STUDIA UNIVERSITATIS BABES - BOLYAI, CHEMIA, LI, 2, 2006

Dedicated to professor Gh. Marcu at his 80th anniversary

THE INFLUENCE OF ADHESIVE ON COMPOSITE MATERIALS BONDED JOINTS ASSEMBLIES

O. NEMEŞ[1*], F. LACHAUD[2], A. MOJTABI[3], V. SOPORAN[1], O. TĂTARU[1]

ABSTRACT. The paper presents a theoretical calculation model of plane assemblies joined with adhesive, based on an energy method. After the determination of the cinematically acceptable field of stresses, according to the applied load, a variational calculus on the expression of elastic potential energy leads to the complete expression of the stress field in the whole assembly. A first parametric analysis (geometrical and physical parameters) is carried out on a double-lap plane assembly and makes it possible to deduce the optimal length and thickness of the adhesive. For the assembly, the total force-displacement behavior is well defined. Thus the analytical model makes it possible to determine the rigidity of the assembly and to obtain a simple formulation very rapidly which gives the total behavior of the assembly.

Keywords: Stress Analysis, Bonded joints, Numerical modeling

INTRODUCTION

The increase in use of adhesive bonded joints is due to the many advantages of this method compared to the traditional methods. This assembling method distributes the stresses over the whole joining surface and removes the concentrations of stresses to the boundary of holes generated by bolting or riveting assemblies. The mechanical performance of an adhesive bonded joint is related to the distribution of the stresses in the adhesive layer. Consequently it is essential to know this distribution, which, because of its complexity, makes prediction of fractures difficult. From the first works of Volkersen (1938) when only a distribution of the shear stress in the adhesive joint was taken into account to the more recent studies by finite elements, many formulations have made it possible to define the field of stresses in such assemblies better and better.

Since the first work of Volkersen until the more recent studies by finite elements many formulations allowed to better define the stress field in such assemblies. Among those, we can mention in principal the works due to:

[*]e-mail: ovidiu.nemes@sim.utcluj.ro
[1] Technical University of Cluj-Napoca
[2] ENSICA Toulouse
[3] “Paul Sabatier” University Toulouse

- Goland and Reissner (1944) – dealing with the description of the peeling stress. They showed that the effects of flexion create additional stresses which are superimposed on shear stresses,
- Hart-Smith (1973) - studied the elastoplastic behavior of the joint of adhesive and took into account the effects of temperature,
- Renton and Vinson (1977) have checked certain results by experimental way.

One should mention also the theoretical work of Oljado and Eidinoff (1978), or those due to Bigwood (1989), and Allman (1977),

Following Goland and Reissner (1944), Volkersen (1965) introduced into his new analysis the normal stress "stress of shearing" (peeling stress) which is variable in the thickness of the adhesive layer. This assumption enabled him to build a stress field observing the boundary conditions of the assembly. However, due to the complexity and difficulty of its implementation, this analytical formulation is not easily applicable.

Gilibert and Rigolot (1979 – 1991) propose, based on the method of the asymptotic developments connected in the vicinity of the ends, an analytical formulation of the stress field over the entire covering length. If this formulation constitutes a clear improvement of the modeling of the field of the constraints on the level of the ends and represents experimental reality better, it is however not valid near the free edges.

Other more recent studies, due to Liyong (1994) present an analytical formulation making it possible to calculate the mechanical strength of the double-lap joints by taking into account the effects of temperature; however the author disregards the stress distribution and calculates only the joint shearing deformation energy.

Adams and Peppiatt (1974), in their finite element analysis, circumvented this difficulty by studying a joint modified by the addition of a regularizing part. However even this study is not satisfactory on the level of the ends.

Tsai and Oplinger (1998) develop the existing traditional solutions by the inclusion of shearing strains, neglected until there. The solutions obtained ensure a better forecast of the distribution and intensity of the shear stress.

Mortensen and Thomsen (2002) developed the approach for the analysis and the design of the joints adhesive bonded. They held into account the influence of the interface effects between the adherents and they modeled the adhesive layer by assimilating it to a spring.

