensitivity analysis is important for guiding the complexity of the analysis; such sensitivity analysis plays a central role in the tiered process for risk assessment.

There are some alternative methods of dealing with uncertainty (fuzzy theory, possibility theory), but the present discussion will be limited to those methods based on probability theory. Generally, uncertainties typically fall into one of two categories: probabilistic or possibilistic. Probabilistic techniques are characterized by the use of random variables to describe the various sources of uncertainty and are often referred to as reliability methods by structural engineers. These techniques are typically applied when the systems are of small to moderate: complexity from 100 to150 random variables. Possibilistic techniques involve the use of fuzzy set theory or possibility theory to model uncertainty and variability and are particularly useful when dealing with large, complex systems.

In the light of the previous discussion, the aims of this paper are:

♦ to present some methods available for performing reliability analysis;

♦ to present the importance of sensitivity and probabilistic uncertainty analysis on complex systems;

♦ to present one's own developed procedure, suitable for engineers in the stage of a preliminary risk analyze;

2. THEORETICAL CONSIDERATIONS

Briefly, the reliability of an engineering system can be defined as its ability to fulfill its design purpose for a time period. So the reliability of a structure can be viewed as the probability of its satisfactory performance under specific service conditions within a stated time period. There are two major categories of analysis methods used to estimate the probability of failure:

- analytical techniques
- random sampling methods

The first category is characterized by the use of analytical techniques to find a particular point in design space that can be related (at least approximately) to the probability of system failure. This point is often referred to as the most probable point (*MPP*) or the design point [5, 6, 8, 9,11].

The *First-Order Reliability* method (*FORM*) is widely used in reliability analysis due to its simplicity and speed. However, for problems with nonlinear limit states, the accuracy or convergence of *FORM* cannot be assured satisfactory. The *Second-Order Reliability* method (*SORM*) can improve the reliability estimation for nonlinear problems. However, it is found that reliability estimates from *SORM* may be far from the accurate solution as well. Moreover, for problems with a large number of random variables and implicit limit state, each *FORM* or *SORM* procedures will need many function evaluations and advanced second moment method (*ASM*) and computer-based simulation methods were developed.

The second category includes a broad class of random sampling methods characterized by the random selection of observations of each system parameter [6, 8, 11]. This category is dominated by traditional Monte Carlo methods (*MCS*) as well as numerous variations such as stratified sampling, *Latin Hypercube Sampling - LHS*, *Importance Sampling* (*IS*) and *Adaptive Importance Sampling* (*AIS*).

Monte Carlo methods have a long history in reliability and uncertainty analysis as function integrators. The basic concept of *Monte Carlo Simulation* is to replace a continuous average by a discrete approximation for that average. *Iterative Monte Carlo Simulation* (*IMCS*) procedure, utilizes results from simulation to adapt the importance sampling density. This method is especially suitable for system reliability analysis to estimate failure probability since multiple failure modes need not be treated separately. However, *Monte Carlo Simulation* or *Latin Hypercube Sampling* often requires prohibitively large computational effort, especially for low probability of failure, although the number of simulations is independent of the number of basic variables. Thus, in reliability and risk assessment, several more efficient and accurate calculation algorithms for analyzing complicated models have been proposed.

The *Adaptive Importance Sampling* (*AIS*) method can be used to compute component and system reliability and reliability sensitivities. The *AIS* approach uses a sampling density that is proportional to the joint probability density function of the random variables. Starting from an initial approximate failure domain, sampling proceeds adaptively and incrementally to reach a sampling domain that is slightly greater than the failure domain to minimize oversampling in the safe region. The *Robust Importance Sampling* method (*RISM*) calculates the reliability or its converse, the probability of failure. *RISM* first uses a tracking scheme to locate the failure domain. Further, an efficient adaptive sampling scheme is used to calculate the reliability with minimum computational effort. Fundamental to the approach, for both two previous categories of analysis methods used to estimate the probability of failure are the following goals:

- ♦ the concept of a limit state function;
- ♦ the characterization of uncertainty in system response due to uncertainty in either internal system parameters;
- ♦ the sensitivity analysis of system response to uncertainties in the system variables;

2.1 Limit State Concept

The starting point for all these methods is a performance function, which gives the relation between the chosen performance and the inputs of the model. This function represents the total performance of the structure and includes the main operating and dimensional parameters. Usually it is named as the *performance function,* the *limit state function* (*LSF=G*), or *system response function;* this function may be explicit or implicit.

