

## ON THE SCHULTZ, MODIFIED SCHULTZ AND HOSOYA POLYNOMIALS AND DERIVED INDICES OF CAPRA-DESIGNED PLANAR BEZENOIDS

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**ABSTRACT.** In this paper, Schultz, Modified Schultz and Hosoya polynomial and their topological indices of a benzenoid molecular graph constructed by Capra-map operation,  $Ca(C_6)$ , are calculated. Several examples are given.

**Keywords:** *Schultz polynomial; Modified Schultz polynomial; Hosoya polynomial; Wiener index; Capra-operated benzenoid.*

### INTRODUCTION

Let  $G=(V,E)$  be a simple connected graph of finite order  $n=|V|$ , such that it has the vertex set  $V=V(G)$  and edge set  $E=E(G)$ . A general reference for the notation in Graph Theory is [1]. The distance between vertices  $u$  and  $v$  of  $G$ , denoted  $d(u,v)$ , is the number of edges in a shortest path connecting them. The largest distance in  $G$  is called the diameter,  $d(G)$ . Another invariant in graph is degree of a vertex  $v \in V(G)$  that it is the number of edges incident in it and is denoted by  $\delta_v$ .

In graph theory, several counting polynomials are known: Schultz polynomial  $Sc(G,x)$ , Modified Schultz polynomial  $Sc^*(G,x)$ , Hosoya polynomial  $H(G,x)$ , etc. Their first derivative (in  $x=1$ ) define, in general, the corresponding topological indices.

Definitions of the above polynomials and indices are as follows:

$$Sc(G, x) = \frac{1}{2} \sum_{\{u,v\} \in E(G)} (\delta_u + \delta_v) x^{d(u,v)}$$

$$Sc^*(G, x) = \frac{1}{2} \sum_{\{u,v\} \in E(G)} (\delta_u \delta_v) x^{d(u,v)}$$

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$$H(G, x) = \sum_{i=0}^{d(G)} d(G, i) x^{d(u, v)}$$

$$Sc(G) = \frac{1}{2} \sum_{\{u, v\} \in V'(G)} (\delta_u + \delta_v) d(u, v)$$

$$Sc^*(G) = \frac{1}{2} \sum_{\{u, v\} \in V'(G)} (\delta_u \times \delta_v) d(u, v)$$

$$W(G) = \frac{1}{2} \sum_{v \in V'(G)} \sum_{u \in V'(G)} d(u, v) = \sum_{i=0}^{d(G)} d(G, i) d(u, v)$$

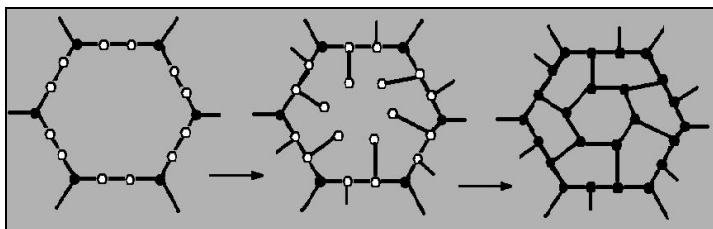
$$WW(G) = H'(1) + (1/2)H''(1)$$

The Schultz index was introduced by *Schultz* in 1989 [2] while the Modified Schultz index was defined by *Klavžar* and *Gutman* in 1997 [3]. The Schultz index, also called molecular topological index, was studied in many papers [2-17]. These indices have been computed in some nanotubes [12-14, 17-21].

Hosoya polynomial was introduced by *H. Hosoya*, in 1988 [16]. The first derivative of Hosoya polynomial is just the Wiener index; a Hyper-Wiener index, denoted  $WW(G)$  (see above) can be computed from the first and second derivative of Hosoya polynomial. Wiener index had found numerous application and was reported in [8, 16, 21-32].

The coefficients of Hosoya polynomial can be calculated from layer/shell matrices, as shown by *Diudea* [33-35], who gave a “chemical” generalization in Hosoya-Diudea weighted polynomials.

