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EDGE GEOMETRIC-ARTHMETIC INDEX OF SOME GRAPHS

M. SAKI[a], A. IRANMANESH[b]* O. KHORMALI[c]

ABSTRACT. The edge version of geometric–arithmetic index of graphs is introduced based on the end-vertex degrees of edges of their line graphs. In this paper we compute this index for product graphs $P_n \times P_m$ and $p_n \times C_m$, and a dendrimer nanostar D_n.

Keywords: *edge geometric–arithmetic index, line graph, vertex degree, dendrimer*

1. INTRODUCTION

A single number that can be used to characterize some properties of molecular graphs is called a topological index. There are numerous topological descriptors that have found applications in the theoretical chemistry, especially in QSPR/QSAR research [1]. The oldest topological index, introduced by Harold Wiener in 1947, is the ordinary (vertex) version of Wiener index [2], which is the sum of all distances between vertices of a graph. There is also known an edge versions of Wiener index, based on distance between edges, introduced by Iranmanesh et al. in 2008 [3].

One of the most important topological indices is the Randić connectivity index [4], defined as the sum of certain bond contributions calculated from the vertex degree of the hydrogen suppressed molecular graphs.

Inspired by the Randic index in a graph $G(V,E)$, with $V(G)$ being the vertex/atom set and $E(G)$ the edge/bond set, [5,6], Vukicevic and Furtula [7] proposed a topological index named the geometric-arithmetic index (shortly GA) as

[a] *Department of Mathematics, Science and Research Branch, Islamic Azad University, Tehran, Iran*
[b] *Department of Pure Mathematics, Faculty of Mathematical Sciences, Tarbiat Modares University, Tehran, Iran*
[c] *Mathematics and Informatics Research Group, ACECR, Tarbiat Modares University, Tehran, Iran*
* *Corresponding author: iranmanesh@modares.ac.ir*

$$GA(G) = \sum_{uv \in E(G)} \frac{2\sqrt{d_G(u)d_G(v)}}{d_G(u)+d_G(v)}$$

where $d_G(u)$ denotes the degree of the vertex u in G. The reader can find more information about geometric-arithmetic index in [7-9].

In [10], the edge version of geometric-arithmetic index (edge GA index) was introduced, based on the end-vertex degrees of edges in a line graph of G; it is a derived graph such that each vertex of L(G) represents an edge in G and two vertices of L(G) are adjacent if and only if their corresponding edges share a common endpoint in G

$$GA_e(G) = \sum_{ef \in E(L(G))} \frac{2\sqrt{d_{L(G)}(e)d_{L(G)}(f)}}{d_{L(G)}(e)+d_{L(G)}(f)}$$

where $d_{L(G)}(e)$ denotes the vertex degree in L(G) or the degree of edge e in the original graph G.
We can calculate the edge GA index as

$$\mathrm{GA(G)} = \sum_{i=1}^{|E(G)|} \xi_i \text{ ,}$$

where $\xi_i = \frac{2\sqrt{du_i dv_i}}{du_i + dv_i}$ .Also, we have $d_e = d_u + d_v - 2$ where $e = uv \in E(G)$.
Then the number of edges in a line graph is

$$\left|E(L(G))\right| = \frac{1}{2} \sum_{e_i = u_i v_i \in E(G)} \left(d_{u_i} + d_{v_i} - 2\right) \times \left|E_i\right|$$

where $\left|E_i\right| = \left|\left\{e_i \left| e_i \in E(G), e_i = (du_i, dv_i)\right.\right\}\right|$.

In this paper, we compute this index for product graphs $P_n \times P_m$ and $P_n \times C_m$ anda dendrimer nanostar D_n.

2. EDGE GA INDEX OF PRODUCT GRAPHS $P_n \times P_m$ AND $p_n \times C_m$

At first, we compute the edge GA index for the product graph of two paths P_n and P_m, i.e., $P_n \times P_m$. In Figure 1, the graph of $P_7 \times P_6$ is shown. According to this figure, we have

$$\begin{aligned} |E(P_n \times P_m)| &= |E(P_n)||V(P_m)| + |E(P_m)||V(P_n)| \\ &= (n-1)(m) + (m-1)(n) = 2mn - (n+m) \end{aligned}$$

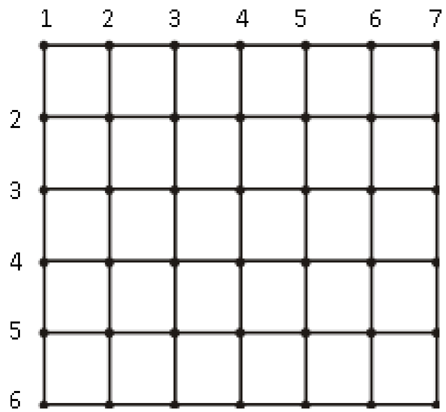

Figure 1. The graph of $P_7 \times P_6$

In Figure 2, the line graph of $P_7 \times P_6$ is shown. Accordingly, we have $\left|E\left(L\left(P_n \times P_m\right)\right)\right| = 6mn - 6\left(n+m\right) + 4$.