For simplicity of discussion, in this section we use ***z*** to represent any element of the system output vector $\vec{z}_i$, x_i to represent all the input of random variables generally referred as *design* variables and *G* to represent the analysis corresponding to $\vec{z}_i$. Therefore, outputs of interest have formal functional relationship $z = G(x)$. In the reliability field, $z = LSF(x) = G(x)$ characterizes the function of a specific performance criterion *z* named a *limit state function*. The failure surface or the limit state (a set defined by the locus of points *G(x))* is defined as $G(x) = g_0$ or simply $G(x) = 0$ [5, 6, 9, 10, 11].

This is the boundary between the safe and failure regions in the random variables space: a region Ω where combinations of system parameters lead to an unacceptable or unsafe system response and a safe region Ω where system response is acceptable. When *G(x)* > *0*, the system is considered safe and when *G(x)* < *0*, the system can no longer fulfill the function for which it was designed. Only for simplicity Fig. 1 shows the limit state for a particular state, a two dimensional problem. The use of the terms “failure” is also customary, since only the likelihood of a particular system state may be of interest rather than system failure.

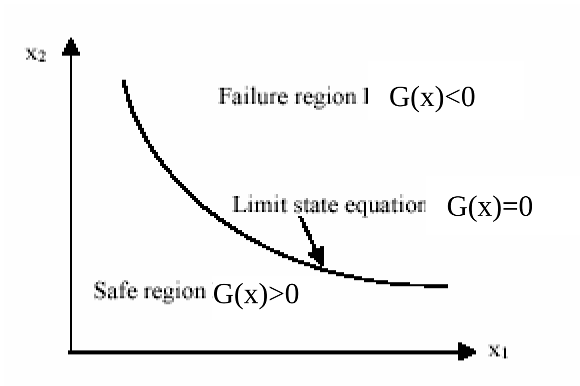

Fig. 1. Limit State Concept

The probability of system failure p_f is defined as the probability of the event that the system can no longer fulfill its function and is given by the expression:

$$p_f = P\{G(x) < 0\} \tag{2}$$

generally calculated by the integral

$$p_f = \iint\limits_{G(x)<0} \dots \int f(x)dx \tag{3}$$

where *f(x)* is the joint probability density function (*PDF*) of *x* and the probability is evaluated by the multidimensional integrals over the failure region *G(x)* < *0*.

Because the reliability *R* is the probability that the system works properly, it is given by the expression:

$$R = P\{G(x) > 0\} = 1 - p_f \tag{4}$$

In many situations it is very difficult or even impossible to analytically compute the multidimensional integrals (2). Alternative methods to evaluate the integration are random sampling methods. However, when the probability of failure p_f is very small the computational efforts of random sampling methods are extremely expensive.

To overcome this difficulty, Hasofer and Lind had proposed the concept of the *Most Probable Point* (*MPP*) to approximate the integration [6]. This *MPP* is the point on the limit state that lies closest to the origin, $u^* = (u^*_1 \ldots u^*_n)$. There is a direct relationship between the safety index and the probability of failure:

$$p_f = \Phi(-\beta) \tag{5}$$

where β is the safety index defined (Hasofer and Lind) as the shortest distance, in normal space *u*, from the origin to a point on the limit state surface (Fig. 2). When it is used in the context of $p_f = \Phi(-\beta)$ it is assumed that $\beta > 0$. In general, the relationship (5) is only approximate, but in the unique case of a linear combination of Gaussian distributed random variables where $\Phi(\ldots)$ is the cumulative normal density function, the relationship is exact.

Using the *MPP* concept, the input random variables $x = \{x_1 \ldots x_n\}$ - in the original design space x_n - must be transformed into an independent standardized normal space $u = \{u_1 \ldots u_n\}$. Rackwitz -Fiessler - Rosenblatt in addressing statistical dependency between the random variables [6, 11,14] gives the most commonly used transformation. The methods imply that the transformation forces the two cumulative densities function and joint probability density function to have equivalent similar statistical properties or to be identical both in x_n-space and u_n-space. The limit state function may be now rewritten as:

$$G(x) = G(u) = 0 \tag{6}$$

Searching for β can be formulated as a minimization problem with an equality constraint:

$$\left\{\beta = \min(u^T u)^{0.5} \ldots subjected..to..G(u) = 0\right\} \tag{7}$$

The joint probability density function on the limit state surface has its highest value at the *MPP* (Fig. 2) and therefore the *MPP* has the property that in the standard normal space it has the highest probability of producing the value of limit state function *G(u)*.

