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ADHESIVE ASSEMBLIES OPTIMIZATION

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ABSTRACT. This work presents a theoretical model of calculation of stuck cylindrical assemblies, based on an energy method. After the determination of the acceptable field of constraints, according to the applied load, a variational calculus on the expression of elastic potential energy makes it possible to lead to the complete expression of the stress field in the whole assembly. A first parametric analysis (geometrical and physical parameters) is carried out on an assembly of tubes and makes it possible to deduce the optimal length and the thickness of joining.

Keywords: Adhesion and adhesives; Energy methods; Variational calculus.

INTRODUCTION

Joining is a powerful technique of assembling which makes it possible more and more to replace or to supplement the other traditional methods of assembly (welding, riveting or the bolting). However, the optimization of this type of assembly passes by the determination of the constraints in the adhesive and the substrates; stresses are strongly influenced by the geometrical and physical parameters of the assembly.

The mechanical performance of an adhesive bonded joint is related to the distribution of the stresses in the adhesive film [1]. Consequently it is essential to know this distribution which, because of its complexity makes prediction of fracture difficult.

The first studies are developed on plane assemblies with simple covering loaded in traction. The work of Volkersen [2] developed into 1944 leads to a false evaluation of the maximum level of constraint because the effects of inflection of the supports are not taken into account.

Compared to the number of recent scientific publications concerning plane joints, with simple or double lap [3, 4], there are only a few theoretical publications concerning the study of the mechanical behavior in adhesive bonded joints, with symmetry of revolution, subjected to traction [5, 6].

The first theoretical studies concerning the cylindrical adhesive bonded joints are due to Lubkin [7] and Volkersen [8]. They suppose that the supports do not become deformed.

Therekhova and Skorkyi, [9], considered the influence of the pressures which are exerted inside and outside the tubes.

Kukovyankin and Skorkyi, [10], were interested in the action of the moments and the axisymmetric forces which make it possible to introduce the inflection into the tubes. In this work the orthoradiale constraints are not taken into account.

The most recent work, concerning the type of assembly considered, is by Shi and Cheng, [6]. They build a first stress field using the equilibrium equations and the conditions of continuity of the stresses at the interfaces using an equation of compatibility. They then calculated the potential energy associated with this field and using the theorem of the minimal complementary energy, they obtain a system of differential equations whose solutions allow the determination of the optimal stress field.

However, if the field of the constraints obtained checks well a part of the equations of compatibility, this does not check therefore the totality of the equations of compatibility.

The treated numerical examples propose the zones of constraints which appear at the ends of the joints. The others also provide an outline of the consequence of the variations of the various geometrical and physical parameters on the distribution and the intensity of shear stresses in the adhesive.

All the developed techniques, based on the resolution of the associated differential equations, encounter a not overcome difficulty which is the taking into account of the boundary conditions at the ends of the joint. The damage declaring itself in these zones, it is thus important to model the effects edge.

Armengaud, [11] and Nemes [12] used a technique based on the minimization of the potential energy. The first stage consists in building a statically acceptable stress field, i.e. checking the boundary conditions and the equilibrium equations. The second stage consists in calculating the potential energy generated by such a stress field. In the third stage the use of the second theorem of the potential energy results in minimizing this energy, in order to determine the stress distributions.

The analytical approaches which relate to the cylindrical stuck interfaces are applicable at the time of estimated calculations, therefore for preparatory project, but there is not any doubt that, under complexes loading, the digital simulation is a stage impossible to circumvent if we want to optimize an adhesive assembly.

RESULTS

Analytical formulation

All works showed some difficulties encountered in modeling the stress field in the vicinity of the ends of the join.

The method used to obtain the optimal field for this type of assembly consists of:

- the construction of a statically acceptable field,
- the calculation of the potential energy associated to this stress field,
- the minimization of this energy by variational calculus,
- the resolution of the differential equation obtained.

Definition of the statically acceptable field

In this work we consider an assembly of stuck tubes subjected to a tensile load whose geometrical definitions are represented in fig. 1. The parameters of the assembly are: E_{it}, E_{il} - Transversal and longitudinal Young's modulus, v_{tli} - Poisson's ratio, r_i - Internal radius of the inner tube, r_{ic} - External radius of the inner tube, r_{ec} - Internal radius of the external tube, r_e - External radius of the external tube, L - Joining length, f - Tensile stress following z axis on the inner tube, q - Tensile stress following z axis on the external tube.