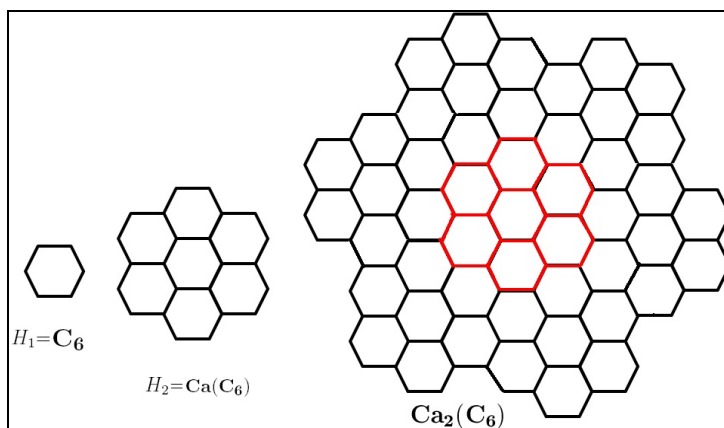
In chemistry, physics and nanoscience, there are especially symmetric structures. Such molecular graphs are *Capra-designed planar benzenoids*. Capra  $Ca$  map operation (also called Septupling  $S_1$ ) is a method of drawing and modifying the covering of a polyhedral structure, introduced by *Diudea* [36,37]. A detailed example is given in Figure 1.



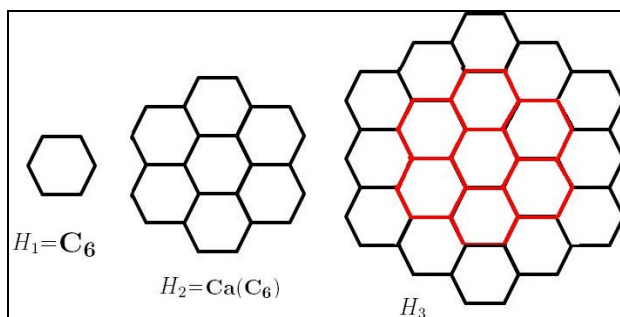
**Figure 1.** An example of Capra map operation on the hexagon face.

In this paper, we applied Capra operation on the benzene molecular graph  $C_6$  to design planar benzenoid structures; the  $k$ -iterated benzenoids are denoted  $Ca_k(C_6)$ . The two first members of this series are shown in Figure 2.

Also,  $Ca(C_6)$  is called Coronene  $H_2$  and is the second member of the circumcoronene series of benzenoids  $H_k$ ,  $k \geq 1$ . The first three members of circumcoronene series are shown in Figure 3.



**Figure 2.** Benzenoid molecular graphs  $H_2 = Ca(C_6)$  and  $Ca_2(C_6)$ , representing the first two members of Capra-designed planar benzenoids.



**Figure 3.** The first three graphs  $H_1$ ,  $H_2$  and  $H_3$  of the Circumcoronene series.

Within this paper, we focused on the Schultz, Modified Schultz and Hosoya polynomials and their topological indices of the Coronene  $Ca(C_6)$  planar benzenoid structure.

**Theorem 1.** Let  $G = Ca(C_6)$  be a Capra-designed planar benzenoid. Then the Schultz polynomial of  $G$  is equal to:

$$Sc(G, x) = 156x + 252x^2 + 294x^3 + 276x^4 + 222x^5 + 132x^6 + 48x^7$$

and the Schultz index  $Sc(G)=4884$ .

The Modified Schultz polynomial of  $G$  is equal to:

$$Sc^*(G,x)=204x+330x^2+381x^3+348x^4+267x^5+144x^6+48x^7$$

and the Modified Schultz index  $Sc^*(G)=5934$ .

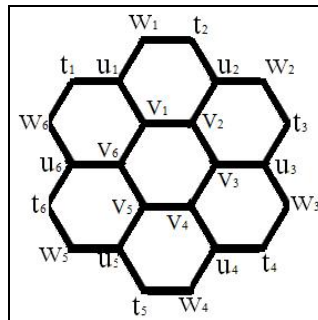
**Theorem 2.** Let  $G=Ca(C_6)$  be a Capra-designed planar benzenoid. Then, the Hosoya polynomial of  $G$  is equal to:

$$H(G,x)=24+30x^1+48x^2+57x^3+54x^4+45x^5+30x^6+12x^7$$

Also, the Wiener index is  $W(G)=1002$ , and Hyper Wiener index  $WW(G)=2697$ .

