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IS IT POSSIBLE TO APPLY UNITARY MATHEMATICAL TREATEMENT TO FLUID FLOW?

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ABSTRACT. In the paper are presented the fundamental ideeas of modified general dimensional analysis and its application for to the fluid flow in contact with solids having diferent geometries. A mathematical study is performed and a correletion between the working parameters is proposed resulting equation deduced by application of the modified general dimensional analysis. Comparation between the obtained and clasical relationship is also presented.

Keywords: dimensional analysis, fluid flow, dimensionless relations

Cases in which fluids, liquids and gases are contacted by solids through chemical reactions, the transfer phenomenon intensification, or due to the necessity of physical presence of parting walls of conducting pipes are often encountered in the chemical practice. This phenomenon is interesting from the point of view for transfer phenomena intensification as well for the materials transport through pipes. Due to the large number of possible situations: fluid flow in a pipe, contacting into packed columns, the presence of other interfaces, the passage through different pipe details: elbows, tees, valves, derivations, entrances and exists from storage tanks, etc… even though there are specific parameters precisely determined for each particular situation, the question raises: is it possible to apply a unitary treatment to this phenomenon, irrespective of obstacle's geometrical form.

In this work we propose a way to present the unitary treatment of hydrodynamic process of fluid flow by means of the general dimensional analysis method. Thus, we try to give an answer to the question in the title of this article. In this sense, in conformity with the General Dimensional Analysis Theory, (GDAT), presented by Staicu, [1], there are three fundamental stages that must be followed to find the solution:

1. – establish the list of variables that influence the process directly or in reverse sense;
2. – establish the monomial type relationship describing the treated phenomenon;

3 – determine the numerical coefficient describing the process.

I propose to introduce another working stage in the methodology [10]. This new stage should be placed between the two first stages and is expected to lead to a certain hierarchy upon the importance of variables describing the process. Thus, the real mathematical basis may be revealed by neglecting certain variables that are empirically considered to be less important, a situation often encountered in the practical and experimental studies. Moreover, after establishing the monomial type relation, we may operate mathematically upon it to obtain some criteria relations describing the process. Criteria expressions show the influence that different types of forces exert on the system, presenting details about this process phenomenology that would not be relevant otherwise, being dissimulated by several aspects.

The model bellow describes pure fluid flow through spaces with any geometry and develops the working algorithm to find the general relation to calculate fluid flow and all explanations that derive from here in the light of the question in the title.

In the 1st stage, the matrix line of the variables describing the fluid flow process and their directly or reverse influence is formed. The list of all possible variables and the way of their distribution is presented bellow:

$$// \Delta p, d_{ech}, ;\ w, H, \rho, \eta, \sigma, g // \quad (1).$$

The following may appear on the list of variables: pressure drop, Δp, the equivalent diameter, $d_{ech,,}$ flow length or the height of packed bed, H, the flow speed , w, the dynamic viscosity, η, interfacial strength, σ and earth acceleration, g. Into the matrix line, variables are distributed according to the influence they have upon the process as follows:

- equivalent diameter of flow, d_{ech} operates reversely upon the pressure loss, which represents the studied parameter;
- length or height of flow and speed, density, dynamic viscosity of fluids and earth acceleration have a direct effect upon the increase of the pressure drop.

The grouping of certain physical particularities of the solid can also be noticed. These are as follows: specific surface, $S_{p,}$ and void fraction, ε, in a single one, $d_{ech} = 4\varepsilon/S_{sp}$, which is in fact the specific dimension of fluid flow, or considering the fluid speed as the real one, w, either the fluid fictive one w_f in the packed bed or on the geometrical area causing fluid flow deformation. Other measures describing the flow can also be taken into account as mass or volumetric flow rate. These parameters are considered to be composed of simpler ones:

$Q_M = Q_V * \rho = w * A * \rho$, so the result of calculus is the same.

