STUDIA UBB CHEMIA, LIX, 4, 2014 (p. 113-128)
(RECOMMENDED CITATION)

ON TOPOLOGICAL PROPERTIES OF NANOCONES $CNC_k[n]$

SAKANDER HAYAT[a], MUHAMMAD IMRAN[a,*]

ABSTRACT. In this paper, we compute fourth atom-bond connectivity indices and fifth geometric-arithmetic indices for conical graphite. We also compute atom-bond conncetivity (ABC) and geometric-arithmetic (GA) indices for these conical graphite.

2010 Mathematics Subject Classification: *05C12, 05C90*

Keywords: *Atom-bond connectivity (ABC) index, Geometric-arithmetic (GA) index, ABC_4 index, GA_5 index, $CNC_k[n]$ nanocones*

INTRODUCTION

Mathematical calculations are of much importance to investigate essential concepts in chemistry. There is a substantial use of graph theory in chemistry. *Chemical graph theory* is the subject in which we model chemical structures and then study these structures by using graph theoretical properties/invariants. In the last few decades there is a lot of research which has been done in this field. A *moleculer/chemical* graph is a simple finite hydrogen depleted graph in which vertices denote the atoms and edges denote the chemical bonds in underlying chemical structure.

A *topological index* is a function "Top" from ∑ to the set of real numbers, where ∑ is the set of finite simple graphs with the property that $Top(G) = Top(H)$ if both G and H are isomorphic. Obviously, the number of edges and vertices of a graph are topological indices also. A graph can be recognized by a numeric number, a polynomial, a sequence of numbers or a matrix which represents the whole graph, and these representations are aimed to be uniquely defined for that graph. Topological indices are graph invariants and are used for Quantitative Structure - Activity Relationship (QSAR) and Quantitative Structure-Property Relationship (QSPR) studies [1]. Many

[a] *Department of Mathematics,School of Natural Sciences (SNS), National University of Sciences and Technology (NUST), Sector H-12, Islamabad, Pakistan*
** Corresponding author: imrandhab@gmail.com*

topological indices have been defined and several of them have found applications as means to model physical, chemical, pharmaceutical and other properties of molecules.

A *nanostructure* is an object of intermediate size between microscopic and molecular structures. It is a product derived through engineering at molecular scale. *Carbon nanocones* are conical structures which are allotropes of carbon having at least one dimension of the order one micrometer or smaller. Carbon cones have also been observed, since 1968 or even earlier, on the surface of naturally occurring graphite. Their bases are attached to the graphite and their height varies between less than 1 and 40 micrometers. The analytical applications of carbon nanocones are still quite limited, however, and fall in the field of solid-phase extraction, in which surpassed carbon nanotubes thanks to their lower aggregation tendency.

Throughout this article, *G* is considered to be a connected graph with the vertex set $V(G)$ and edge set $E(G)$, d_u is the degree of vertex $u \in V(G)$ and $S_u = \sum_{v \in N_G(u)} d_G(v)$ where $N_G(u) = \{v \in V(G) \mid uv \in E(G)\}$. The notations used in this paper are mainly taken from books [2,3].

The first degree-based connectivity index for the graphs, constructed on the ground of vertex degrees is *Randić* index [4]. The Randić index of graph G is defined as

$$R_{-\frac{1}{2}}(G) = \sum_{uv \in E(G)} \frac{1}{\sqrt{d_u d_v}}$$

The general *Randić* connectivity index $R_\alpha(G)$ is the sum of $(d_u d_v)^\alpha$ over all edges $e = uv \in E(G)$ defined as

$$R_\alpha(G) = \sum_{uv \in E(G)} (d_u d_v)^\alpha$$

Obviously $R_{-\frac{1}{2}}(G)$ is the particular case of $R_\alpha(G)$ when $\alpha = -\frac{1}{2}$.

One of the well-known connectivity topological index is *atom-bond connectivity* (ABC) index, introduced by *Estrada* et al. in [5]. The ABC index of graph G is defined as

$$ABC(G) = \sum_{uv \in E(G)} \sqrt{\frac{d_u + d_v - 2}{d_u d_v}}$$

Another well-known connectivity topological descriptor is *geometric-arithmetic* (GA) index, introduced by *Vukičević* et al. in [6]. The GA index for graph G is defined by

$$GA(G)=\sum_{uv\in E(G)}\frac{2\sqrt{d_u d_v}}{d_u+d_v}$$

The fourth version of ABC index was introduced by *Ghorbani* et al. [7] in 2010. For graph G , the ABC_4 index is defined as