Nemeş (2004) use a technique based on the minimization of the potential energy. The first stage consists in building a statically acceptable stress field, i.e. verifying the boundary conditions and the equilibrium equations. Then, the potential energy generated by such a stress field is calculated. In the third stage, the potential energy is minimized in order to determine the stress distributions. As we have just seen, the analytical formulations and the finite element analysis provide a stress field satisfying for the median part of the joints. On the other hand, these two approaches provide results that do not satisfy the boundary conditions imposed at the ends of covering. However it is in the vicinity of these ends that one observes the majority of the phenomena of degradation (non-linear behavior, damage, cracking, even fracture). The analytical study that follows gives a first solution of the field of the constraints respecting the whole of these conditions.

RESULTS AND DISCUSSION

THEORETICAL MODEL

All work has encountered difficulties in modeling the stress field in the vicinity of the ends of the joint. The method used to obtain the optimal field for this type of assembly consists of:

- Construction of a statically acceptable field,
- Calculation of the potential energy associated with the stress field,
- Minimization of this energy by a variational method,
- Resolution of the differential equation obtained.

Definitions and hypothesis. Let us consider a plane joining with double covering (Figure 1) whose supports are maintained stuck by a marked elastic adhesive of the index ©.

The whole joining is in balance under the action of a tensile load. The two adherents are subjected to the same load of F/2 traction following axis X, the median plate it being subjected to an opposite load of intensity F. Considering the geometrical symmetries of the problem, our analysis will be limited to the study of the higher half of the assembly represented Figure **2.**

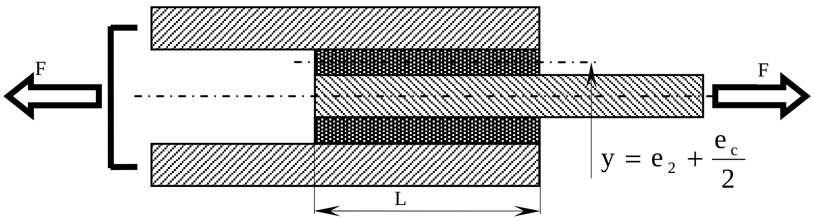

Figure 1. Double-lap adhesive assembly

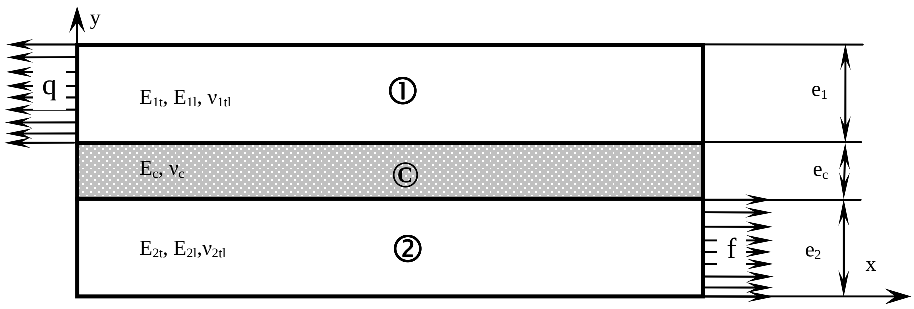

Figure 2. Geometrical and material definition of the double-lap joint

Where:

- $\mathbf{E_c, \nu_c}$, Young's modulus and Poisson's ratio of the adhesive ©,
- $\mathbf{E_{tl}, E_{ll}, \nu_{tll}}$, longitudinal, transverse modulus and Poisson's ratio of the inner tube,

- **E_{2t}, E_{2l}, ν_{tl2}**, longitudinal, transverse modulus and Poisson's ratio of the external tube,
- **e_c**, adhesive © thickness,
- **e_1, e_2,** adherents ① and ② thickness,
- **L**, joining length,
- **f** and **q**, tensile stresses following x axis, on the adherents.