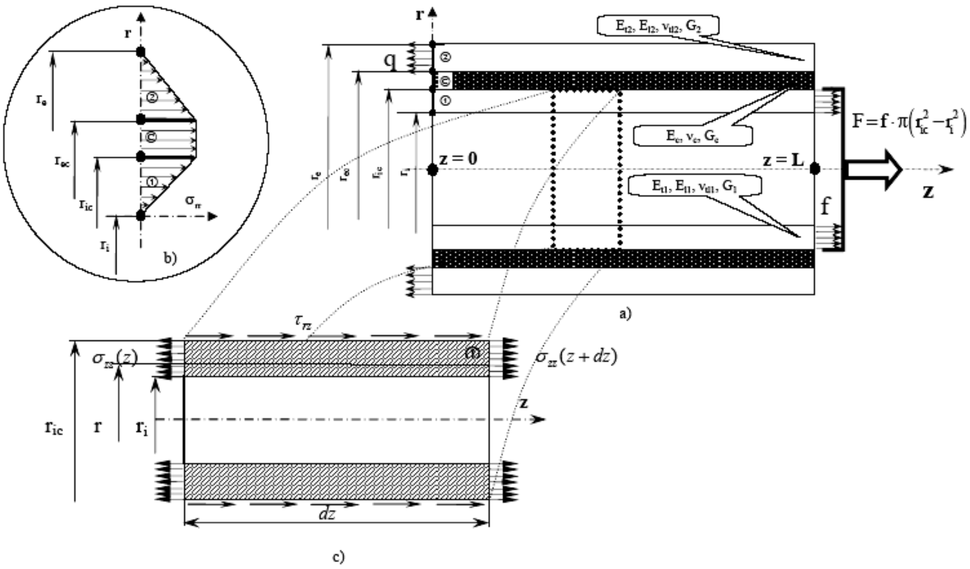


Figure 1. Cylindrical assemblies:

a) Diagram of the adhesive bonded joint;

b) σ_{rr} distribution in the cylindrical assembly;

c) Equilibrium of an elementary section of the inner tube, $r \in [r_i,\ r_{ic}]$.

The stresses in various materials will be located by the index (i), (i = ① for the inner tube, © for the adhesive and ② for the external tube).

To build the statically acceptable field, we will adopt the following hypothesis:

- the symmetry of revolution imposes that the shear stresses are nulls:

$$\tau_{r\theta} = \tau_{z\theta} = 0 \tag{1}$$

- the normal stress in the adhesive will be neglected:

$$\sigma_{zz}^{(©)} = 0 \tag{2}$$

- the axial stress will be a function only of variable z.

The stress field is reduced to the following components:

- for the inner tube (①):

$$\sigma_{zz}^{(1)}(z),\ \tau_{rz}^{(1)}(r,z),\ \sigma_{\theta\theta}^{(1)}(r,z),\ \sigma_{rr}^{(1)}(r) \tag{3}$$

- for the adhesive (©):

$$\sigma_{\theta\theta}^{(©)}(z),\ \tau_{rz}^{(©)}(r,z),\ \sigma_{rr}^{(©)} = \text{cst.} \tag{4}$$

- for the external tube (②):

$$\sigma_{zz}^{(2)}(z),\ \tau_{rz}^{(2)}(r,z),\ \sigma_{\theta\theta}^{(2)}(r,z),\ \sigma_{rr}^{(2)}(r) \tag{5}$$

Table 1 shows the components of the stress field developed in the references. We can notice that only the formulations of Armengaud [11] and Nemes [12] do not take into account the radial stress, by supposing them null.

Table 1

Comparative table of the stress fields.