$$ABC_4(G)=\sum_{uv\in E(G)}\sqrt{\frac{S_u+S_v-2}{S_u S_v}}$$

Recently, the fifth version of GA index was proposed by *Graovac* et al. [8] in 2011. The GA_5 index for graph G is defined as follows

$$GA_5(G)=\sum_{uv\in E(G)}\frac{2\sqrt{S_u S_v}}{(S_u+S_v)}$$

In this paper, we discuss two topological descriptors, namely ABC_4 and GA_5 indices for $CNC_k[n]$, $3\le k\le 6$ nanocones. We also present the two important types of partitions of $CNC_k[n]$ nanocones in two parameters k and n, and then apply them on nanocones $CNC_k[n]$ to compute certain topological indices.

RESULTS AND DISCUSSON

In this paper, we find general partitions of the edge set of $CNC_k[n]$ nanocones for $n\ge 1, k\ge 3$, based on the degrees sum of neighbors of each edge and degrees of end vertices for each. We used these partitions to computed ABC_4, GA_5, *ABC* and *GA* indices of these nanocones.

Results for *CNC₃[n]* Nanocones

In this section, we compute exact formulas of ABC_4 and GA_5 indices of $CNC_3[n]$ nanocones. A $CNC_3[n]$ nanocone consists of a triangle as its core and encompassing the layers of hexagons on its conical surface. If there are n layers of hexagons on the conical surface around triangle, then we denote the graph of that nanocones as $CNC_3[n]$ in which n denotes the number of layers of hexagons while the subscript number shows the sides of polygon which acts as the core of nanocones. The $CNC_3[2]$ nanocone is shown in Figure 1. We have $|V(CNC_3[n])| = 3(n+1)^2$ and $|E(CNC_3[n])| = \frac{9}{2}n^2 + \frac{15}{2}n + 3$. In the next theorem, we compute the ABC_4 index of $CNC_3[n]$ nanocones.

Theorem 1. *Consider the graph of $CNC_3[n]$ nanocones, for $n \geq 1$, then their ABC_4 index is equal to*

$$ABC_4(CNC_3[n]) = 2n^2 + (\frac{\sqrt{462}}{7} + \frac{3\sqrt{2}-2}{3})n + \frac{6\sqrt{2}}{5} + \frac{6\sqrt{14}}{17} - \frac{\sqrt{462}}{7}$$

Proof. Let G be the graph of $CNC_3[n]$ nanocones. We find the edge partition of $CNC_3[n]$ nanocones based on the degree sum of vertices lying at the unit distance from end vertices of each edge, as in Table 1.

Table 1. The edge partition of $CNC_3[n]$

(S_u, S_v) where $uv \in E(G)$	$(5,5)$	$(5,7)$	$(6,7)$	$(7,9)$	$(9,9)$
Number of edges	3	6	$6(n-1)$	$3n$	$\frac{9}{2}n^2 - \frac{3}{2}n$

Now we can apply the formula of ABC_4 index to compute it for G. Since

$$ABC_4(G) = \sum_{uv \in E(G)} \sqrt{\frac{S_u + S_v - 2}{S_u S_v}}$$, then

$$ABC_4(G) = (3)\sqrt{\frac{5+5-2}{5\times 5}} + (6)\sqrt{\frac{5+7-2}{5\times 7}} + 6(n-1)\sqrt{\frac{6+7-2}{6\times 7}} + (3n)\sqrt{\frac{7+9-2}{7\times 9}} + (\frac{9}{2}n^2 - \frac{3}{2}n)\sqrt{\frac{9+9-2}{9\times 9}}$$

After simplification, we get

$$ABC_4(G) = 2n^2 + (\frac{\sqrt{462}}{7} + \frac{3\sqrt{2}-2}{3})n + \frac{6\sqrt{2}}{5} + \frac{6\sqrt{14}}{17} - \frac{\sqrt{462}}{7}$$ □

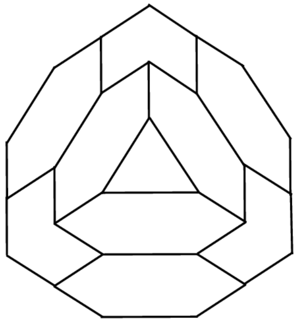

Figure 1. Graph of $CNC_3[2]$ nanocone.

The GA_5 index for $CNC_3[n]$ nanocones is computed in the following theorem.

