# **ENTROPY PREDICTION OF BENZENE DERIVATIVES USING TOPOLOGICAL INDICES**

# **HOSSEIN HOSSEINI and FATEMEH SHAFIEIa \***

**ABSTRACT.** In this study, a QSPR study relating topological indices to the entropy of 69 benzene derivatives is reported. The entropy values were calculated at HF level of theory (6-31 G basis sets) by Gussian 98.

Multiple linear regression (MLR) provided good models with three to seven independent variables. The best model obtained is based on three descriptors: Randić, Wiener and Szeged topological indices

*Keywords***:** *Topological indices; benzene derivatives; QSPR; MLR method*.

# **INTRODUCTION**

One of the most important purposes in application of mathematical and statistical methods is to find a relationship between molecular structure and values of physical properties, chemical reactivity or biological activity. As a result, quantitive structure-property relationship (QSPR) and quantitative structure-activity (QSAR) studies have been promoted.

Topological indices (TIs), as molecular descriptors, are important tools in QSPR/QSAR studies [1-11]. A topological index is a graph invariant number calculated from a graph representing a molecule.

The physicochemical properties of compounds are important in many fields, including pharmaceutics, chemistry, biochemistry and environmental sciences. Property estimations can help to minimize time and cost in producing new chemical materials with desired properties.

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Predictive methods for estimating thermodynamic properties, such as enthalpies of formation, Gibbs free energy and entropies of acyclic and aromatic compounds, on the basis of fundamental concepts on molecular structure have been reported [12].

Artificial Neural Networks were also used in developing QSPR models for prediction of physicochemical properties [13-16].

Prediction of entropies and enthalpies of organic compounds by using group contribution methods was also published [17-19]. Prediction of standard absolute entropy  $(S_{298} K)$  of gaseous organic and inorganic compounds was reported in [20, 21].

In thermodynamics, entropy (usual symbol S) is a measurement of the randomness or disorder of a system.

In the present work, we developed QSPR models for entropy estimation of benzene derivatives by describing the chemical structure by the aid of topological indices. Benzene derivatives are used in a wide range of technological applications.

Experimental data of benzene derivatives are often scarce, and at this point, topological descriptors provide powerful tools for modeling and extrapolating experimental data.

 The main aim of this study is to illustrate the usefulness of topological indices in QSPR study of entropy (S) of benzene derivatives. As far as we are aware, this is the first QSPR study for prediction of benzene derivatives entropies using topological indices.

# **METHODS**

The entropy(S) of 69 benzene derivatives (benzene included) was computed at the Hartree-Fock (HF) level of theory, using the ab initio 6-31G basis sets. The benzene derivatives in this set have seven different substituents, each substituent being present in at least six compounds. These substituents are amino, bromo, chloro, hydroxyl, methyl, methoxyl and nitro groups. Studied benzene derivatives and their entropy are listed in Table 1. To obtain an appropriate QSPR model we used multiple linear regression (MLR) procedure, by SPSS software, version 16, and backward stepwise regression was used to construct the QSPR models.

For drawing the graphs of our results, we used the Microsoft Office Excel – 2003 program.



# **Table 1.** Benzene derivatives and their entropy.



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#### **TOPOLOGICAL INDICES**

A large number of topological indices Tis have been defined and used, majority of them being calculated from the various matrices corresponding to molecular graphs. The Adjacency matrix (A) and the Distance matrix (D) of the molecular graph have been most widely used in the definition of topological indices. The most used TIs are presented below.

Randić index (1975),  $1\chi(G)$ , was introduced as the connectivity index  $[22,23]$  and is defined as  $(1)$ :

$$
{}^{1}\chi = \sum_{\text{all edges}} (d(i)d(j))^{-0.5}
$$
 (1)

where  $d(i)$  and  $d(i)$  are the valencies of the vertices i and j defining the edge  $(i, j).$ 

Wiener index (1947), W(G), can be defined by (2):

$$
W(G) = \frac{1}{2} \sum_{i} \sum_{j} [D(i,j)]
$$
 (2)

where  $D(i,j)$  is the number of edges on the shortest path joining vertex i and vertex j (i.e., the topological distance) in the graph [24].

Hyper-Wiener index, WW(G), can be defined [25,26] as (3):

$$
WW(G) = \frac{1}{2} (\sum d(u,v) + \sum (d(u,v))^2)
$$
 (3)

where  $d(u,v)$  denotes the distance between the vertices u and  $v$  in the graph G and the summations run over all pairs of vertices of G.