## MAIN RESULTS

In this section we will prove the two above theorems. At first, we introduce some notations, related to Figure 4.



**Figure 4.** Capra-designed planar molecular graph: Coronene,  $Ca(C_6)=H_2$  and the notation used in the text.

Let  $V(G)$  be the vertex set of  $G=Ca(C_6)$  with cardinality 24 and  $E(G)$  the edge set, of cardinality 30. We describe each vertex of  $G$  by automorphism  $f$ , such that:

$$f: V(G) \rightarrow \{u_i, v_i, w_i, t_i \mid i \in \mathbb{Z}_6\}.$$

and

$$f: E(G) \rightarrow \{v_i v_{i+1}, v_i u_i, u_i w_i, u_i t_i, w_i t_{i+1} \mid i \in \mathbb{Z}_6\}.$$

According to the Figure 4, we have the vertices  $u_i, v_i$  of degree 3 and vertices  $w_i, t_i$  of degree 2, for all  $\mathbb{Z}_6$ .  $\mathbb{Z}_6 = \{0, 1, 2, 3, 4, 5\}$  is the cycle finite group of order 6 (or integer number of module 6).

*Proof of Theorem 1:* Let  $G=Ca(C_6)$  be the Coronene graph. Since there exists 24 distinct vertices, we have  $\binom{n}{2} = 276$  distinct shortest paths between vertices  $u$  and  $v$  of  $G$ . Also, in Coronene there are distances from one to seven, for every vertices  $u, v \in V(G)$ . In other words,

$$\forall u, v \in V(G), \exists d(u, v) \in \{1, 2, \dots, 7\}.$$

So, we will have seven partitions for proof.

*I .* If  $d(u, v)=1$ , then  $D_1 = \{(v_i, v_{i+1}), (v_i, u_i), (u_i, w_i), (u_i, t_i), (w_i, t_{i+1}) \mid i \in \mathbb{Z}_6\}$  and  $|D_1|=30$  (that is equal to  $|E(G)|$ ). So, we have three subsets of it.

*I -1.*  $\forall i \in \mathbb{Z}_6$ , let  $v=v_i$  and  $u=u_i=v_{i+1}$ . Since  $\delta_{v_i} = \delta_{u_i} = 3$ , hence  $\delta_v + \delta_u = 6$  and  $\delta_v \times \delta_u = 9$ . Therefore  $|\{(u, v) \mid u, v \in V(G), d(u, v) = 1 \& \delta_v + \delta_u = 6, \delta_v \times \delta_u = 9\}| = 6 \times 2$ .

So, we have two terms  $72x^1$ ,  $108x^1$  of the Schultz polynomial and Modified Schultz polynomial, respectively.

*I -2.*  $\forall i \in \mathbb{Z}_6$ , let  $v=t_i$ ,  $w_i$  and  $u=u_i$ . Since  $\delta_{u_i} = 3$  and  $\delta_{t_i} = \delta_{w_i} = 2$ . So,  $\delta_v + \delta_u = 5$  and  $\delta_v \times \delta_u = 6$ . Therefore  $|\{(u, v) \mid u, v \in V(G), d(u, v) = 1 \& \delta_v + \delta_u = 5, \delta_v \times \delta_u = 6\}| = 6 \times 2$ .

So, we have two sentences  $60x^1$ ,  $72x^1$  of the above polynomials.

*I -3.*  $\forall i \in \mathbb{Z}_6$ , let  $v=w_i$  and  $u=t_i$ . Since  $\delta_{t_i} = \delta_{w_i} = 2$ . So,  $\delta_v + \delta_u = 4$  and  $\delta_v \times \delta_u = 4$ .

Therefore  $|\{(u, v) \mid u, v \in V(G), d(u, v) = 1 \& \delta_v + \delta_u = 4, \delta_v \times \delta_u = 4\}| = 6$ .

In general, we have two terms  $156x^1$ ,  $204x^1$  for the Schultz polynomial and Modified Schultz polynomial, respectively.