Theorem 2. *Consider the graph of $CNC_3[n]$ nanocones, for $n \geq 1$, then their GA_5 index is equal to*

$$GA_5(CNC_3[n]) = \frac{9}{2}n^2 + (\frac{12\sqrt{42}}{13} + \frac{9\sqrt{7}}{8} - \frac{3}{2})n + \sqrt{35} - \frac{12\sqrt{42}}{13} + 3$$

Proof. Let G be the graph of $CNC_3[n]$ nanocones. The edge partition of $CNC_3[n]$ nanocones based on the degree sum of vertices lying at the unit distance from end vertices of each edge is given in Table 1.

Now we apply the formula of GA_5 index to compute this index for G. Since

$GA_5(G) = \sum_{uv \in E(G)} \frac{2\sqrt{S_u S_v}}{S_u + S_v}$, then

$$GA_5(G) = (3)\frac{2\sqrt{5\times 5}}{5+5} + (6)\frac{2\sqrt{5\times 7}}{5+7} + 6(n-1)\frac{2\sqrt{6\times 7}}{6+7} + (3n)\frac{2\sqrt{7\times 9}}{7+9} + (\frac{9}{2}n^2 - \frac{3}{2}n)\frac{2\sqrt{9\times 9}}{9+9}$$

After simplification, we get

$$GA_5(G) = \frac{9}{2}n^2 + (\frac{12\sqrt{42}}{13} + \frac{9\sqrt{7}}{8} - \frac{3}{2})n + \sqrt{35} - \frac{12\sqrt{42}}{13} + 3$$ □

Results for *CNC4[n]* Nanocones

In this section, we compute the ABC_4 and GA_5 indices of $CNC_4[n]$ nanocones. These $CNC_4[n]$ nanocones consist of a square as the core and tiling of hexagonal layers on its conical surface. A $CNC_4[2]$ nanocone is shown in Figure 2. The vertex and edge cardinalities are $|V(CNC_4[n])|=4(n+1)^2$ and respectively $|E(CNC_4[n])|=6n^2+10n+4$.

Now we compute the closed formula for ABC_4 index of $CNC_4[n]$ nanocones in the following theorem.

Theorem 3. *Consider the graph of $CNC_4[n]$ nanocones, for $n\geq 1$, then their ABC_4 index is equal to*

$$ABC_4(CNC_4[n])=\frac{8}{3}n^2+(\frac{4\sqrt{462}}{21}+\frac{4\sqrt{2}}{3}-\frac{8}{9})n+\frac{8\sqrt{2}}{5}+\frac{8\sqrt{14}}{7}-\frac{4\sqrt{462}}{21}$$

Proof. Let G be the graph of $CNC_4[n]$ nanocones. We determine the edge partition of $CNC_4[n]$ based on the degree sum of neighbors of end vertices of each edge.

Table 2. The edge partition of $CNC_4[n]$

(S_u,S_v) where $uv\in E(G)$	$(5,5)$	$(5,7)$	$(6,7)$	$(7,9)$	$(9,9)$
Number of edges	4	8	$8(n-1)$	$4n$	$6n^2-2n$

Now we use this partition to compute ABC_4 index of $CNC_4[n]$ nanocones. Since

$$ABC_4(G)=\sum_{uv\in E(G)}\sqrt{\frac{S_u+S_v-2}{S_uS_v}}$$, then

$$ABC_4(G)=(4)\sqrt{\frac{5+5-2}{5\times 5}}+(8)\sqrt{\frac{5+7-2}{5\times 7}}+8(n-1)\sqrt{\frac{6+7-2}{6\times 7}}+$$
$$(4n)\sqrt{\frac{7+9-2}{7\times 9}}+(6n^2-2n)\sqrt{\frac{9+9-2}{9\times 9}}$$

After an easy simplification, we get

$$ABC_4(G) = \frac{8}{3}n^2 + (\frac{4\sqrt{462}}{21} + \frac{4\sqrt{2}}{3} - \frac{8}{9})n + \frac{8\sqrt{2}}{5} + \frac{8\sqrt{14}}{7} - \frac{4\sqrt{462}}{21} \quad \square$$

Theorem 4. *Consider the graph of $CNC_4[n]$ nanocones, for $n \geq 1$, then their GA_5 index is equal to*

$$GA_5(CNC_4[n]) = 6n^2 + (\frac{16\sqrt{42}}{13} + \frac{3\sqrt{2}-4}{2})n + \frac{4\sqrt{35}}{3} - \frac{16\sqrt{42}}{13} + 4$$

***Proof*.** Let G be the graph of $CNC_4[n]$ nanocones. The edge partition of $CNC_3[n]$ nanocones based on the degree sum of vertices lying at the unit distance from end vertices of each edge is given in Table 2.