The constraints in various materials will be located by the index (i), where i = ①, © or ②. We are in the case of plane constraints and we will adopt the following assumptions:

- the state of plane stresses: $\tau_{zx}^{(i)} = \tau_{zy}^{(i)} = \sigma_{zz}^{(i)} = 0$
- the $\sigma_{xx}^{(i)}$, $\tau_{xy}^{(i)}$ et $\sigma_{yy}^{(i)}$ stresses are independent of z variable
- the $\sigma_{xx}^{(1)}$ et $\sigma_{xx}^{(2)}$ stresses are function only of x variable
- the normal stress in the adhesive will be considered null: $\sigma_{xx}^{(c)} = 0$

The stress field is thus reduced to:

- adherent ①: $\sigma_{xx}^{(1)}(x)$, $\tau_{xy}^{(1)}(x,y)$, $\sigma_{yy}^{(1)}(x,y)$,
- adhesive ©: $\tau_{xy}^{(c)}(x)$, $\sigma_{yy}^{(c)}(x,y)$,
- adherent ②: $\sigma_{xx}^{(2)}(x)$, $\tau_{xy}^{(2)}(x,y)$, $\sigma_{yy}^{(2)}(x,y)$.

The stress field definition. We build a statically acceptable stress field by respecting the preceding assumptions. i.e. checking the local equilibrium equations, the boundary conditions as well as the conditions of continuity to the interfaces with the adhesive. We write the balance of the forces which act on the whole of joining by carrying out a fictitious cut of the assembly following y axis.

The stress components for each three component must satisfy the equilibrium equations, the conditions of continuity of the vector forced with the crossing of the interfaces as well as the boundary conditions in x = 0, X = L, y = 0 and $y = (e_1 + e_2 + e_c)$. Starting from equilibrium equations, in order to determine the various component the of the stress vectors, we must write the boundary conditions as well as the conditions of continuity of the stress vectors with the interfaces.

On the free edges, the normal stress on the surface and shear stresses are null (equal with zero value). With the crossing of the interfaces, these same stresses must be continuous. The nullity of the shear stress for y = 0 is due to the symmetry of the problem. All these conditions were gathered below:

- For x = 0: $\sigma_{xx}^{(1)} = q$, $\sigma_{xx}^{(2)} = 0$, $\tau_{xy}^{(1)} = 0$, $\tau_{xy}^{(2)} = 0$
- For x = L: $\sigma_{xx}^{(1)} = 0$, $\sigma_{xx}^{(2)} = f$, $\tau_{xy}^{(1)} = 0$, $\tau_{xy}^{(2)} = 0$
- For y = 0: $\tau_{xy}^{(2)} = 0$
- For $y = e_2$: $\tau_{xy}^{(2)} = \tau_{xy}^{(c)}$, $\sigma_{yy}^{(2)} = \sigma_{yy}^{(c)}$
- For $y = e_2 + e_c$: $\tau_{xy}^{(1)} = \tau_{xy}^{(c)}$, $\sigma_{yy}^{(1)} = \sigma_{yy}^{(c)}$
- For $y = e_1 + e_c + e_2$: $\tau_{xy}^{(1)} = 0$, $\sigma_{yy}^{(1)} = 0$

The stress field is thus reduced to the following components:

$$\tau_{xy}^{(1)}(x,y)=\left[\left(e_1+e_2+e_c\right)-y\right]\frac{d\sigma_{xx}^{(1)}}{dx},$$
$$\sigma_{yy}^{(1)}(x,y)=\frac{1}{2}\left[y-\left(e_1+e_2+e_c\right)\right]^2\frac{d^2\sigma_{xx}^{(1)}}{dx^2}$$
$$\tau_{xy}^{(c)}(x)=e_1\frac{d\sigma_{xx}^{(1)}}{dx},$$
$$\sigma_{yy}^{(c)}(x,y)=e_1\left[\left(\frac{e_1}{2}+e_2+e_c\right)-y\right]\frac{d^2\sigma_{xx}^{(1)}}{dx^2}, \qquad (1)$$
$$\sigma_{xx}^{(2)}(x)=f-\frac{e_1}{e_2}\sigma_{xx}^{(1)}$$
$$\tau_{xy}^{(2)}(x,y)=\frac{e_1}{e_2}y\frac{d\sigma_{xx}^{(1)}}{dx},$$
$$\sigma_{yy}^{(2)}(x,y)=\frac{e_1}{2}\left[\left(e_1+e_2+2e_c\right)-\frac{y^2}{e_2}\right]\frac{d^2\sigma_{xx}^{(1)}}{dx^2}$$