*II .* If  $d(u, v)=2$ , then

$D_2 = \{(v_i, v_{i+2}), (v_i, u_{i+1}), (v_i, w_i), (v_i, t_i), (v_i, u_{i-1}), (u_i, t_{i+1}), (u_i, w_{i-1}), (w_i, t_i) \mid i \in \mathbb{Z}_6\}$  and  $|D_2|=48$ . Similarly, we have three subsets of it.

*II -1.*  $\forall i \in \mathbb{Z}_6$ , let  $v=v_i$  and  $u=v_{i+2}, u_{i+1}, u_{i-1}$  (or  $u_{i+5}$ ). Since  $\delta_{v_i} = \delta_{u_i} = 3$ , hence  $\delta_v + \delta_u = 6$  and  $\delta_v \times \delta_u = 9$ . Therefore  $|\{(u, v) \mid u, v \in V(G), d(u, v) = 2 \& \delta_v + \delta_u = 6, \delta_v \times \delta_u = 9\}| = 6 \times 3$ .

So, we have two terms  $108x^2$ ,  $162x^2$  for the Schultz polynomial and Modified Schultz polynomial, respectively.

*II -2.*  $\forall i \in \mathbb{Z}_6$ , let  $(u=t_i, w_i \& v=v_i)$  or  $(v=t_{i+1}, w_{i-1} \& u=u_i)$ . Since  $\delta_{v_i} = \delta_{u_i} = 3$  and  $\delta_{t_i} = \delta_{w_i} = 2$ , therefore  $|\{(u, v) \mid u, v \in V(G), d(u, v) = 2 \& \delta_v + \delta_u = 5, \delta_v \times \delta_u = 6\}| = 24$ . So, we have two sentences  $120x^2$ ,  $144x^2$  of these polynomials.

*II -3.*  $\forall i \in \mathbb{Z}_6$ , let  $v=w_i$  and  $u=t_i$ . Since  $\delta_{t_i} = \delta_{w_i} = 2$ , therefore

$$|\{(u, v) \mid u, v \in V(G), d(u, v) = 2 \& \delta_v + \delta_u = 4, \delta_v \times \delta_u = 4\}| = 6.$$

Generally, we have two terms  $252x^2$ ,  $330x^2$  for the Schultz polynomial and Modified Schultz polynomial, respectively.

III. If  $d(u, v) = 3$ , then  $D_3 = \{(v_i, v_{i+3}), (v_i, u_{i+2}), (v_i, u_{i-2}), (v_i, w_{i+1}), (v_i, t_{i+1}), (v_i, w_{i-1}), (v_i, t_{i-1}), (u_i, u_{i+1}), (t_i, t_{i+1}), (w_i, w_{i+1}) \mid i \in \mathbb{Z}_6\}$  and  $|D_3| = 57$ .

Similarly, we have three subsets of it.

III-1.  $\forall i \in \mathbb{Z}_6$ , let  $v=v_i$  &  $u=v_{i+3}$ ,  $u_{i+2}$ ,  $u_{i-2}$  or  $v=v_{i+1}$  &  $u=u_i$ . Hence  $\delta_v + \delta_u = 6$  and  $\delta_v \times \delta_u = 9$ . Therefore  $|\{(u, v) \mid u, v \in V(G), d(u, v) = 3 \& \delta_v + \delta_u = 6, \delta_v \times \delta_u = 9\}| = 6 \times 3 + 3 = 21$

Then, we have  $126x^3$  and  $189x^3$  in these polynomials.

III-2.  $\forall i \in \mathbb{Z}_6$ , let  $v=v_i$  &  $u=t_{i+1}, t_{i-1}, w_{i+1}, w_{i-1}$ . Thus

$|\{(u, v) \mid u, v \in V(G), d(u, v) = 3 \& \delta_v + \delta_u = 5, \delta_v \delta_u = 6\}| = 6 \times 4 = 24$ .

So, we have  $120x^3$  and  $144x^3$ .

III-3.  $\forall i \in \mathbb{Z}_6$ , let  $v=w_i$  &  $u=w_{i+1}$  or  $v=t_i$  &  $u=t_{i+1}$ . Since  $\delta_{t_i} = \delta_{w_i} = 2$ , then

$|\{(u, v) \mid u, v \in V(G), d(u, v) = 3 \& \delta_v + \delta_u = 4, \delta_v \times \delta_u = 4\}| = 12$ . Overall, there are the terms  $294x^3, 381x^3$  for the Schultz polynomial and Modified Schultz polynomial, respectively.