Now we apply the formula of GA_5 index to compute this index for G.

Since

$$GA_5(G) = \sum_{uv \in E(G)} \frac{2\sqrt{S_u S_v}}{S_u + S_v},$$

then

$$GA_5(G) = (4)\frac{2\sqrt{5\times5}}{5+5} + (8)\frac{2\sqrt{5\times7}}{5+7} + 8(n-1)\frac{2\sqrt{6\times7}}{6+7} + (4n)\frac{2\sqrt{7\times9}}{7+9} + (6n^2 - 2n)\frac{2\sqrt{9\times9}}{9+9}$$

After simplification, we get

$$GA_5(G) = 6n^2 + (\frac{16\sqrt{42}}{13} + \frac{3\sqrt{2}-4}{2})n + \frac{4\sqrt{35}}{3} - \frac{16\sqrt{42}}{13} + 4 \quad \square$$

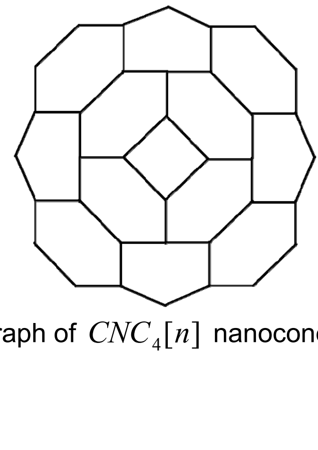

Figure 2. Graph of $CNC_4[n]$ nanocone with $n = 2$.

Results for *CNC₅[n]* Nanocones

In this section, we determine ABC_4 and GA_5 indices of $CNC_5[n]$ nanocones. The vertex and edge cardinalities are $|V(CNC_5[n])| = 5(n+1)^2$ and $|E(CNC_5[n])| = \frac{15}{2}n^2 + \frac{25}{2}n + 5$. This family of nanocones are often called *one pentagonal* nanocones, is depicted in Figure 3. In the following theorem, the ABC_4 index of $CNC_5[n]$ nanocones is computed.

Theorem 5. *Consider the graph of $CNC_5[n]$ nanocones, for $n \geq 1$, then their ABC_4 index is equal to*

$$ABC_4(CNC_5[n]) = \frac{10}{3}n^2 + (\frac{5\sqrt{462}}{21} + \frac{5\sqrt{2}}{3} - \frac{10}{9})n + \frac{10\sqrt{4}}{7} - \frac{5\sqrt{462}}{21} + 2\sqrt{2}$$

Proof. Consider G be the graph of $CNC_5[n]$ nanocones. We find the partition of edge set of $CNC_5[n]$ nanocones based on the degree sum of vertices lying at the unit distance from end vertices of each edge. Table 3 shows the data for theabove discussed edge partition of $CNC_5[n]$ nanocones.

Table 3. The edge partition of the graph of $CNC_5[n]$ nanocones

(S_u, S_v) where $uv \in E(G)$	(5,5)	(5,7)	(6,7)	(7,9)	(9,9)
Number of edges	5	10	$10(n-1)$	$5n$	$\frac{15}{2}n^2 - \frac{5}{2}n$

Now we use this partition to compute ABC_4 index of $CNC_4[n]$ nanocones. Since

$$ABC_4(G) = \sum_{uv \in E(G)} \sqrt{\frac{S_u + S_v - 2}{S_u S_v}} \text{ then,}$$

$$ABC_4(G) = (5)\sqrt{\frac{5+5-2}{5\times5}} + (10)\sqrt{\frac{5+7-2}{5\times7}} + 10(n-1)\sqrt{\frac{6+7-2}{6\times7}} +$$
$$(5n)\sqrt{\frac{7+9-2}{7\times9}} + (\frac{15}{2}n^2 - \frac{5}{2}n)\sqrt{\frac{9+9-2}{9\times9}}$$

After an easy simplification, we get

$$ABC_4(G)=\frac{10}{3}n^2+(\frac{5\sqrt{462}}{21}+\frac{5\sqrt{2}}{3}-\frac{10}{9})n+\frac{10\sqrt{4}}{7}-\frac{5\sqrt{462}}{21}+2\sqrt{2}$$ □

In the following theorem, the GA_5 index of $CNC_5[n]$ nanocones is computed.