The additional proposed stage development is, that of evaluation of the importance of functional parameters and we may start form the minimum list of variables describing the process. This can be determined from the solving condition of the system of undetermined diophantian equations imposed by GDAT: minimum, integer, positive and non-null solution. The only combination of variables which complies with the imposed condition is the one bellow:

$$// \Delta p, d_{ech}, ;\ w, H, \rho //, \quad (2),$$

and the dimensional matrix and the system of non-determined equations attached to the matrix are presented hereinafter:

	Δp^a	d_{ech}^b	w^c	H^e	ρ^f
L	- 1	1	1	1	- 3
M	1	0	0	0	1
T	- 2	0	- 1	0	0

$$\begin{cases} -a+b=c+e-3f \\ a=f \\ -2a=-c \end{cases}$$

The system of undetermined equations can be solved by the method of progressive homogenization of diophantian relations:

The solution is as follows: a = 1, f = 1; c = 2; b = 1, e = 1; and the monomial type relation generated by it, according to GDAT is:

$$\Delta p = k_1 \frac{w^2\ H\ \rho}{d_{ech}}\ , \qquad (3).$$

The relation is dimensionally homogenous and after the mathematical operations have been performed upon it, the result is an expression also accepted by the classical method of dimensional analysis, Buckingham:

$$\frac{\Delta p}{w^2\ \rho} = k_1 \cdot \frac{H}{d_{ech}}\ ;\ (4), \quad \text{or} \quad Eu = k_1 \frac{H}{d_{ech}}\ , \qquad (5).$$

We may notice that the fluid flow over a solid is, in the most general case, caused by the following fundamental parameters: pressure drop, speed, density and geometrical configuration of the flow space. Moreover, under constant conditions, the pressure drop Δp is constant. As these are basic parameters, they will be also present in all the other relations, and therefore they, as such, would present their most “rude” influence on fluid flow.

If we add to the minimum list of variables some other additional variables, one by one extracted from the general list, we can obtain the following lists of variables:

// Δp, d_{ech}, ; w, H, ρ, η //, (6),
// Δp, d_{ech}, ; w, H, ρ, σ //, (7),
// Δp, d_{ech}, ; w, H, ρ, g // (8).

For each list there’s a specific relation:

$$\Delta p^2 = k_2 \frac{w^3\ H\ \rho\,\eta}{d_{ech}^{\ 2}}\ ,\ (9), \quad \frac{\Delta p^2}{w^4\ \rho^2} = k_2 \frac{\eta}{w\, d_{ech}\ \rho} \cdot \frac{H}{d_{ech}}\ ,\ (10),$$

$$Eu^2 = k_2\ Re^{-1}\ \frac{H}{d_{ech}}\ , \qquad (11);$$

$$\Delta p^2 = k_3 \frac{w^2\ H\ \rho\,\sigma}{d_{ech}^{\ 2}}\ ,\ (12), \quad \frac{\Delta p^2}{w^4\ \rho^2} = k_3 \frac{\sigma}{w^2\ d_{ech}\ \rho} \cdot \frac{H}{d_{ech}}\ ,\ (13),$$

$$Eu^2 = k_3\ We^{-1}\ \frac{H}{d_{ech}}\ , \qquad (14).$$

In the case of the third list, (8), NO solution is accepted by GDAT, which means that the variable introduced additionally does not influence the process directly and fundamentally. This proves that earth acceleration is a secondary parameter for the flow process. In fact, the pressure drop is the key element for the fluid movement. From the practical and phenomenological point of view, we may ascertain that fluid movement is only sometimes influenced by gravitation or either acceleration forms, which is proved by particular calculation relations.

After the introduction in the matrix line of the variables that were not studied, we obtain the following result:

$// \Delta p, d_{ech}, ;\ w, H, \rho, \eta, \sigma //,$ (15),

$// \Delta p, d_{ech}, ;\ w, H, \rho, \eta, g //$ (16).