References	zones	σ_{zz}	$\sigma_{\theta\theta}$	σ_{rr}	τ_{rz}	$\tau_{\theta z}$
Armengaud [11] Nemes [12]	Adhesive	/	$\sigma_{\theta\theta}(r,z)$	/	$\tau_{rz}(r,z)$	/
	Substrates	$\sigma_{zz}(z)$	$\sigma_{\theta\theta}(r,z)$	/	$\tau_{rz}(r,z)$	/
Shi and Cheng [6]	Adhesive	/	$\sigma_{\theta\theta}(r,z)$	$\sigma_{rr}(r,z)$	$\tau_{rz}(r,z)$	/
	Substrates	$\sigma_{zz}(r,z)$	$\sigma_{\theta\theta}(r,z)$	$\sigma_{rr}(z)$	$\tau_{rz}(r,z)$	/
Lubkin and Reissner [7]	Adhesive	/	/	$\sigma_{rr}(z)$	$\tau_{rz}(z)$	/
	Substrates	$\sigma_{zz}(z)$	/	$\sigma_{rr}(z)$	$\tau_{rz}(z)$	/
This study	Adhesive	/	$\sigma_{\theta\theta}(z)$	$\sigma_{rr}(z)$	$\tau_{rz}(z)$	/
	Substrates	$\sigma_{zz}(z)$	$\sigma_{\theta\theta}(r,z)$	$\sigma_{rr}(r,z)$	$\tau_{rz}(z)$	/

The equilibrium equations of an elementary volume of adhesive bonded joint length dz are:

$$\frac{\partial}{\partial r}[r\sigma_{rr}] + \frac{\partial}{\partial z}[r\tau_{rz}] = \sigma_{\theta\theta} \tag{6}$$

$$\frac{\partial}{\partial r}[r\tau_{rz}] + \frac{\partial}{\partial z}[r\sigma_{zz}] = 0 \tag{7}$$

The radial stress field (Fig. 1) is supposed equal to:

$$\sigma_{rr}^{(1)} = \alpha_1[r - r_i]\,;\ \sigma_{rr}^{(c)} = \beta_c = \text{cst.}\,;\ \sigma_{rr}^{(2)} = \alpha_2[r - r_e] \tag{8}$$

The continuity of σ_{rr} makes it possible to write:

$$r = r_{ic} \ \rightarrow\ \beta_c = \alpha_1[r_{ic} - r_i] \tag{9}$$

$$r = r_{ec} \rightarrow\ \beta_c = \alpha_2[r_{ec} - r_e] \tag{10}$$

$$\beta_c = \alpha_1[r_{ic} - r_i] = \alpha_2[r_{ec} - r_e] \tag{11}$$

Expressions of the stresses in the adhesive bonded joint

In the inner tube (①):

The equilibrium of an elementary section of the tube enables us to express the shear stress $\tau_{rz}^{(1)}$:

$$\tau_{rz}^{(1)}(r,z) = \frac{\left(r_i^2 - r^2\right)}{2r}\frac{d\sigma_{zz}^{(1)}}{dz} \tag{12}$$

From expression (12) and equilibrium equation (6), we express directly the orthoradial stress in the inner tube ①:

$$\sigma_{\theta\theta}^{(1)}\left(r,z\right)=\frac{r_i^2-r^2}{2}\frac{d^2\sigma_{zz}^{(1)}}{dz^2}+\alpha_1[2r-r_i] \quad (13)$$

In the adhesive ©:

Using the equilibrium equation:

$$\frac{\partial\tau_{rz}}{\partial\, r}+\frac{1}{r}\tau_{rz}+\frac{\partial\sigma_{zz}}{\partial\, z}=0 \quad (14)$$

and the continuity condition of the shear stress for $r = r_{ic}$, enable us to obtain the expression for the shear stress $\tau_{rz}^{(c)}$:

$$\tau_{rz}^{(c)}(r,z)=\frac{\left(r_i^2-r_{ic}{}^2\right)}{2r}\frac{d\sigma_{zz}^{(1)}}{dz} \quad (15)$$

The expression of the orthoradial stress in the adhesive is obtained by the same manner like for the inner tube ①:

$$\sigma_{\theta\theta}^{(c)}\left(z\right)=\frac{r_i^2-r_{ic}{}^2}{2}\frac{d^2\sigma_{zz}^{(1)}}{dz^2}+\alpha_1[r_{ic}-r_i] \quad (16)$$

In the external tube (②):