Theorem 6. *Consider the graph of $CNC_5[n]$ nanocones, for $n \geq 1$, then their GA_5 index is equal to*

$$GA_5(CNC_5[n])=\frac{15}{2}n^2+(\frac{20\sqrt{42}}{13}+\frac{15\sqrt{7}}{8}-\frac{5}{2})n+\frac{5\sqrt{35}}{3}-\frac{20\sqrt{42}}{13}+5$$

Proof. Let G be the graph of $CNC_5[n]$ nanocones. The edge partition of $CNC_5[n]$ based on the degree sum of neighbors of end vertices of each edge is given in Table 3. Since

$GA_5(G)=\sum_{uv\in E(G)}\frac{2\sqrt{S_uS_v}}{S_u+S_v}$ then,

$$GA_5(G)=(5)\frac{2\sqrt{5\times5}}{5+5}+(10)\frac{2\sqrt{5\times7}}{5+7}+10(n-1)\frac{2\sqrt{6\times7}}{6+7}+(5n)\frac{2\sqrt{7\times9}}{7+9}$$
$$+(\frac{15}{2}n^2-\frac{5}{2}n)\frac{2\sqrt{9\times9}}{9+9}$$

After simplification, we get

$$GA_5(G)=\frac{15}{2}n^2+(\frac{20\sqrt{42}}{13}+\frac{15\sqrt{7}}{8}-\frac{5}{2})n+\frac{5\sqrt{35}}{3}-\frac{20\sqrt{42}}{13}+5$$ □

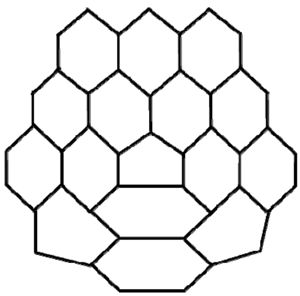

Figure 3. Graph of one pentagonal nanocone $CNC_5[n]$ with $n=2$.

Results for *CNC₆[n]* Nanocones

Now we compute ABC_4 and GA_5 indices of $CNC_6[n]$ nanocones. In this family of nanocones the hexagon acts as the core for the surrounding hexagonal layers; in fact, it is a coronene family. For such a plane graph we have $|V(CNC_6[n])| = 6(n+1)^2$ and $|E(CNC_6[n])| = 9n^2 + 15n + 6$. A graph of $CNC_6[3]$ nanocone is depicted in Figure 4.In the following theorem, ABC_4 index of $CNC_6[n]$ nanocones is exhibited.

Theorem 7. *Consider the graph of* $CNC_6[n]$ *nanocones, for* $n \geq 1$ *then their* ABC_4 *index is equal to*

$$ABC_4(CNC_6[n]) = 4n^2 + (\frac{2\sqrt{462}}{7} + 2\sqrt{2} - \frac{4}{3})n + \frac{12\sqrt{2}}{5} + \frac{12\sqrt{14}}{17} - \frac{2\sqrt{462}}{7}$$

Proof. Let G be the graph of $CNC_6[n]$ nanocones. We first compute the edge partition of $CNC_6[n]$ nanocones based on the degree sum of neighbors of end vertices of each edge (Table 4).

Table 4. The edge partition of $CNC_6[n]$

(S_u, S_v) where $uv \in E(G)$	$(5,5)$	$(5,7)$	$(6,7)$	$(7,9)$	$(9,9)$
Number of edges	6	12	$12(n-1)$	$6n$	$9n^2 - 3n$

Now we use this partition to compute ABC_4 index of $CNC_4[n]$ nanocones. Since

$$ABC_4(G) = \sum_{uv \in E(G)} \sqrt{\frac{S_u + S_v - 2}{S_u S_v}}$$ then,

$$ABC_4(G) = (6)\sqrt{\frac{5+5-2}{5\times 5}} + (12)\sqrt{\frac{5+7-2}{5\times 7}} + 12(n-1)\sqrt{\frac{6+7-2}{6\times 7}} +$$
$$(6n)\sqrt{\frac{7+9-2}{7\times 9}} + (9n^2 - 3n)\sqrt{\frac{9+9-2}{9\times 9}}$$

After an easy simplification, we get

$$ABC_4(G) = 4n^2 + (\frac{2\sqrt{462}}{7} + 2\sqrt{2} - \frac{4}{3})n + \frac{12\sqrt{2}}{5} + \frac{12\sqrt{14}}{17} - \frac{2\sqrt{462}}{7} \quad \square$$

In the following theorem, we compute GA_5 index of $CNC_6[n]$ nanocones.