The result of the general dimensional analysis is:

$$\Delta p^3 = k_4 \frac{w^3\ H\ \rho\, \eta\, \sigma}{d_{ech}^{\ 3}}\ , \qquad (17),$$

$$\frac{\Delta p^3}{w^6 \rho^3} = k_4 \frac{\eta}{w\, d_{ech}\ \rho} \cdot \frac{\sigma}{w^2\ d_{ech}\ \rho} \cdot \frac{H}{d_{ech}}\ , \qquad (18),$$

or $Eu^3 = k_4\ Re^{-1}\ We^{-1}\ \frac{H}{d_{ech}}$, (19).

$$\Delta p^2 = k_5 \frac{w\, H\ \rho\ g}{d_{ech}}\ ;\ (20),\quad \frac{\Delta p^2}{w^4 \rho^2} = k_2 \frac{\eta}{w\, d_{ech}\ \rho} \cdot \frac{H\ g}{w^2\ d_{ech}} \quad (21),$$

or $Eu^2 = k_5\ Re^{-1}\ Fr^{-1}$, (22).

The fact that earth acceleration can complete fluid flow relation only in the presence of other parameters is now relevant.

For the complete list of variables:

$// \Delta p, d_{ech}, ;\ w, H, \rho, \eta, \sigma, g //,$ (23).

the following are obtained:

$$\Delta p^3 = k_6 \frac{w\, H\ \rho\, \eta\, \sigma\, g}{d_{ech}^{\ 2}}\ , \qquad (24),$$

$$\frac{\Delta p^3}{w^6 \rho^3} = k_6 \frac{\eta}{w\, d_{ech}\ \rho} \cdot \frac{\sigma}{w^2\ d_{ech}\ \rho} \cdot \frac{H\ g}{w^2}\ , \qquad (25),$$

$Eu^3 = k_6\ Re^{-1}\ We^{-1}\ Fr^{-1}$, (26).

The relation generated by all considered parameters is in agreement with the bases of the dimensional analysis, the л theorem of Buckingham, which means that a physical phenomenon is described by a product of dimensionless groups at a certain index. Moreover, as compared to the initial theorem, the general dimensional analysis may indicate the form of function, except the numerical factor for the analyzed case:

$Eu^3 * Re* We* Fr =$ constant, (27).

The literature [2,3,4] shows in the case of fluid flow over solids, dimensionless relations in the following form:

Eu = f(Re), (28), Eu = f(Re, d_{ech}/e), (29), Eu = f(Re, We, Fr), (30), etc…

similarly to those inferred by general modified dimensional analysis, but defining the form of functions f only after the experimental determination of all numerical factors.

In the case of bi or multiphase fluids flow, the problem is very much the same, only the point of view has shifted to determine the drop or ascension speed of particle, this being the main parameter of the process. In fact, we are dealing again with liquid displacement, this time as compared to a mobile point of reference, the particle. In this particular case, the list of variables also include, besides the principal parameter, w, the diameter of the particle, d_{ech} the density difference between phases Δρ, fluid viscosity, η, and acceleration, g.

The matrix line of distributed variables can be written as follows:

// w, η ; d_{ech}, Δ ρ, g //, (31).

We can now notice that the place of some variables was reversed, due to the different phenomenology of processes. The lack of interfacial strength in the line matrix is due to absence of experimental data to quantify the model.

The dimensional matrix and the system of undetermined equations are shown bellow:

	w^m	η^n	$d_{ech}^{\ p}$	$\Delta\rho^q$	g^r
L	1	- 1	1	- 3	1
M	0	1	0	1	0
T	- 1	- 1	0	0	- 2

The system attached to the matrix line becomes as follows:

$$\begin{cases} m - n = p - 3q + r \\ n = q \\ -m - n = -2r \end{cases}$$

and has the solution accepted by GDAT: n = 1, q = 1; r = 1, m = 1; p = 2; with monomial-type relation:

$$w = k_7 \frac{d^2{}_{ech}\ \Delta\rho\ g}{\eta}, \quad (32).$$

The mathematical transformation of monomial type relation leads to the dimensionless relation:

$$\frac{w\ d_{ech}\ \rho}{\eta} = k_7 \frac{d^3{}_{ech}\ \Delta\rho\ \rho\ g}{\eta^2}, \quad (33), \text{ or } \quad Re = k_7\, Ar, \quad (34),$$

Thus, a correlation between Reynolds and Archimedes's criterion is shown as a final relation. The well-known expressions in literature are particularized as follows:

1. Re = $\frac{Ar}{18}$ Ar ≤ 36, Stockes's relation, [2,3],

2. $\text{Re} = \left(\frac{Ar}{13{,}9}\right)^{5/7}$ 36 ≤ Ar ≤ 84 000, Allen's relation, [3,4],

3. $\text{Re} = 1{,}73\sqrt{Ar}$ Ar ≥ 84 000, Newton's relation, [3,4],

4. $\text{Re}_{cr} = \frac{Ar}{1400 + 5{,}29\sqrt{Ar}}$, Thodes's relation for determination of minimum fluidization speed, [5,6],

5. $\text{Re}^2{}_{pl} = \frac{4\,Ar}{3\,C_f}$ Relation for floating speed of some monodispersed particles, [8],

6. Re_{pl} = f(Ar, D/d_{ech}, β, φ,) Relation for floating speed of some polydispersed irregular particles, [7],

7. $\text{Re}_{antr} = \frac{Ar}{18 + 0{,}61\sqrt{Ar}}$ Relation for minimum transport speed from exhaust particles of fluidized bed, [6],

8. $w_{\min} = 8 \cdot 10^{-4}\,\frac{d^2{}_{ech}\,\Delta\rho\,g}{\eta}$, Rowe's relation for minimum fluidization speed, [7,8],

9. $w_{\min} = 9{,}35 \cdot 10^{-3}\,\frac{d_{ech}^{1{,}88}}{\upsilon_{fluid}^{0{,}88}} \cdot \left(\frac{\Delta\rho}{\rho_{fluid}}\right)^{0{,}94}$, Leva's relation for minimum fluidization speed of some polydispersed non spherical particles, [7],

10. $w_{\min} = \frac{1}{200} \cdot \frac{\varepsilon}{\varepsilon - 1} \cdot \frac{d_p^2\,\rho_p\,g}{\eta}$; Lewis's relation for minimum fluidization speed of some monodispersed non spherical particles, [7],

11. $\text{Re} = 0{,}049\,Ar\,(1-\varepsilon)^{0{,}8}\left(\frac{d_M}{d_m}\right)^{0{,}48}\left(\frac{\rho_f}{\rho_p}\right)^{0{,}2}$, Carpov's relation for fluidization speed of some poly dispersed non spherical particles, [8],

12. $$\text{Re}_m = \frac{Ar_M}{150 \cdot \frac{1-\varepsilon}{\varepsilon} \cdot \left[1 + \sum_{i=1}^{n-1} m_i\,(d_i - 1)\right]^2 + \sqrt{\frac{1{,}75}{\varepsilon_0^3}\left[1 + \sum_{i=1}^{n-1} m_i\,(d_i - 1)\right] \cdot Ar}},$$

Petrov's relation for fluidization speed of some screening non spherical particles, [8],

13. $Re = 5,1 * 10^{-2} Ar^{0,59} (d/D)^{0,1} (H/D)^{0,25}$, Nicolaev's and Golubev's relation for spouted bed speed of mono-dispersed particles, [8].

We can thus notice a similitude between the relation obtained trough application of modified general dimensional analysis and those particularized formulas generated by means of specific experiments. The differences consist in the value of numerical constants. This fact proves the existence of identical phenomenology, differentiated only by strict particularities of the systems which provided us with the experimental data.

To conclude, we can now assert that the method of general dimensional analysis, modified by the introduction of some differentiating criteria between variables, can be used for a global approach of some phenomena which have a common basis phenomenology.

NOTATIONS

L, M, T = the symbols for measurement units for length, mass and time;

a, b, c, d, e, f, h, i, j, m, n, p, q, r = exponents for measurement units;

$Eu = \frac{\Delta p}{w^2 \rho}$ Euler number; $Re = \frac{w d \rho}{\eta}$ Reynolds number;

$Fr = \frac{w^2}{d_{ech} g}$ Froude number; $We = \frac{d_{ech} w^2 \rho}{\sigma}$ Weber number;

$Ar = \frac{d_{ech}^3 g \rho \Delta\rho}{\eta^2}$ Archimede number.

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