IV. If  $d(u, v) = 4$ , then  $D_4 = \{(v_i, u_{i+3}), (v_i, w_{i+2}), (v_i, w_{i-2}), (v_i, t_{i+2}),$

$(v_i, t_{i-2}), (u_i, w_{i+1}), (u_i, t_{i-1}), (u_i, u_{i+2}), (w_i, t_{i+2}) \mid i \in \mathbb{Z}_6\}$  and  $|D_4| = 54$ . Similarly, we have three subsets of  $D_4$ .

IV-1.  $\forall i \in \mathbb{Z}_6$ , since  $d(v_i, u_{i+3}) = d(u_i, u_{i+2}) = 4$ , thus  $|\{(u, v) \in V(G) \mid d(u, v) = 4 \& \delta_v + \delta_u = 6, \delta_v \times \delta_u = 9\}| = 12$ . Thus, we have the term  $72x^4$  for the Schultz polynomial and  $108x^4$  for the Modified Schultz polynomial.

IV-2.  $\forall i \in \mathbb{Z}_6$ , let  $(u=t_{i+2}, t_{i-2}, w_{i+2}, w_{i-2} \& v=v_i)$  or  $(v=t_{i-1}, w_{i-1}, w_{i+1} \& u=u_i)$ . Thus,

$|\{(u, v) \mid u, v \in V(G), d(u, v) = 4 \& \delta_v + \delta_u = 5, \delta_v \delta_u = 6\}| = 36$ . So, we have the term  $180x^4$  for the Schultz polynomial and  $216x^4$  for the Modified Schultz polynomial.

IV-3.  $\forall i \in \mathbb{Z}_6$ , let  $v=w_i$  &  $u=w_{i+2}$ . Thus,

$|\{(u, v) \in V(G) \mid u, v \in V(G), d(u, v) = 4 \& \delta_v + \delta_u = 4, \delta_v \delta_u = 4\}| = 6$ .

Generally, the two terms for the Schultz polynomial and Modified Schultz polynomial are  $276x^4, 348x^4$ , respectively.

V. If  $d(u, v) = 5$ , then  $D_5 = \{(v_i, t_{i+3}), (v_i, w_{i+3}), (u_i, w_{i+2}), (u_i, w_{i-2}), (u_i, t_{i+2}),$

$(u_i, t_{i-2}), (u_i, u_{i+2}), (w_i, t_{i-1}) \mid i \in \mathbb{Z}_6\}$  and  $|D_5| = 45$ . Again, we have three subsets of  $D_5$  as above.

V-1. We have  $|\{(u_i, u_{i+3}) \mid \forall i \in \mathbb{Z}_6, d(u_i, u_{i+3}) = 5 \& \delta_v + \delta_u = 5, \delta_v \times \delta_u = 9\}| = 3$ . Hence,  $18x^5$  and  $27x^5$  are the terms for the Schultz polynomial and Modified Schultz polynomial, respectively.

V-2.  $\forall i \in \mathbb{Z}_6$ , let  $(v=t_{i+2}, t_{i-2}, w_{i+2}, w_{i-2} \& u=u_i)$  or  $(u=t_{i+3}, w_{i+3} \& v=v_i)$ , therefore  $|\{(u, v) \mid u, v \in V(G), d(u, v) = 5 \& \delta_v + \delta_u = 5, \delta_v \delta_u = 6\}| = 36$ . Thus, there is the term  $180x^5$  for the Schultz polynomial and  $261x^5$  for the Modified Schultz polynomial.

V -3.  $\forall i \in \mathbb{Z}_6$  since  $d(w_i, t_{i-1})=5$ ,  $|\{(u,v) \in V(G) | d(u,v)=5 \& \delta_v + \delta_u = \delta_v \delta_u = 4\}|=6$ .

In general, the terms are  $222x^5$ ,  $267x^5$  for these polynomials.