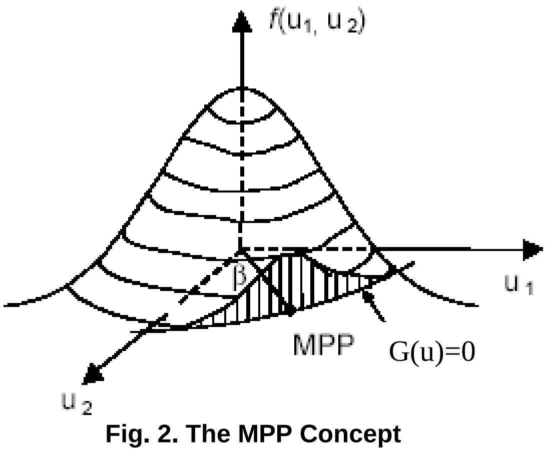

Fig. 2. The MPP Concept

If, in the u_n space, the limit state function *G(u)* is linear or the principal curvatures of the limit state surface is not so large, *FORM* give an accurate estimate for probability of system failure p_f by the expression (5). Contrary, other optimization algorithms or searching methods well known in structural reliability [6, 9, 10, 14] must be used to solve (7). Assume *"n"* the number of the system inputs – random variables " x_i " in the ***LSF*** and a formal expression for this limit state function in real Euclidean R^n space as:

$$LSF = g_j(x_i) : R^n \rightarrow R \quad (i = 1,2,\ldots n \;;\; j = 1,2,\ldots m, \text{ and } m<n) \quad (8)$$

Because the input random variables $x_i = \{x_1..x_n\}$, in the original design space x_n, must be transformed into a standardized normal space $u_i = \{u_1\ldots u_n\}$ the new limit function in terms of reduced variables, is given by the expression:

$$LSF = g_j(u_i) : R^n \rightarrow R \quad (i = 1,2,\ldots n \;;\; j = 1,2,\ldots m, \text{ and } m<n) \quad (9)$$

The point on the limit state that lies closes to the origin, $u^* = (u^*_1, u^*_2, \ldots, u^*_n)$ referred as *MPP*, must be evaluated. In general [6, 11] the distance from the point u^* to the limit state $g_j(u_i) = 0$ is given by the expression:

$$d = \left[\sum_{1}^{n} u_i^*\right]^{0.5} \quad (10)$$

The difficulty then lies in determining the minimum distance for a general nonlinear function. The reduced variables corresponding to the *MPP* can be found in a various number of ways [9, 10, 12, 14], following approaches involving iterative, vectorial or gradient solutions. It is obviously a straightforward nonlinear constrained optimization problem:

$$\text{Minimize: } d = \left[\sum_{1}^{n} u_i^*\right]^{0.5} = \left(u^{*T} u^*\right)^{0.5}$$

$$\text{Subject to : } g_j(u_i) = 0 \quad (11)$$

2.2 Characterization of uncertainty and sensitivity analysis

In the risk and structural analysis there are many variabilities and uncertainties in loading, material properties, geometry, and environmental conditions. These uncertainties should be taken into consideration carefully in order to ensure that the design performs its function within the desired confidence limit without failure. In robust design, it is important not only to achieve design objectives but also to maintain the robustness of design feasibility under the effects of variations caused by uncertainties. In the light of these the main demands of these previous mentioned methods are:

♦ characterization of variability and uncertainty in system;

♦ sensitivity analysis of system response to uncertainties in the system variables;

In response to the problem, methods have been developed to deal with the random nature of loads material properties and environmental conditions. ***Variability and Uncertainties*** are typically modeled in terms of the mean, the variance and the distribution. Various reliability estimation techniques use part or all of this information in different ways [4, 5, 6, 8].