Deformation energy calculation. In the defined stress field, the only unknown factor is the expression of the normal stress $\sigma_{xx}^{(1)}$, expression which we will determine using the principle of minimum of complementary energy. The potential energy associated with the statically acceptable field previously given is written for a joining length 1 and width unit on axis z:

After integration following y, for the potential energy was obtained the following form:

$$\xi_P=\int_0^L\underbrace{\left[A\sigma_{xx}^{(1)^2}+B\sigma_{xx}^{(1)}\frac{d^2\sigma_{xx}^{(1)}}{dx^2}+C\left(\frac{d\sigma_{xx}^{(1)}}{dx}\right)^2+D\sigma_{xx}^{(1)}+E\left(\frac{d^2\sigma_{xx}^{(1)}}{dx^2}\right)^2+F\frac{d^2\sigma_{xx}^{(1)}}{dx^2}+K\right]}_{\Gamma}dx \qquad (2)$$

the constants A, B, C, D, E, F and K depend on the geometrical and material characteristics of the three components as well as the applied loading. The expressions of these various constants according to the geometrical and physical characteristics of the assembly are given by the following equations.

By applying to the functional ξ_P a variational calculus and by using the boundary conditions which one also writes in the form:

$$\sigma_{xx}^{(1)}(x=0)=q=\frac{e_2}{e_1}f\,,\ \frac{d\sigma_{xx}^{(1)}}{dx}(x=0)=0\,,\ \sigma_{xx}^{(1)}(x=L)=0\,,\ \frac{d\sigma_{xx}^{(1)}}{dx}(x=L)=0 \qquad (3)$$

we obtain that the energy is minimal when the stress function $\sigma_{xx}^{(1)}(x)$ is solution of the following differential equation:

$$E\frac{d^4\sigma_{xx}^{(1)}(x)}{dx^4}+(B-C)\frac{d^2\sigma_{xx}^{(1)}(x)}{dx^2}+A\sigma_{xx}^{(1)}(x)+\frac{D}{2}=0 \qquad (4)$$

Parametric study of the adhesive bonded joints

Stress distribution. Figure 3 shows the stress distributions in the adhesive for the analyzed configurations. These data show that the peeling stresses are more important than shear stresses. This observation joined the remarks of Volkersen (1965), Gilibert and Rigolot (1985) who also argument that the peeling stress are most important. We notice that: for σ_{yy}, the maximum values are obtained on the free edges (z = 0, z = L). These values are much localized at the edges; however, the maximum constraint σ_{yymax} is obtained in compression, for τ_{xy}, we raises two peaks of stresses located at equal distance of the two free edges. The peaks do not have the same intensity because of the difference of rigidities of the two stuck adherents.