Theorem 8. *Consider the graph of $CNC_6[n]$ nanocones, for $n \geq 1$ then their GA_5 index is equal to*

$$GA_5(CNC_6[n]) = 9n^2 + (\frac{24\sqrt{42}}{13} + \frac{9\sqrt{7}}{4} - 3)n + 2\sqrt{35} - \frac{24\sqrt{42}}{13} + 6$$

Proof. Let G be the graph of $CNC_6[n]$ nanocones. The required edge partition to compute GA_5 index is in Table 4. Since

$GA_5(G) = \sum_{uv \in E(G)} \frac{2\sqrt{S_u S_v}}{S_u + S_v}$ then,

$$GA_5(G) = (6)\frac{2\sqrt{5\times5}}{5+5} + (12)\frac{2\sqrt{5\times7}}{5+7} + 12(n-1)\frac{2\sqrt{6\times7}}{6+7} + (6n)\frac{2\sqrt{7\times9}}{7+9} + (9n^2 - 3n)\frac{2\sqrt{9\times9}}{9+9}$$

After simplification, we get

$$GA_5(CNC_6[n]) = 9n^2 + (\frac{24\sqrt{42}}{13} + \frac{9\sqrt{7}}{4} - 3)n + 2\sqrt{35} - \frac{24\sqrt{42}}{13} + 6$$

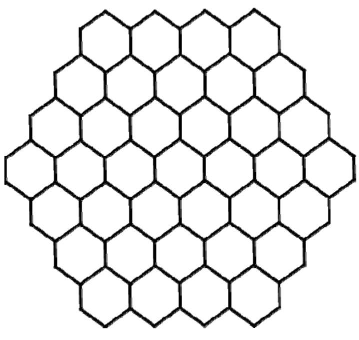

Figure 4. Graph of $CNC_6[n]$ nanocone with $n = 3$.

Results for *$CNC_k[n]$* Nanocones

Now we determine the ABC_4 and GA_5 indices of $CNC_k[n], k \geq 3, n \geq 1$ nanocones. A general representation of $CNC_k[n]$ nanocones is shown in Figure 5 in which parameters k and n are shown. We have $|V(CNC_k[n])| = k(n+1)^2$ and $|E(CNC_k[n])| = \frac{3}{2}(kn^2) + \frac{5}{2}(kn) + k$. For further study of nanocones, see [9,10,11,12,13,14,15].

Table 5. The edge partition of $CNC_k[n]$ based on the degree sum of neighbors of end vertices of each edge

(S_u, S_v) where $uv \in E(G)$	$(5,5)$	$(5,7)$	$(6,7)$	$(7,9)$	$(9,9)$
Number of edges	k	$2k$	$2k(n-1)$	kn	$\frac{3}{2}kn^2 - \frac{1}{2}kn$

In the following theorem, we present exact formula to calculate ABC_4 index of $CNC_k[n], k \geq 3, n \geq 1$ nanocones.

Theorem 9. *Consider the graph of $CNC_k[n], k \geq 3, n \geq 1$ nanocones, then their ABC_4 index is equal to*

$$ABC_4(CNC_k[n]) = \frac{2\sqrt{2}}{5}(k) + \frac{\sqrt{14}}{7}(2k) + \frac{\sqrt{462}}{42}(2k(n-1)) + \frac{\sqrt{2}}{3}(kn) + \frac{4}{9}(\frac{3}{2}(kn^2) - \frac{1}{2}(kn))$$

Proof. Consider the graph of $CNC_k[n], k \geq 3, n \geq 1$ nanocones. In order to compute the ABC_4 index of $CNC_k[n]$ nanocones, we find the general partition of $CNC_k[n]$ nanocones in two parameters k and n based on the degree sum of vertices lying at unit distance from end vertices of each edge. Table 5 shows such partition.

Now by using the edge partition given in Table 5, we compute the ABC_4 index of $CNC_k[n]$ nanocones. Since

$$ABC_4(G) = \sum_{uv\in E(G)} \sqrt{\frac{S_u + S_v - 2}{S_u S_v}} \text{ then,}$$

$$ABC_4(CNC_k[n]) = (k)\sqrt{\frac{5+5-2}{5\times 5}} + (2k)\sqrt{\frac{5+7-2}{5\times 7}} + 2k(n-1)\sqrt{\frac{6+7-2}{6\times 7}} + (kn)\sqrt{\frac{7+9-2}{7\times 9}} + (\frac{3}{2}kn^2 - \frac{1}{2}kn)\sqrt{\frac{9+9-2}{9\times 9}}$$

After an easy simplification, we get

$$ABC_4(CNC_k[n]) = \frac{2\sqrt{2}}{5}(k) + \frac{\sqrt{14}}{7}(2k) + \frac{\sqrt{462}}{42}(2k(n-1)) + \frac{\sqrt{2}}{3}(kn) + \frac{4}{9}(\frac{3}{2}(kn^2) - \frac{1}{2}(kn))$$

Following theorem exhibits the GA_5 index of $CNC_k[n]$ nanocones.