Randić's original definition (1993) [27] of the hyper-Wiener index is applicable to trees only.

Wiener polarity index (1947),  $W_p(G)$ , of G is the number of unordered pairs of vertices  $(u,v)$  of G lying at distance 3 to each other. The Wiener polarity index [28,29] is defined as (4):

$$
WP(G) = |\{(u, v) | d(u, v) = 3, u, v \in V \}|.
$$
 (4)

Balaban index (1982), J(G) of G was introduced in 1982 [30,31] as one of the less degenerated indices. It calculates the average distance sum connectivity index, according to eq.(5):

$$
J = \frac{M}{\mu + 1} \sum_{\text{all edges}} (D_i D_j)^{0.5}
$$
 (5)

where M is the number of the edges in G; and  $D_i$  is the distance sum from the vertex i to all the other vertices in G (i.e., the sum of all entries in the  $i<sup>th</sup>$ row of the distance matrix D).

The cyclomatic number  $\mu = \mu$  (G) of a polycyclic graph G is equal to the minimum number of edges that must be removed from G to transform it to the related acyclic graph. For trees,  $μ=0$ ; for monocycles,  $μ=1$ .

Harary number, H(G), was introduced in 1993 [32].This index is defined by eq. (6)

$$
H = \frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{n} (1/D_{i,j})
$$
 (6)

Within this paper, a version of this index is calculated from the inverse of the squared elements of the distance matrix, according to eq.(7):

$$
H = \frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{n} (D_{ij})^{-2}
$$
 (7)

where  $D_{i,j}$  is the entry in the distance matrix D.

Szeged index, Sz(G), was introduced by Gutman [33,34] as (8):

$$
Sz_v(G) = \sum_e n_u(e|G) \cdot n_v(e|G) \tag{8}
$$

 $n_{\rm w}$  = $n_{\rm w}$  (e | G) is the number of vertices of G whose distance to the vertex v is smaller than the distance to the vertex u. Note that vertices equidistant to u and v are not counted.

All the used topological indices were calculated in hydrogen suppressed graphs. The descriptors were calculated with Chemicalize program [35]. Seven topological indices tested in the present study are listed in Table 2.

#### **STATISTICAL ANALYSIS**

Structure-Property models (MLR models) are generated using the multi linear regression procedure of SPSS, version 16. The entropy (S, J/mol K) is used as the dependent variable and  $1\chi$ , J, H, Sz, WW, W<sub>p</sub> and W indices are used as the independent variables. The models are assessed with r value (correlation coefficient), the  $r^2$  (coefficient of determination), the  $r^2$ - adjusted, the s value (root of the mean square of errors), the F value (Fischer statistic), the D value (Durbin-Watson) and the Sig (significant).

### **RESULTS AND DISCUSSION**

 Several linear QSPR models involving three to seven descriptors were established and the strongest multivariable correlations were identified by the backward method, with significant at the 0.05 level and regression analysis of the SPSS program. In the first of this study we drown scattering plots of S versus the seven topological indices  $(1<sub>\gamma</sub>, J, H, Sz, WW, Wp, W)$ . Some of these plots are given in Figs. 1 to 3, respectively.



**Figure 1.** Plot of the Randić index  $(1\gamma)$  versus entropy of 69 benzene derivatives.



**Figure 2.** Plot of the Szeged index (Sz) versus entropy of 69 benzene derivatives.



**Figure 3.** Plot of the Wiener (W) versus entropy of 69 benzene derivatives.

A	$^1\chi$	J	Н	W	<b>WW</b>	$W_{p}$	Sz
1	3.39	1.82	12.92	42	71	5	78
2	3.39	1.82	12.92	42	71	5	78
3	3.80	2.28	16.17	60	106	8	106
4	3.79	2.23	16.08	61	110	7	108
5	3.79	2.23	16.08	61	110	7	108
6	4.33	1.98	19.15	88	176	9	146
7	5.11	2.25	26.67	148	315	14	232
8	4.20	2.09	19.53	84	160	10	144
9	4.22	2.15	19.67	82	151	11	140
10	4.70	2.32	22.73	117	245	11	186
11	6.04	2.40	34.60	234	545	19	348
12	6.43	2.70	39.02	282	669	21	420

**Table 2.** Benzene derivatives and their topological indices, used in present study.



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 Distribution of the dependent variable against the independent variable for 69 chemicals was employed in developing quantitative structure- properties relationships. For obtaining appropriate QSPR models we used maximum R2 method and followed backward regression analysis. The predictive ability of the model is discussed on the basis of predictive correlation coefficient.