The expression of the normal stress $\sigma_{zz}^{(2)}$ can be given starting from the equation:

$$\left(r_{ic}^2-r_i^2\right)\sigma_{zz}^{(1)}+\left(r_{ec}^2-r_{ic}^2\right)\underbrace{\sigma_{zz}^{(C)}}_{=0}+\left(r_e^2-r_{ec}^2\right)\sigma_{zz}^{(2)}=\left(r_{ic}^2-r_i^2\right)f=\left(r_e^2-r_{ec}^2\right)q \quad (17)$$

that is to say:

$$\sigma_{zz}^{(2)}\left(r,z\right)=\left(\frac{r_{ic}^2-r_i^2}{r_e^2-r_{ec}^2}\right)\left(f-\sigma_{zz}^{(1)}\right) \quad (18)$$

The expression of the shear stress in the external tube can be given in two ways, either by considering the equilibrium of a section of tube, or using the equilibrium equation (7) and the condition of continuity of the same constraint at the interface with the adhesive.

These two methods lead to the same expression:

$$\tau_{rz}^{(2)}(r,z)=\frac{\left(r_e^2-r^2\right)\left(r_{ic}^2-r_i^2\right)}{2r(r_{ec}^2-r_e{}^2)}\frac{d\sigma_{zz}^{(1)}}{dz} \quad (19)$$

The orthoradial stress is obtained immediately and is written:

$$\sigma_{\theta\theta}^{(2)}\left(r,z\right)=\frac{(r_e^2-r^2)(r_{ic}^2-r_i^2)}{2(r_{ec}^2-r_e^2)}\frac{d^2\sigma_{zz}^{(1)}}{dz^2}+\underbrace{\alpha_1\frac{r_{ic}-r_i}{r_{ec}-r_e}}_{\alpha_2}[2r-r_e] \quad (20)$$

The field is entirely determined and its components are written according to the axial stress in the inner tube ($\sigma_{zz}^{(1)}$).

Expression of the deformation energy

The expression of the deformation energy is:

$$\xi_P = \pi\int_0^L\int_{r_i}^{r_{ic}}\left[\frac{\sigma_{\theta\theta}^{(1)2}}{E_{1t}}+\frac{\sigma_{zz}^{(1)2}}{E_{1l}}-\frac{2\nu_{tl1}}{E_{1t}}\sigma_{zz}^{(1)}\sigma_{\theta\theta}^{(1)}+\frac{\tau_{rz}^{(1)2}}{G_1}\right]rdrdz + \\ +\pi\int_0^L\int_{r_{ic}}^{r_{ec}}\left[\frac{\sigma_{\theta\theta}^{(c)2}}{E_c}+\frac{2(1+\nu_c)}{E_c}\tau_{rz}^{(c)2}\right]rdrdz + \\ +\pi\int_0^L\int_{r_{ec}}^{r_e}\left[\frac{\sigma_{\theta\theta}^{(2)2}}{E_{2t}}+\frac{\sigma_{zz}^{(1)2}}{E_{2l}}-\frac{2\nu_{tl2}}{E_{2t}}\sigma_{zz}^{(2)}\sigma_{\theta\theta}^{(2)}+\frac{\tau_{rz}^{(2)2}}{G_2}\right]rdrdz \tag{21}$$

The expressions of the stresses given by the equations (12) to (20) makes it possible to simply write the deformation energy according to $\sigma_{zz}^{(1)}$:

$$\xi_P = \pi\int_0^L\underbrace{\left[A\sigma_{zz}^{(1)2}+B\sigma_{zz}^{(1)}\frac{d^2\sigma_{zz}^{(1)}}{dz^2}+C\left(\frac{d\sigma_{zz}^{(1)}}{dz}\right)^2+\tilde{D}\sigma_{zz}^{(1)}+E\left(\frac{d^2\sigma_{zz}^{(1)}}{dz^2}\right)^2+\tilde{F}\frac{d^2\sigma_{zz}^{(1)}}{dz^2}+\tilde{K}\right]}_{\Gamma}dz \tag{22}$$

where: $\tilde{D}=D+\alpha_1 k$, $\tilde{F}=F+\alpha_1 h$, $\tilde{K}=K+\alpha_1^2 m$ (23)

The A, B, C, D, E, F, K constants (the same ones as those obtained for the case $\sigma_{rr} = 0$) and k, h, m are depending on the load and on the dimensional and mechanical specifications of the two tubes and adhesive [8].