Sensitivity Analysis can involve more complex mathematical and statistical techniques such as correlation and regression analysis to determine which factors in a risk or structural model contribute most to the variance in the final estimate. These techniques have their specific advantages and limitations. Because optimization algorithms or searching methods in structural reliability, based on *MPP* method, are developed in independent standardized normal space $\boldsymbol{u} = \{u_1 \ldots u_n\}$, this paper provides additional information only on the underlying principles of *Sensitivities of Limit State Function* and *Probabilistic Sensitivities of the Safety Index.* Nevertheless, it is not a comprehensive summary and is not intended to substitute for the numerous statistical books and journal articles on sensitivity analysis [1, 2]. The basic concept of this sensitivity analysis is to understand how variability and uncertainty in the reliability problems influence the estimates. So, sensitivity analysis can provide information to support additional testing in an efficient manner. Sensitivity measures when used in the context of the previous analytical methods are often referred to as *importance factors*. The magnitudes of these factors characterize the impact of each of the random variables on the safety index and thereby, their impact on the probability of failure. The importance factors represent the direction cosines of the individual random variables in reduced space and are defined as:

$$\gamma_i = \left(\frac{\partial G(u)}{\partial u_i}\right)_{u=u^*} \Bigg/ \left(\left(\sum_1^n \left(\frac{\partial G(u)}{\partial u_i}\right)^2\right)^{0.5}\right)_{u=u^*} \tag{12}$$

where u is the current variable in reduce space and u^* is the variable corresponding to the possible *MPP* on limit state surface. As a computation check it is noted that:

$$\Sigma\gamma^2_i = 1 \text{ and } 1 \le \gamma_i \le 1 \tag{13}$$

The partial derivative of *G* (Fig. 3) with respect to each random variable *u* gives a measure of its sensitivity to that variable. If the probabilistic characteristics of the response of a system are not significantly impacted by statistical variation in certain variables, those variables can be omitted from the probabilistic analysis.

The sensitivity of Y ($Y=G(u_i, X_i)$) denotes a model output with respect to *u or X* calculated as the slope at a specific point, or the partial derivative.

Thus the generality of the analyze diminishes, dominant and secondary variables may be establish and the searching for safety index β are simplified. Information from sensitivity analysis can be important when trying to determine where to focus additional resources. If the input variables are all discrete and take a small number of values, it is possible to evaluate the influence of the various input variables at each of the defined points by considering all possible combinations of the inputs by computing normalized partial derivatives at each point. This detailed approach is limited to situations where the number of inputs and the number of possible values for each input are relatively small or moderate.

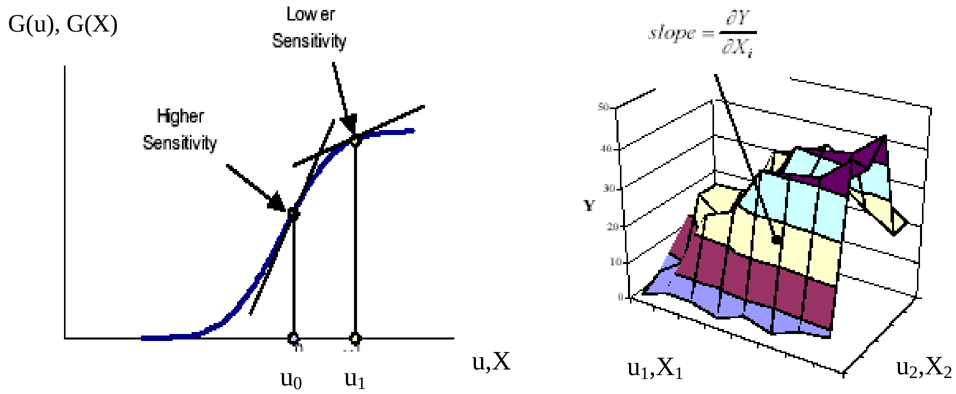

Fig. 3. Exemplary sensitivity state.

A similar approach may be used to analyze inputs that are continuous variables. Generally, in a *PRA*, many of the inputs will be random variables described by probability distributions and it will be necessary to quantify the influence of each input, u_i over the entire range of $\boldsymbol{u}$. As previously noted, if the relationship between the model output and all of the inputs is nonlinear, the influence of a particular input may vary depending on the value of that input. One approach to this problem is to consider a range of values for the input and to examine the influence over that range. If the input is considered to be a random variable following some specified probability distribution, then it may be desirable to look at the influence that the random input has on the model output across the distribution of input values.