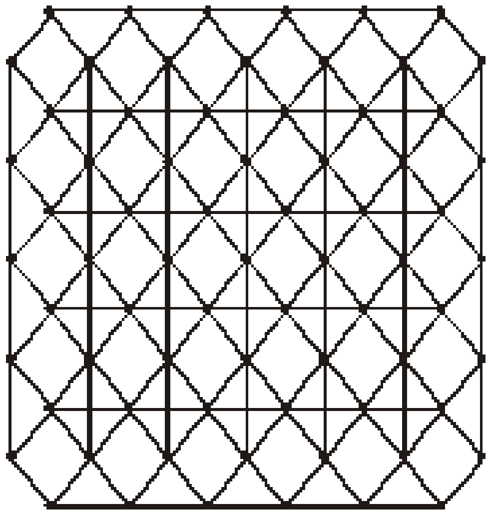

Figure 2. The graph of L($P_7 \times P_6$)

In Table 1, the type of edges, their numbers and amount of ξ_i are computed.

Table 1. Type of edges, their numbers and amount of ξ_i of $L(P_n \times P_m)$

Number of edges	ξ_i	Type of edges
4	1	(3,3)
8	$\frac{2\sqrt{12}}{7}$	(3,4)
8	$\frac{2\sqrt{15}}{8}$	(3,5)
4	1	(5,5)
2(n-4)+2(m-4)	1	(4,4)
4(n-3)+4(m-3)	$\frac{2\sqrt{20}}{9}$	(4,5)
6(n-4)+8(m-2)	$\frac{2\sqrt{30}}{11}$	(5,6)
6mn-18n-20m+60	1	(6,6)

Then, we have the following Theorem:
Theorem 1. The edge GA index of $P_n \times P_m$ is

$$GA_e(P_n \times P_m) = 6mn + \left(\frac{12\sqrt{30}}{11} + \frac{16\sqrt{5}}{9} - 16\right)n + \left(\frac{16\sqrt{30}}{11} + \frac{16\sqrt{5}}{9} - 18\right)m - \frac{96\sqrt{5}}{9} - \frac{80\sqrt{3}}{11} + 2\sqrt{15} + \frac{32\sqrt{3}}{7} + 52.$$

Proof. According to Table 1 and Figure 2, we obtain the result.

Now, we compute the edge GA index of $P_n \times C_m$, with C_m being a cycle of size m. In Figure 3, the graph of $P_6 \times C_4$ is indicated. According to the following figure, we have $\left|E(P_n \times C_m)\right| = (n-1)m + n.m = 2nm - m$.

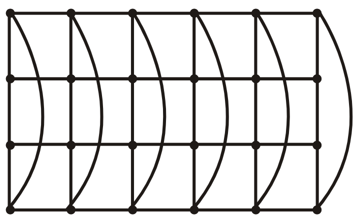

Figure 3. The graph of $P_6 \times C_4$

In Figure 4, the line graph of $P_6 \times C_4$ is shown. Accordingly, we have :

$$\left|E\left(L\left(P_n \times C_m\right)\right)\right| = \frac{1}{2}\left[(3+3-2)(2m)+(3+4-2)(2m)+(2mn-5m)(4+4-2)\right]$$
$$= 6mn - 6m$$

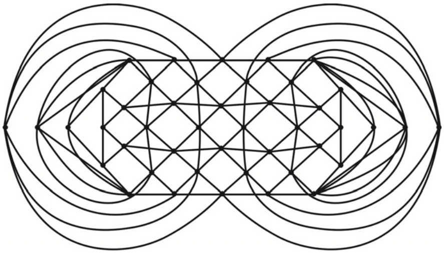

Figure 4. The graph of L($P_6 \times C_4$)

In Table 2, the type of edges, their numbers and amount of ξ_i are computed.