Theorem 10. *Consider the graph of $CNC_k[n], k \geq 3, n \geq 1$ nanocones, then their GA_5 index is equal to*

$$GA_5(CNC_k[n]) = k + \frac{\sqrt{35}}{6}(2k) + \frac{2\sqrt{42}}{13}(2k(n-1)) + \frac{3\sqrt{7}}{8}(kn) + (\frac{3}{2}(kn^2) - \frac{1}{2}(kn))$$

Proof. Consider G be the graph of $CNC_k[n], k \geq 3, n \geq 1$ nanocones. We have $|V(CNC_k[n])| = k(n+1)^2$ and $|E(CNC_k[n])| = \frac{3}{2}(kn^2) + \frac{5}{2}(kn) + k$. By using the edge partition given in Table 5, we compute the ABC_4 index of $CNC_k[n]$ nanocones. Since

$$GA_5(G) = \sum_{uv\in E(G)} \frac{2\sqrt{S_u S_v}}{S_u + S_v} \text{ then,}$$

$$GA_5(G) = (k)\frac{2\sqrt{5\times 5}}{5+5} + (2k)\frac{2\sqrt{5\times 7}}{5+7} + 2k(n-1)\frac{2\sqrt{6\times 7}}{6+7} + (kn)\frac{2\sqrt{7\times 9}}{7+9} + (\frac{3}{2}kn^2 - \frac{1}{2}kn)\frac{2\sqrt{9\times 9}}{9+9}$$

After simplification, we get

$$GA_5(CNC_k[n]) = k + \frac{\sqrt{35}}{6}(2k) + \frac{2\sqrt{42}}{13}(2k(n-1)) + \frac{3\sqrt{7}}{8}(kn) + (\frac{3}{2}(kn^2) - \frac{1}{2}(kn))$$

Now we compute the edge partition of $CNC_k[n]$ nanocones with respect to degree of end vertices of edges. Table 6 shows such a partition of $CNC_k[n]$ nanocones.

Table 6. The edge partition of $CNC_k[n]$ based on the degrees of end vertices of each edge.

(d_u, d_v) where $uv \in E(G)$	$(2,2)$	$(2,3)$	$(3,3)$
Number of edges	*k*	2*kn*	$\frac{3}{2}kn^2 + \frac{1}{2}kn$

In the following theorem, ABC index of $CNC_k[n]$ nanocones is presented.

Theorem 11. *Consider the graph of* $CNC_k[n], k \geq 3, n \geq 1$ *nanocones, then their* ABC *index is equal to*

$$ABC(CNC_k[n]) = \frac{\sqrt{2}}{2}(k(1+2n)) + \frac{2}{3}(\frac{3}{2}kn^2 + \frac{1}{2}kn)$$

Proof. By using the edge partition based on the degrees of end vertices of each edge of $CNC_k[n]$ nanocones given in Table 6 we compute the ABC index of $CNC_k[n]$ nanocones. Since

$$ABC(G) = \sum_{uv \in E(G)} \sqrt{\frac{d_u + d_v - 2}{d_u d_v}}$$ then,

$$ABC(CNC_k[n]) = (k)\sqrt{\frac{2+2-2}{2\times 2}} + (2kn)\sqrt{\frac{2+3-2}{2\times 3}} + (\frac{3}{2}kn^2 + \frac{1}{2}kn)\sqrt{\frac{3+3-2}{3\times 3}}$$

After an easy simplification, we get

$$ABC(CNC_k[n]) = \frac{\sqrt{2}}{2}(k(1+2n)) + \frac{2}{3}(\frac{3}{2}kn^2 + \frac{1}{2}kn)$$

The GA index of $CNC_k[n]$ nanocones is computed in the following theorem.