To improve the efficiency of sensitivity analysis, when evaluating the influence at different points on the input-output simultaneously, it is important to take into account the probability associated with each of those points. The fact that a particular input has a large influence on the model output at a particular point would be discounted if the probability associated with that particular point is very low. Changes to the distribution of a variable with a high sensitivity could have a profound impact on the risk estimate, whereas even large changes to the distribution of a low sensitivity variable may have a minimal impact on the final result. To overcome this difficulty we propose one hybrid cyclic recursive method "*sensitivity analysis - safety index - sensitivity analysis*" conducted by random sampling points across the distribution of input values.

2.3. Cyclic recursive method and reliability analysis

By principle, this cyclic recursive algorithm belongs to the *Most Probable Point* (*MPP*) methods. Roughly it is a hybrid sampling-analytical procedure, conducted by random sampling points across the distribution of input values.

The proposed method will be presented in this section and demonstrative examples will be given in the next section. This particular method has its own specific advantages and limitations. As was shown in previous section the starting point of the method is to establish a performance function- named as the *limit state function* (*LSF=g*). This function may be of explicit or implicit form. For simplicity we chose one explicit form, to be presented. This function, giving the relation between the chosen performance-or critical state and the inputs of the model, must represent the total performance of the structure and includes the main operating and dimensional parameters.

To avoid some cumbersome approaches, unnecessary for the purpose of this paper the main assumptions are mentioned:

• we consider a series type structural system;

• basic variables: material properties, design parameters, operating parameters, environmental conditions, etc. are assumed to be random variables;

• at any stage the active loads are assumed to be stationary and ergodic;

• any estimator is statistic, hence any estimated parameter is a random variable;

• the random variables may be statistically independent or dependent-correlated;

• in the u_n space the limit state function g(u) may be nonlinear or the magnitude of the curvatures of the limit state may be large;

• we consider the functions that define the curvatures of the limit-state surface as a convex one and belonging at least to category C^1;

• the technique considers information regarding the underlying density function by random sampling points across the distribution of input values;

Based on these assumptions and the previous approach outlined in this section the main stages of this approach are presented.

2.3.1. Establishing the LSF

The first step is a random sampling procedure in agreement to the original distribution of all the input variables. An ample number of random sampling points [2,12,13] are necessary to establish a values domain, enough to reflect the variability and uncertainty of each variable. Thus the original inputs that are continuous variables described by probability distributions are transformed in some equivalent discrete, thus the number of combinations to be evaluated will be reasonable managed. The ranges of values for these inputs are considered intervals between sampling points. These statements are necessary both on sensitivity analysis to establish the model output across the distribution of input values and for searching or solving the *MPP*.

2.3.2. Searching for β or the MPP

At this stage we propose a dual mathematical approach involving *MPP* searching. The proposed approach consists in the following intermediate steps:

♦ Perform a sensitivity analysis to establish dominant *"k"* variables, thus new reduced Gaussian space becomes $u_k = \{u_1 \ldots u_k\}$, where $k < n$;

♦ All the other variables are expressed in terms of the mean of their distributions;

♦ Focus on these new variables a new limit function in terms of only these variables is given:

$$LSF = g_j\,(u_k) : R^k \to R \quad (i = 1,2,\ldots k\ ;\ j = 1,2,\ldots m,\ and\ \ m < k) \tag{14}$$

♦ Transform the problem of *MPP* searching into one dual problem:

• Consider the limit state function as one closed domain –surface in u_k space:

$$S : g_j\,(u_k) = 0 \tag{15}$$

• Consider the objective function *f(u)* a distance function from one point $O(u_{01},u_{02},\ldots,u_{0n})$ in u_k -space (particularly identical with the origin of standardized normal space u_i) to the previous mentioned closed domain – surface, $S(u_1,u_2,\ldots,u_k)$ as one strict convex function:

$$f(u) = d^2(P,S) \ = \ d^2(u_0,u\ \ ) = \sum_{1}^{n} (u - u_0)^2 \tag{16}$$

♦ Searching for *MPP* is then formulated as an extreme searching problem, with a nonlinear equality constraint:

$$\min\left(f \mid g_i = 0;\ldots i = 1,2\ldots m;\ldots m < n\right) \tag{17}$$

These statements suggest that the extreme point - associated *MPP* - are placed at the base of a normal straight line through the point *"P"* belonging to the surface *"S"*. The range of values of the input variables – in multiple random combinations – are considered as searching points. Multiple cyclic searching approaches are done to solve the system of equations (17). Thus the location of a lot of possible *MPP*, has been found. These points may represent a vector of the safety index $\boldsymbol{u_{jk}}$, associated with their importance factors at each step. Based on direct relationship like (5), the probability of failure for all possible *MPP* may be established.