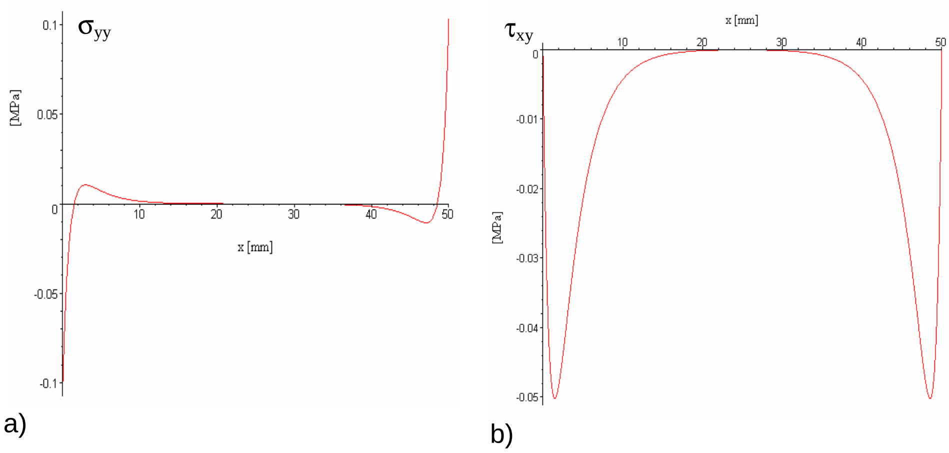

Figure 3. Distribution of the stresses in the adhesive of a VE ±45°-AV 119-VE ±45° assembly for F = 1 N/mm. a) the peeling stress (σ_{yy}), b) the shear stress (τ_{xy}).

Based on previous analysis of distributions we can note that the peeling stress are more important than the shear stresses thus the use of a criterion of rupture of the adhesive bonded joint must take into account not only the stress shear τ_{xy} but also the peeling stress σ_{yy}.

Parametric study

Covering length influence. Figure 4 shows the influence of covering length on the distribution and the intensity of shear stresses. The fact of increasing the length of joining beyond a certain value does not have any influence on the maximum stresses in the adhesive. Indeed, for the assembly with double covering there is an optimal length of covering beyond which the added length is not under load. By increasing gradually the covering length we observed:

- the reduction of the values of the shear stress in the middle of the joint,
- the displacement of the stress peaks towards the free edges.

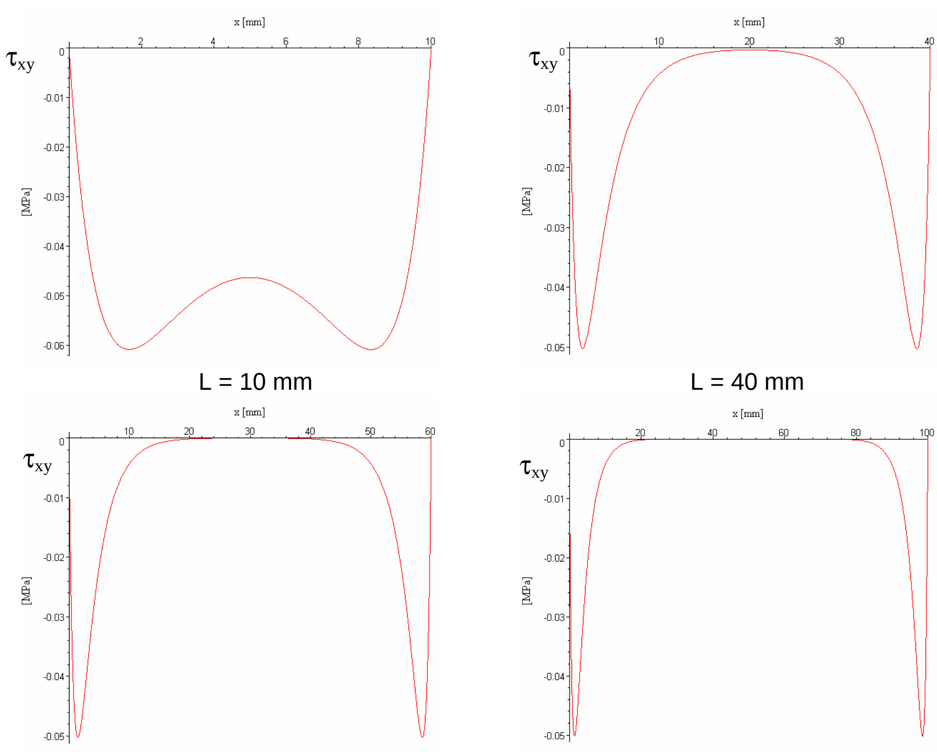

Figure 4. Variation of τ_{xy} in the adhesive according to the covering length (L = 10÷100 mm) and F = 1 N/mm, for a VE ±45°-AV 119-VE ±45° assembly.

Influence of rigidities. Figure 5 represents the influence of the elastic module of the adhesive on the shear stress in the adhesive. The maximum peaks will increase slightly when the elastic module grows.