VI. If  $d(u,v)=6$ , then  $D_6 = \{(u_i, t_{i+3}), (u_i, w_{i+3}), (w_i, w_{i+2}), (w_i, t_{i-2}), (t_i, t_{i+2}) | i \in \mathbb{Z}_6\}$  and  $|D_6|=30$ . It means that, we have two subsets of  $D_6$ .

VI-1. We have  $|\{(u_i, w_{i+3}), (u_i, t_{i+3}) | \forall i \in \mathbb{Z}_6, d(u,v)=6 \& \delta_v + \delta_u = 5, \delta_v \times \delta_u = 6\}|=12$ . Then,  $60x^6$  and  $72x^6$  are the two terms for the Schultz polynomial and Modified Schultz polynomial, respectively.

VI-2.  $\forall i \in \mathbb{Z}_6$ , since  $d(w_i, w_{i+2})=d(w_i, t_{i-2})=d(t_i, t_{i+2})=6$ . Therefore,  $|\{(u,v) | u,v \in V(G), d(u,v)=6 \& \delta_v + \delta_u = \delta_v \delta_u = 4\}|=18$ . In general, we have  $132x^6$  for the Schultz polynomial and  $144x^6$  for the Modified Schultz polynomial.

VII. If  $d(u,v)=7$ , then  $|D_7| = |\{\forall i \in \mathbb{Z}_6, (w_i, t_{i+3}), (w_i, w_{i+3}), (t_i, t_{i+3}) | d(u,v)=7 \& \delta_v + \delta_u = \delta_v \delta_u = 4\}|=12$ . Thus, there is the same term  $48x^7$  for the Schultz polynomial and Modified Schultz polynomial. Now, we enumerate all distinct shortest path of any  $u, v \in V(G)$ . Thus the Schultz polynomial of  $Ca(C_6)$  is:  $Sc(Ca(C_6), x) = 156x + 252x^2 + 294x^3 + 276x^4 + 222x^5 + 132x^6 + 48x^7$  and the Schultz index is  $Sc(Ca(C_6)) = 4884$ .

The Modified Schultz polynomial of  $Ca(C_6)$  is:

$Sc^*(Ca(C_6), x) = 204x + 330x^2 + 381x^3 + 348x^4 + 267x^5 + 144x^6 + 48x^7$   
and the Modified Schultz index  $Sc^*(Ca(C_6)) = 5934$ .

Thus, the *proof* of Theorem 1 is complete.

*Proof of Theorem 2.* Let  $Ca(C_6)$  be the molecular graph of Coronene.

Since  $D_i = D_i(3,3) \cup D_i(3,2) \cup D_i(2,2)$ ,  $\forall i \in \{1, 2, \dots, 7\}$

therefore the set of distances in  $G$  is given by the following relation:  
 $d(G, i) = |D_i| = |D_i(3,3)| + |D_i(3,2)| + |D_i(2,2)|$ . Keeping in mind the definition of Hosoya polynomial and the data provided in the proof of Theorem 1, the formula of this polynomial in Coronene is:

$$H(G, x) = \sum_{i=0}^{d(G)} d(G, i) x^i = 24 + 30x + 48x^2 + 57x^3 + 54x^4 + 45x^5 + 30x^6 + 12x^7.$$

Hence, the Wiener and Hyper Wiener indices of Coronene are:

$$W(G) = \sum_{i=0}^{d(G)} i \times d(G, i) = 24 \times 0 + 30 \times 1 + 48 \times 2 + 57 \times 3 + 54 \times 4 + 45 \times 5 + 30 \times 6 + 12 \times 7 = 1002.$$

$$WW(G) = 1002 + (1/2)(48 \times 2 + 57 \times 3 \times 2 + 54 \times 4 \times 3 + 45 \times 5 \times 4 + 30 \times 6 \times 5 + 12 \times 7 \times 6) = 2697$$

Thus, the *proof* of Theorem 2 is complete.

## CONCLUSIONS

In this paper, Schultz, Modified Schultz and Hosoya polynomials and their topological indices in the molecular graph of Coronene (constructed by Capra-operated benzenoid:  $Ca(C_6)$  (or  $H_2$ )) are calculated. These polynomials and indices could be useful in the topological investigation of benzenoids molecules.

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