The constant α_1 is given by the equation (24):

$$2m\alpha_1 L+\int_0^L\left[k\sigma_{zz}^{(1)}+h\frac{d^2\sigma_{zz}^{(1)}}{dz^2}\right]dz = 0 \tag{24}$$

and with the boundary conditions in z = 0 and z = L we have:

$$2m\alpha_1 L+\int_0^L k\sigma_{zz}^{(1)}dz + h\underbrace{\left[\frac{d\sigma_{zz}^{(1)}}{dz}\right]_0^L}_{=0} = 0 \tag{25}$$

By carrying out a variational calculus on the expression of the potential energy (21) and using the boundary conditions in z = 0 and z = L, we obtain that the complementary energy is minimal when $\sigma_{zz}^{(1)}(z)$ is the solution of the following differential equation:

$$E\frac{d^4\sigma_{zz}^{(1)}(z)}{dz^4}+(B-C)\frac{d^2\sigma_{zz}^{(1)}(z)}{dz^2}+A\sigma_{zz}^{(1)}(z)+\frac{D}{2}+\frac{\alpha_1 k}{2}=0 \tag{26}$$

Parametric study of the adhesive bonded joints

The numerical application developed in this work is presented in the following way: we use the configuration presented in table 2, and the load presented figure 1, figure2 showing the stress distribution in the assembly.

Table 2

Analyzed assembly configuration

Tube 1	Adhesive	Tube 2	r_i [mm]	r_{ic} [mm]	r_{ec} [mm]	r_e [mm]	L [mm]	f [MPa]
Glass/Epoxy +/- 45° E_x = 14470 MPa E_y = 14470 MPa G_{xy} = 12140 MPa υ = 0.508	Araldite Redux 312 E_c = 2500 MPa G_c = 1000 MPa $υ_c$ = 0.35	Carbon/Epoxy +/- 45° E_x = 17090 MPa E_y = 17090 MPa G_{xy} = 36380 MPa υ = 0.781	10	12	12.2	15.2	50	1000

Following, we present an analysis of the influence of various parameters affecting the intensity and the stress distribution. This analysis will be reduced to the study of the influence of the following parameters: the adhesive thickness, the length of adhesive cover, the Young's modulus of the adhesive and the relative rigidity of tubes E_2/E_1.

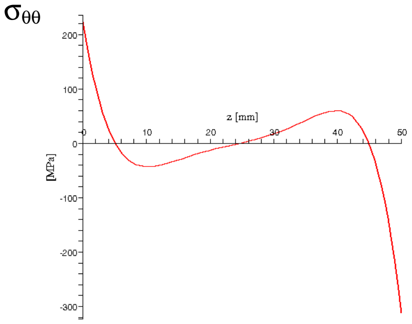
PARAMETRIC STUDY

Stress distribution

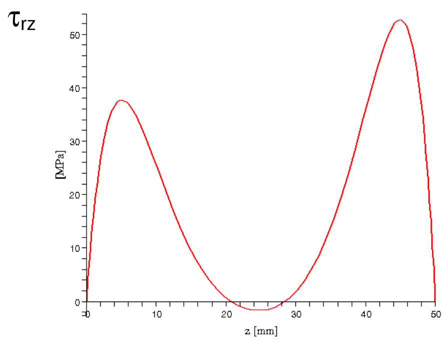
Figure 2 shows the stress distributions in the adhesive for the first analyzed configuration, that is to say the distributions of the orthoradial and shear stresses.

We notice that:

- for $\sigma_{\theta\theta}$, the maximum values are obtained on the free edges (z = 0, z = L). These values are about 60 % of the pressure applied. They are localized at the edges, however, the maximum stress $\sigma_{\theta\theta\ max}$ is obtained in compression,
- for τ_{rz}, we have two peaks of stresses located at equal distance of the two free edges (≈ 6,5 mm). The maximum value is about 12 % of the applied effort.



a)



b)

Figure 2. Stress distributions in the adhesive (f = 1000 MPa, L = 50 mm).
a) The orthoradial stress ($\sigma_{\theta\theta}$); b) The shear stress (τ_{rz}).