The influence of relative rigidity, between the two stuck substrates, is illustrated in Figure 6.

We can notice that the maximum peaks on the two edges are not equaled any more if ratio E_2/E_1 is different from 1.

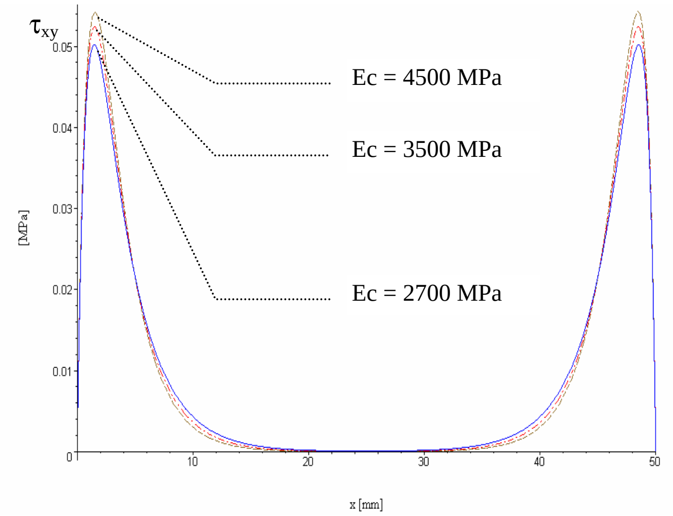

Figure 5. Variation of the shear stress ($-\tau_{xy}$) in the adhesive according to the elastic module of the adhesive.

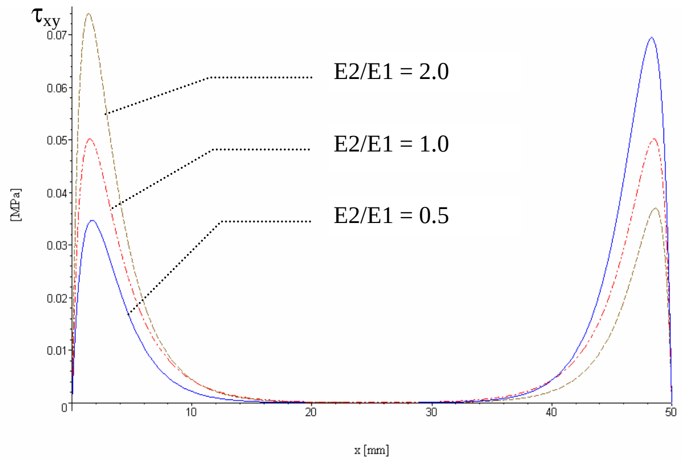

Figure 6. Variation of the shear stress ($-\tau_{xy}$) according to relative rigidity.

Influence of adhesive thickness. Figures 7 - 8 show the influence of adhesive thickness. When the adhesive thickness increases the maximum stresses in the adhesive decrease and the distribution (shear and peeling stress) tends to be uniform over the entire covering length, except in the vicinity of the free edges.

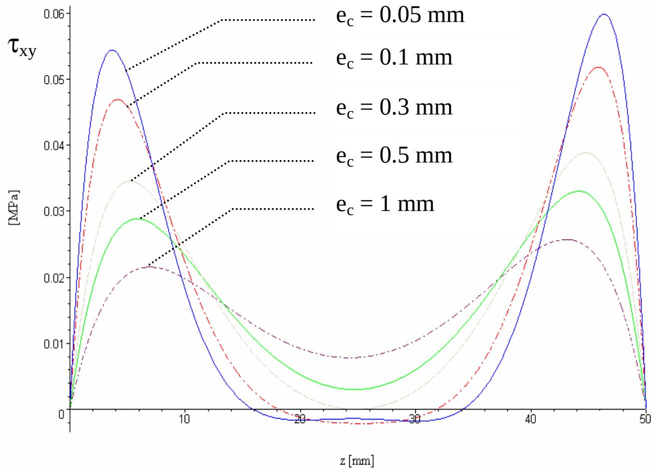

Figure 7. Variation of the shear stress ($-\tau_{xy}$) according to the adhesive thickness.