♦ Checking the concordance between the probability of failure and the importance factors, associated with each of these points, we accept only those points that have not been discounted for low associated probability.

Obviously these points must to belong to the domain of random sampling points and satisfy the system of equations (17). This cyclic algorithm will be stopped when no significant changes in concordance between the probability of failure and the importance factors, will be accomplished.

2.3.3. Establishment of β or the MPP

There are some possibilities to establish the *MPP* or the probability of failure.

♦ One direct possibility is the value expressed as *min (vector of the safety index* $\boldsymbol{u_{jk}}$*)*. This gives a unique assessment for safety and risk. It is well suited for analyses with unique limit state function.

♦ Another possibility to establish the *MPP* and the probability of failure is based on interval probability theory [15, 16].

3. NUMERICAL APPLICATIONS AND DISCUSSIONS

To avoid cumbersome approaches, we present only a demonstrative example based on several already published papers [6, 16]. Accordingly some scenarios the admissible value of a pre-existing flaw (a void on welding joints) in both area of interest [16] *"Area I - II"* is considered as a threshold value for possible fracture propagation. For a specific length of time in storage, the crack size, *"A_i = 2a"*, as a function of initial stress, S_0 and grain size Δ is approximated by [6]:

$$A = S_0 \cdot \left[0.01694 - 0.01353 \cdot \exp\left(-\frac{0.4158}{\Delta} \right) \right] \tag{18}$$

Failure of the welding seams may occur when *"A_i = 2a"* the initial or current size of the flaws, exceeds some critical level, named *Ac*. Thus:

$$p_f = P\{A_C < A\} \quad or \quad p_f = P\{A_C - A < 0\} = P\{g(\mathbf{x}) < 0\} \tag{19}$$

Based on the previous statements the form of the *LSF = g(x)* in this stage is:

$$g = A_C - S_0 \cdot \left[0.01694 - 0.01353 \cdot \exp\left(-\frac{0.4158}{\Delta} \right) \right] \tag{20}$$

Note that here $\mathbf{x} = \{ x_1 , x_2 \} = \{S_0 , \Delta \}$. It will suffice to assume that, critical level named *Ac* , random variables - initial stress S_0 and grain size Δ - are independent statistically with known first and second moments and exact form of the probability density function (Table 1).

Table 1.

The main values of design and simulation parameters

Parameters	Nominal Value	Statistical distribution
Critical width *Ac[mm]*	2.7	Constant
Initial stress S_0 [N/mm^2] (variable x_1)	300	Weibull μ = 300 α = 5.687 η = 324.34
Grain size Δ [mm] (variable x_2)	1.25	LogNormal μ = 300 σ = 60

Since the random variables are independent, the first step is to transform the random variables into reduced Gaussian space $x = x\ (x_1, x_2\ ) \rightarrow u\ (u_1, u_2)$, using the moments of the equivalent normal distribution and transformation given by Rackwitz and Fiessler. That is:

$$\mu' = x - S' \cdot \Phi^{-1}[F(x)] \quad and \quad \sigma' = \frac{\phi\{\Phi^{-1}[F(x)]\}}{f(x)} \tag{21}$$

where $\Phi(...)$ and $\phi(...)$ are the standard normal *cdf* and *pdf*, respectively *F(x)* and *f(x)* are the proper cumulative density function and density function of original variables. The new variables in reduced Gaussian space become:

$$u_{1,2} = x_{1,2} - \mu'_{1,2} / \sigma'_{1,2} \tag{22}$$

An ample number of random sampling points, in accordance with the original distribution for each variable, are sampled to establish domain values. In every points of this domain the previous equations set (22-23) are applied. The new expression for *LSF* in normal reduced space is determined:

$$g(u_{1,2}) = A_C - (\mu_1 + x_1 \cdot \sigma_1) \cdot \left[0.01694 - 0.01353 \cdot \exp\left(-\frac{0.4158}{\mu_2 + x_2 \cdot \sigma_2} \right) \right] \tag{23}$$

To establish the location of *MPP,* the proposed cyclic recursive algorithm is applied for the simultaneous system of equations (17), reduced to a particular form. Since the case of a linear combination of Gaussian distributed random variables is established we can evaluate the probability of failure by the previous expression (5).