Theorem 12. *Consider the graph of* $CNC_k[n], k \geq 3, n \geq 1$ *nanocones, then their* GA *index is equal to*

$$GA(CNC_k[n]) = k + \frac{2\sqrt{6}}{5}(2kn) + \frac{k}{2}(3n^2 + n)$$

Proof. By using the edge partition based on the degrees of end vertices of each edge of $CNC_k[n]$ nanocones given in Table 6 we compute the GA index of $CNC_k[n]$ nanocones. Since

$$GA(G) = \sum_{uv \in E(G)} \frac{2\sqrt{d_u d_v}}{d_u + d_v}$$ then,

$$GA(CNC_k[n]) = (k)\frac{2\sqrt{2\times 2}}{2+2} + (2kn)\frac{2\sqrt{2\times 3}}{2+3} + (\frac{3}{2}kn^2 + \frac{1}{2}kn)\frac{2\sqrt{3\times 3}}{3+3}$$

After simplification, we get

$$GA(CNC_k[n]) = k + \frac{2\sqrt{6}}{5}(2kn) + \frac{k}{2}(3n^2 + n)$$

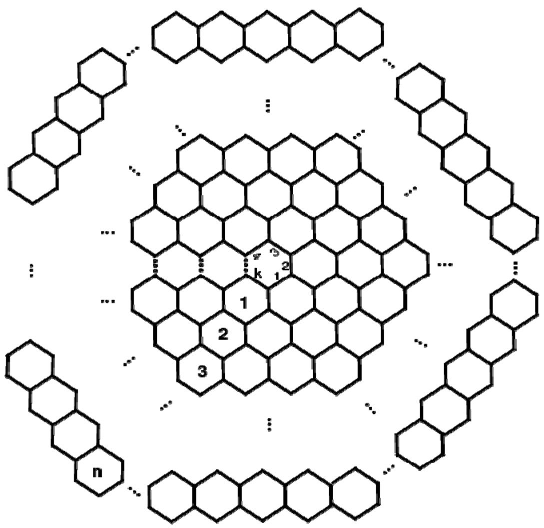

Figure 5. A general representation of graph of $CNC_k[n]$ nanocones.

CONCLUSIONS

In this paper, two new connectivity topological indices ABC_4 and GA_5 of $CNC_k[n], k \geq 3, n \geq 1$ nanocones were studied. We derived closed formulae of these topological indices for them. We found general partitions of the edge set of $CNC_k[n]$ nanocones based on the degrees sum of neighbors of each edge and degrees of end vertices for each. We used these partitions to computed ABC_4, GA_5, *ABC* and *GA* indices of $CNC_k[n], n \geq 1, k \geq 3$ nanocones.

ACKNOWLEDGMENTS

This research is partially supported by National University of Sciences and Technology (NUST), Islamabad, Pakistan. The authors are very grateful to the referees for their useful comments and criticism which improved this paper very much.

REFERENCES

1. M. Saheli, H. Saati, A.R. Ashrafi, *Optoelectron. Adv. Mater. Rapid Commun.*, **2010**, *4*, 896.
2. N. Trinajstić, *CRC Press, Boca Raton, FL,* **1992**.
3. I. Gutman, O.E. Polansky, *Springer-Verlag, New York*, **1986**.
4. M. Randić, *J. Amer. Chem. Soc.,* **1975**, *97*, 6609.
5. E. Estrada, L. Torres, L. Rodríguez, I. Gutman, *Indian J. Chem.*, **1998**, *37*, 849.
6. D. Vukičević, B. Furtula, *J. Math. Chem.,* **2009**, *46*, 1369.
7. M. Ghorbani, M.A. Hosseinzadeh, *Optoelectron. Adv. Mater. Rapid Commun.*, **2010**, *4*, 1419.
8. A. Graovac, M. Ghorbani, M.A. Hosseinzadeh, *J. Math. Nanosciences*, **2011**, *1*, 33.
9. A.E. Vizitiu and M.V. Diudea, *MATCH Commun. Math. Comput. Chem.*, **2008**, *60*, 927.
10. A. Ilić, M.V. Diudea, F. Gholami-Nezhaad, A.R. Ashrafi, Topological indices in nanocones in: I. Gutman, B. Furtula (Eds.), New Molecular Structure Descriptors - Theory and Applications I, *Univ. Kragujevac*, **2010**, 217.
11. Y. Alizadeh, S. Klavzar, M.A. Hosseinzadeh, *MATCH Commun. Math. Comput. Chem.*, **2013**, *69*, 523.
12. A. Khaksar, M. Ghorbani, H.R. Maimani, *Optoelectron. Adv. Mater. Rapid Commun.*, **2007**, *4*, 1868.
13. M.A. Alipour, A.R. Ashrafi, *J. Comput. Theor. Nanosci.,* **2009**, *4*, 1.
14. M. Ghorbani, M. Jalali, *MATCH Commun. Math. Comput. Chem.*, **2009**, *62*, 353.
15. M.H. Khalifeh, M.R. Darafsheh, Hassan Jolany, *J. Cur. Nanosci.*, **2013**, *9*, 557.