At the same time, for the peeling stresses, we can also note a considerable reduction in the maximum values to the level of the free edges.

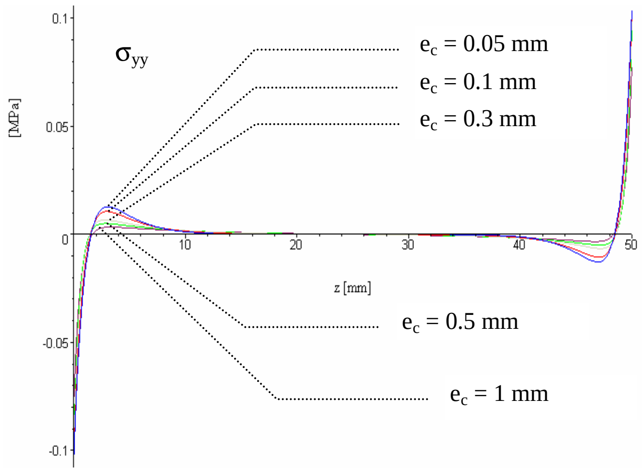

Figure 8. Variation of the peeling stress (σ_{yy}) according to the adhesive thickness.

CONCLUSIONS

The present data point the followings:

- the maximum values of σ_{yy} are obtained on the free edges and the maximum values are localized at the edges,
- concerning τ_{xy}, we found two peaks of stresses located at equal distance of the two free edges,
- the peeling stresses are more important than shear stresses,

- there is an optimal length beyond which the maximum stress do not evolve any more,
- the intensities of the peaks are influenced by the rigidities difference of the two adherents, the maximum peaks increase slightly when the elastic module grows,
- the shear stress in the adhesive increases with the increase in the relative rigidity of the adherents,
- more the thickness of adhesive is increased, more the values of the stresses decrease on the level of the free edges and the distribution tends to being uniform.

REFERENCES

1. Adams, R.D., Peppiat, N.A., *Journal of Strain Analysis*, **1974**, *9*, 185-196.
2. Allman, D.J., *Journal of Mechanical Applied Mathematics*, **1977**, *30*, 415-436.
3. Bigwood, D.A., Crocombe, A.D., *International Journal of Adhesion & Adhesives*, **1989**, *9*, 229-242.
4. Gilibert, Y., Rigolot, A., *Matériaux et Constructions*, **1985**, *18*, 363-387.
5. Gilibert, Y., Rigolot, A., *Mécanique des solides*, **1979**, *288*, 287-290.
6. Gilibert, Y., *Matériaux et techniques*, **1991**, 5-16.
7. Goland, M., Buffalo, N.Y., Reissner, E., *Journal of Applied Mechanics*, **1944**, *66*, A17-A27.
8. Hart-Smith, L.J., Douglas Aircraft Co., *NASA-CR-112236*, **1973**.
9. Liyong Tong, *International Journal of Solids Structures*, **1994**, *31*, 2919-2931.
10. Mortensen, F., Thomsen, O.T., *Composite Structures*, **2002**, *56*, 165-174.
11. Mortensen, F., Thomsen, O.T., *Composite Science and Technology*, **2002**, *62*, 1011-1031.
12. Nemeş, O., *Contribution à l'étude des assemblages collés cylindriques et plans*, PhD. Thesis, INSA Toulouse, France, **2004**.
13. Ojalvo, I.U., Eidinoff, H.L., Bond *AIAA Journal*, **1978**, *16*, 204-211.
14. Renton, W.J., Vinson, J.K., *Journal of Applied Mechanics*, **1977**, 101-106.
15. Tsai, M.Y, Oplinger, D.W., Morton, J., *Int. J. Solid Structures*, **1998**, *35*, 1163-1185.
16. Volkersen O., Die *Luftfahrtforschung*, **1938**, *15*, 41-47.
17. Volkersen, O., Recherché *Construction métallique*, **1965**, *4*, 3-13.