The peaks do not have the same intensity because of the difference of rigidities of the two stuck tubes. We can note that the orthoradial stresses are more important than shear stresses. The use of an fracture criterion of the adhesive bonded joint must take into account not only the shear stress τ_{rz} but also the orthoradial stress $\sigma_{\theta\theta}$.

Influence of covering length

Figure 3 shows the influence of covering length on the shear stresses distribution. We note thus that when the length of covering exceeds 50 mm, a part of this one is not requested any more. This makes it possible to determine a useful maximum length of joining.

We can observe that for low lengths of covering we have a distribution with only one peak (fig. 3). Two peaks of constraint appear starting from approximately 30 mm. By increasing the covering length gradually we observed:

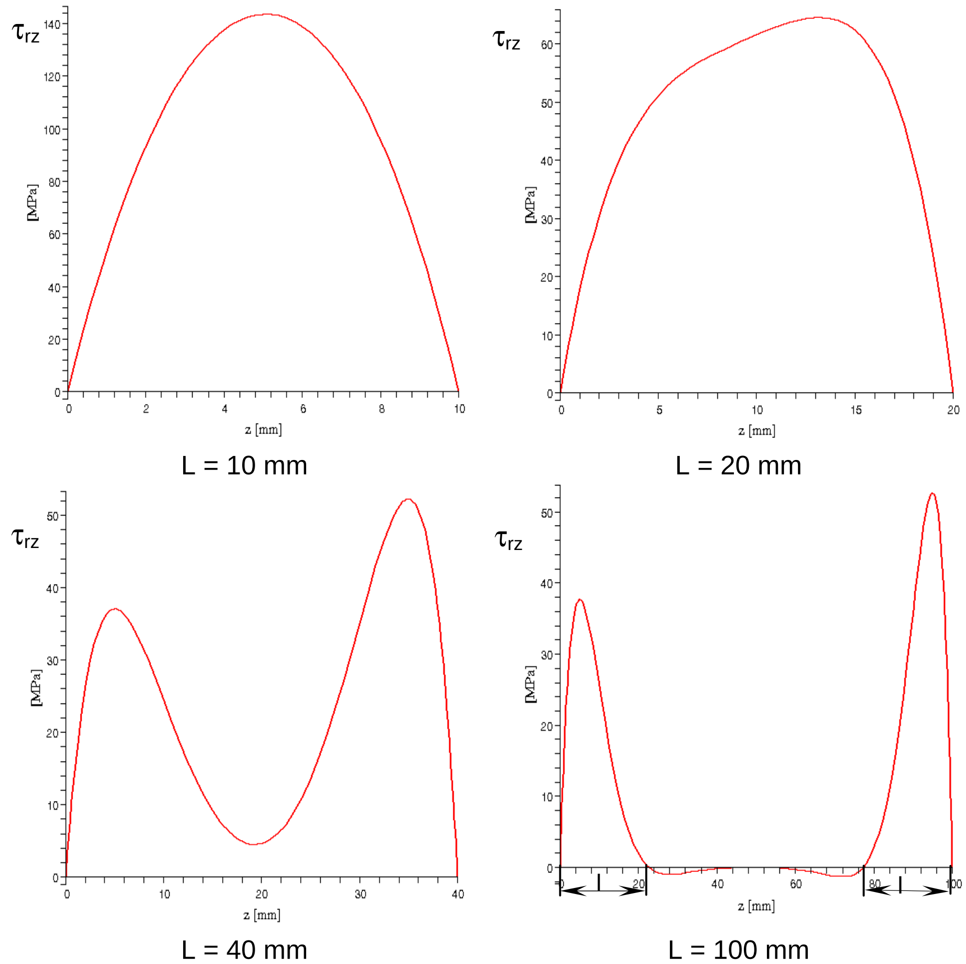


Figure 3. Shear stress (τ_{rz}) distribution according with the covering length (f = 1000 MPa).

- the reduction of the values of the shear stress in the medium of the joint,
- the displacement of the peaks of constraints towards the free edges.

We notice that there is an optimal length beyond which the maximum constraints do not evolve any more.