Table 2. The type of edges, their numbers and amount of ξ_i of $L(P_n \times C_m)$

No. of edges	ξ_i	Type of edges
2m	1	(4,4)
4m	$\frac{4\sqrt{5}}{10}$	(4,5)
6m	$\frac{2\sqrt{30}}{11}$	(5,6)
6mn-18m	1	(6,6)

Then, we have the following Theorem:

Theorem 2. The edge GA index of $P_n \times C_m$ is

$$GA_e\left(P_n \times C_m\right) = 6mn + \left(\frac{16\sqrt{5}}{9} + \frac{12\sqrt{30}}{11} - 16\right)m.$$

Proof. According to Table 2 and Figure 4, we obtain the result.

3. EDGE GA INDEX OF DENDRIMER NANOSTAR D_n

In Figure 5, the dendritic graph D_n is shown. According to this figure, we have $\left|E\left(D_n\right)\right| = 66.2^{n-1} - 45$.

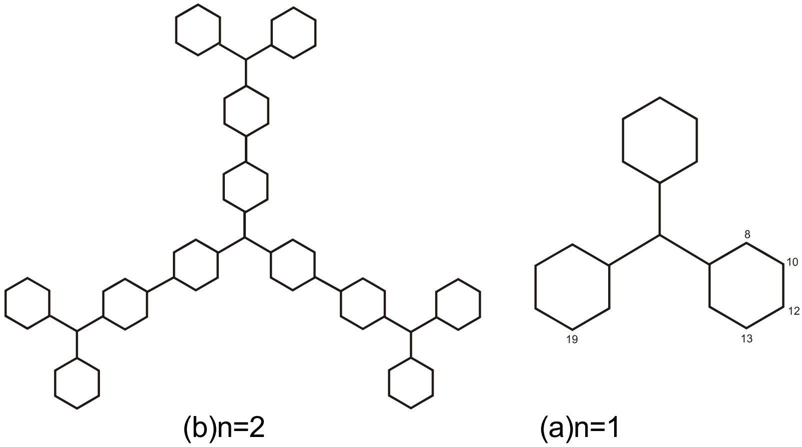

Figure 5. The dendrimer graph D_n

In Figure 6, the line graph of D_2 is shown. According to this figure, we have

$$\left|E(L(D_n))\right| = \frac{1}{2}\left[(2+2-2).\left(24\times 2^{n-1}-12\right)+(2+3-2)\left(3\times 2^{n-1}-24\right)+(3+3-2)\left(12\times 2^{n-1}-9\right)\right]$$
$$= 93\times 2^{n-1} - 66$$

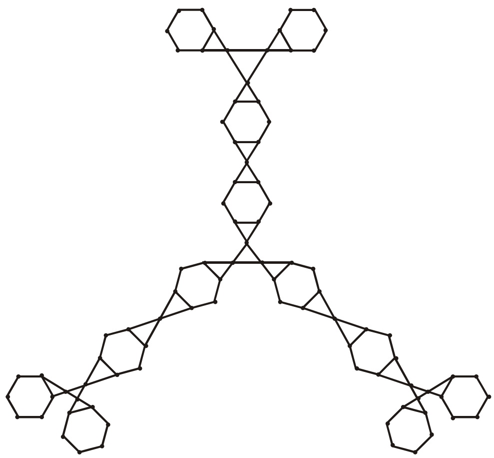

Figure 6. The line graph of D_2

In the Table 3, the type of edges, their numbers and amount of ξ_i for this graph are computed.

Table 3. The type of edges, their numbers and amount of ξ_i of $L(D_n)$

No. of edges	ξ_i	Types of edges
$9\times 2^{n-1}$	1	(2,2)
$15\times 2^{n-1}-12$	1	(3,3)
$30\times 2^{n-1}-24$	$\frac{4\sqrt{3}}{7}$	(3,4)
$30\times 2^{n-1}-24$	$\frac{2\sqrt{6}}{5}$	(2,3)
$9\times 2^{n-1}-6$	1	(4,4)

Then, we have the following

Theorem 3. The edge GA index of D_n is

$$GA_e(D_n)=\left(33+\frac{120\sqrt{3}}{7}+12\sqrt{6}\right)\times 2^{n-1}-\frac{96\sqrt{3}}{7}-\frac{48\sqrt{6}}{5}-18$$

Proof. According to Table 3 and Figure 6, we obtain the result.

CONCLUSIONS

By using the graph theory techniques, we computed the edge GA index for the product graphs $P_n\times P_m$ and $P_n\times C_m$, and a dendrimer nanostar D_n and we expressed their exact values.

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