Based on the approach outlined in previous section, a suitable computer program in *MATLAB* package was implemented on a *PC - AT* microcomputer *IBM* compatible. It allows a complete analysis to estimate the probability of failure. Comparative numerical results are presented in the form of graphs and tables.

The first step, establishing the new limit state function (in reduced space) is depicted in the following figures, Fig. 4. The quantitative information provided by sensitivity analysis (Fig. 5), characterize the impact of each of the random variables on the probability of failure. Thus the *"u1"* variable may be named the principal variable in reduced space and it has the main impact on the safety index β. For a two dimensional problem such this one, the minimum distance to the failure surface - the associated *MPP* and the safety index β is found on the base of the previous mentioned cyclic recursive algorithm.

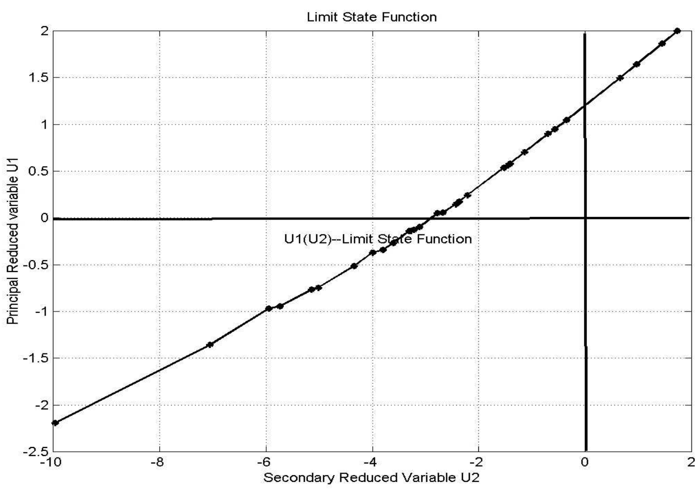

Fig. 4. Limit state function in reduced space

Table 2 and Figures 6-7 outlines the results from the intermediate steps. The best probability associated with each of the safety index β (see Table 2 and Fig. 5-6) arise on the following model outputs:

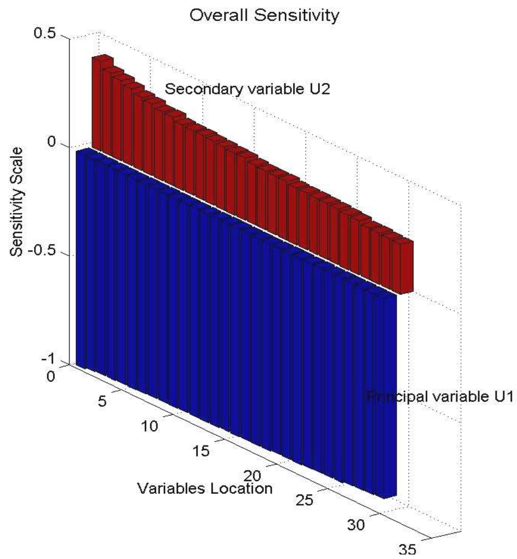

Fig. 5. Sensitivity chart in reduced space

U_1 importance factor	***U_2 importance factor***	***Safety index β.***
- 0.95965	***0.28116***	***9.721010188446312e-001***

Table 2.