Influence of adhesive thickness

The adhesive thickness influence on the intensity and distribution of shear stresses is shown in figure 4. We can notice that more we increases the thickness of adhesive, the more the values of the constraints decrease on the level of the free edges. The distribution tends to being uniform.

CONCLUSIONS

A theoretical model to calculate the cylindrical assemblies is proposed. This model is based on the variational calculus applied on the potential energy in the assembly.

After having determined all the components of the stress field according to the effort $\sigma_{zz}^{(1)}(z)$ in the first tube, by considering the equilibrium of an elementary volume, the use of variational calculus enabled us to obtain the solution of the defined model.

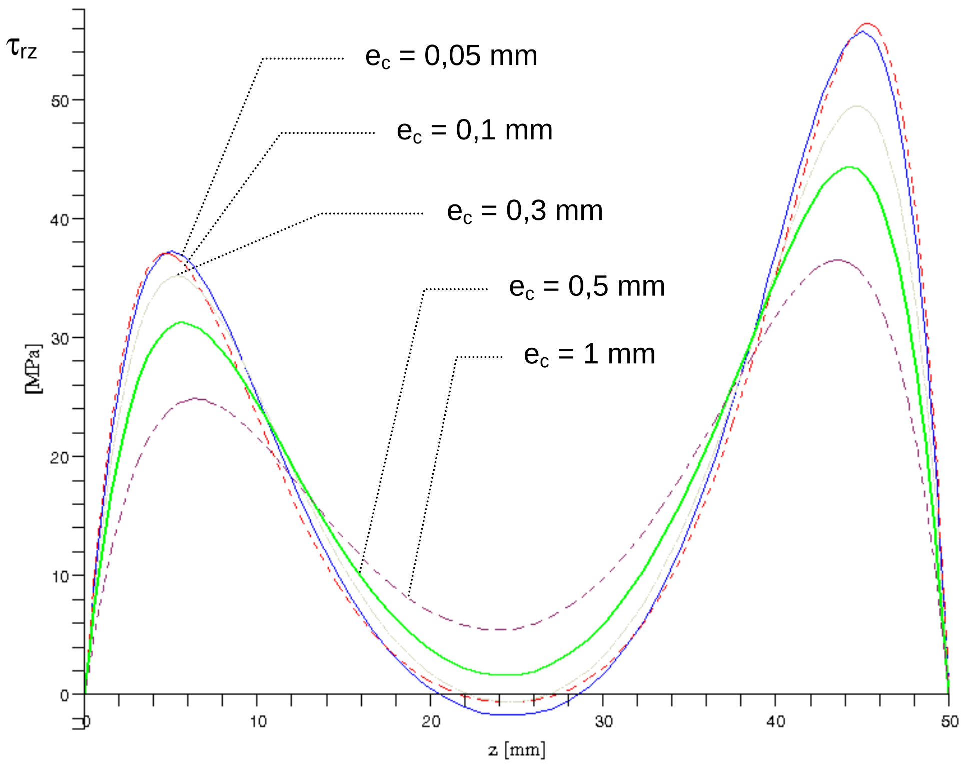


Figure 4. Shear stress (τ_{rz}) variation according with the adhesive thickness (f = 1000 MPa, L = 50 mm).

It comes out from this study the following points: the maximum values of $\sigma_{\theta\theta}$ are obtained on the free edges and the maximum values are about 60 % of the applied effort, moreover they are localized at the edges; concerning τ_{rz}, we have two peaks of constraints located at equal distance of the two free edges (≈ 6,5 mm). The maximum value is approximately 12 % of the applied effort. The taking into account in the model of the σ_{rr} stress influences only the distribution of the $\sigma_{\theta\theta}$ stress, the orthoradiale stresses are more important than shear stresses. There exists an optimal length beyond which the maximum constraints do not evolve any more; the intensities of the peaks are influenced by the difference of rigidities of the two stuck tubes. Te maximum peaks increase slightly when the elastic module grows.

The shear stress in the adhesive increases with the increase of the relative rigidity of the tubes, more we increases the adhesive thickness, plus the values of the constraints decrease on the level of the free edges and the distribution tends to being uniform. The taking into account in the model of the σ_{rr} stress influences only the distribution of the $\sigma_{\theta\theta}$ stress.

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