#### **QSPR MODELS FOR ENTROPY (S)**

 Initial regression analysis indicated that combination of seven topological indices plays a dominating role in modeling the entropy. Table 3 provides the regression parameters and quality of correlation of the proposed models for entropy of 69 benzene derivatives.

**Table 3.** Statistics of models calculated with SPSS software



The best linear model contains three topological descriptors, namely, Randić  $(1\gamma)$ , Wiener (W) and Szeged (Sz) indices.

The regression parameters of the best three descriptor correlation model are gathered in equation 9.

$$
S=70.258+59.966X+2.748Sz-4.163W
$$
\n
$$
r=0.927; r^2=0.859; r^2_{adj} = 0.859;
$$
\n
$$
s=16.430; D=2.033;
$$
\n
$$
F=132.299; mean square = 269.936
$$
\n(9)

This model produced a standard error of  $16.430$  J mol<sup>-1</sup> K<sup>-1</sup>. a correlation coefficient of 0.927, and the adjusted correlation coefficient (adjusted r-squared) was calculated as 0.859.

The result is therefore very satisfactory. Figure 4 shows the linear correlation between the observed and the predicted entropy values obtained using equation (9).



**Figure 4.** Comparison between the predicted and observed entropy by MLR method (cf. eq. 9)

# **The DURBIN-WATSON STATISTIC**

To verify and validate the regression models, we will focus on the Durbin-Watson (D) statistic, unstandardized predicted and residual values.

The Durbin-Watson statistic ranges in value from 0 to 4. A value near 2 indicates non-autocorrelation; a value toward 0 indicates positive autocorrelation; a value toward 4 indicates negative autocorrelation. Therefore the value of Durbin-Watson statistic is close to 2 if the errors are uncorrelated. In our model, the value of Durbin-Watson statistic for model 5 is close to 2 (See Eq. 9) hence the errors are uncorrelated.

# **RESIDUAL VALUES**

The residual values of entropy expressed by equation (9) are shown in Table 4. The residual values show a fairly random pattern (see Figure 5). This random pattern indicates that a linear model provides a decent fit to the data.



**Figure 5.** Plot of residuals against observed values of benzene derivatives entropy (S).

No.	Observed S(J/molK)	Predicted S(J/molK)	Residual	No.	Observed S(J/molK)	Predicted S(J/molK)	Residual
1	317.835	313.051	4.784	36	349.828	341.718	8.110
2	306.457	313.051	$-6.594$	37	445.053	436.089	8.964
3	334.588	339.651	$-5.063$	38	356.019	366.955	$-10.936$
4	364.048	340.385	23.663	39	262.968	286.154	$-23.186$
5	323.616	340.385	$-16.769$	40	367.639	373.379	$-5.740$
6	358.773	364.794	$-6.021$	41	388.619	412.003	$-23.384$
7	394.751	398.127	-3.376	42	441.441	446.622	-5.181
8	379.109	368.154	10.955	43	441.136	441.422	$-0.286$
9	368,007	366.687	1.320	44	425.335	429.461	-4.126
10	380.017	376.180	3.837	45	353.047	389.203	$-36.156$
11	418.029	414.661	3.368	46	415.421	390.944	24.477
12	434.807	436.089	$-1.282$	47	393.149	396.660	$-3.511$
13	435.840	435.955	$-0.115$	48	390.792	403.460	-12.668
14	452.059	442.756	9.303	49	384.928	398.127	-13.199
15	424.529	421.461	3.068	50	364.454	368.154	$-3.700$
16	361.482	380.180	-18.698	51	420.165	394.428	25.737
17	361.720	341.718	20.002	52	374.009	444.296	-70.287
18	390.637	394.428	$-3.791$	53	385,007	395.761	-10.754

**Table 4.** Entropy (S) data of benzene derivatives.





### **CONCLUSIONS**

 In this work, QSPR models for the prediction of entropy for a training set of benzene derivatives using MLR based on topological descriptors calculated from molecular structure have been developed. MLR model is proved to be a useful tool in the prediction of entropy. The aforementioned results and discussion lead us to conclude that combining the three descriptors  $(Sz, W, 1\gamma)$  could be used successfully for modeling and predicting entropy  $(S)$ of compounds. This model contains fewer topological descriptors, maximum of Fischer statistic value (F) and minimum root of the mean square of errors(s).

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