Comparative numerical results

Model values [6]

Safety index β. = 0.977;
U_1 -importance factor = - 0.9619; U_2 -importance factor = 0.2733;

Numerical simulated results

U_1	U_2	β
8.317475978416375e-001	-1.901175118435460e-001	8.531991179231934e-001
7.812844076073880e-001	-1.831978656824919e-001	8.024754099416677e-001
8.351389158766590e-001	-2.057840513927728e-001	8.601186456642274e-001
8.488781805891856e-001	-2.162520608404962e-001	8.759903648432257e-001
8.858156089383533e-001	-2.386643520747695e-001	9.174039273897269e-001
8.923309629711115e-001	-2.454157566989017e-001	9.254639058942524e-001
9.328863757148401e-001	-2.733192288230011e-001	9.721010188446312e-001
8.236437357911779e-001	-4.928923902097007e-001	9.598603605914141e-001

Associated sensitivity

U_1 importance factor	U_2 importance factor	Safety index β.
-0.97485	0.22282	8.531991179231934e-001
-0.97359	0.22829	8.024754099416677e-001
-0.97095	0.23925	8.601186456642274e-001
-0.96904	0.24686	8.759903648432257e-001
-0.96556	0.26015	9.174039273897269e-001
-0.96419	0.26518	9.254639058942524e-001
-0.95965	0.28116	9.721010188446312e-001
-0.85808	0.31350	9.598603605914141e-001

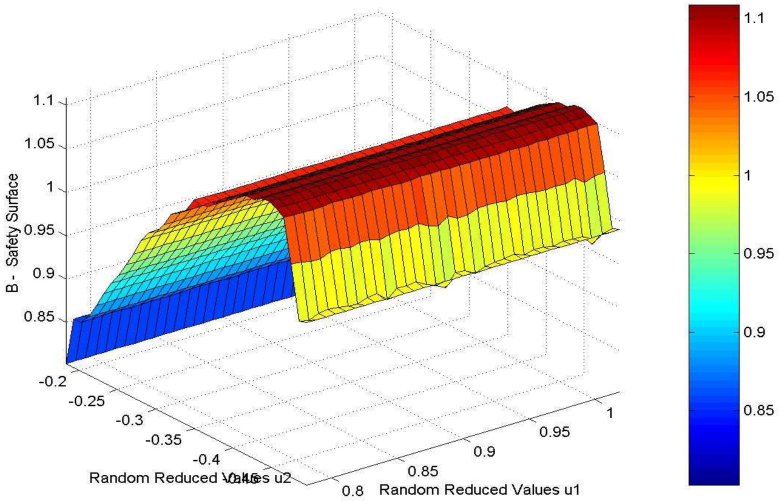

Fig. 5. Safety surface in reduced space

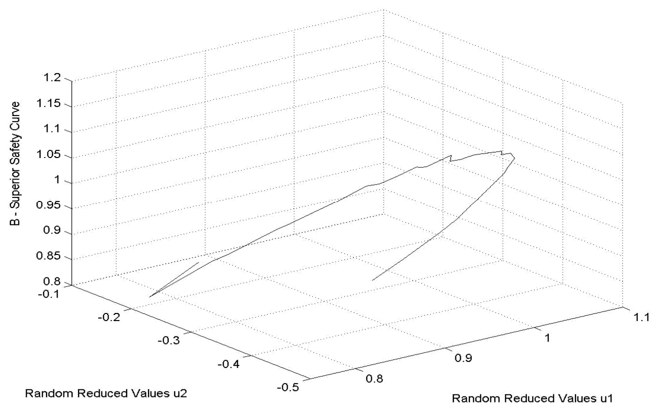

Fig. 6. Safety curve in reduced space

The presented analysis based on the proposed approach produces reasonable accurate results comparatively with the particular model values [6]. Relative errors occur under 1 %. Further the probability of failure obtained using the proposed approach produces not only a single value, butt it produces one inter val of probability values. The bounds of this interval may reflect completeness high uncertainty and variability of variables and may characterize better the probability of failure in the vicinity of tails of real or random variables.

4. CONCLUSIONS

The paper presents a probabilistic cyclic recursive algorithm for calculating the risk of failure, named the risk assessment. Also it accents on the sensitivity analyze as a key factor which could have a profound impact on the risk estimate. The numerical results are observed to be accurate as well as efficient compared with existing methods. The study may offer a greater reliability in life prediction. Highly values for safety factor β lead to low values for the risk of failure. Some approaches, like this, reduce the need for excessive safety margins in design and more cumbersome experimental and analytical approaches. The method can be used to predict the probability of failure, such as the limit-state in risk and reliability analysis. These types of study become not only recommended, but also necessary, for engineers, especially for structural or chemical engineers to work out optimal safety decisions.

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